Title of a Course

Foundations of complex algebraic geometry

M. Verbitsky

Pre-requisites

One would need a good understanding of analysis on manifolds (vector bundles, differential forms, de Rham cohomology, sheaf cohomology, Riemannian manifolds) and differential geometry (connections, parallel transoport along the connections, principal bundles). We assume working knowledge of linear algebra (tensor spaces, Grassmann algebra, Hermitian structures), topology (de Rham cohomology, fundamental groups, coverings), one-dimensional complex analysis (Cauchy
formula) and representation theory (Lie groups and Lie algebras). Also we need some understanding of sheaf
theory: sheaves, resolutions, sheaf cohomology.

Course is elective.

Abstract

Complex algebraic geometry was developed by W. V. D. Hodge in 1930-es and was given its modern form by A. Weil and S.-S. Chern in 1940-ies and 1950-ies.
This is a discipline allowing one to get results of classical algebraic geometry using basic observations of analysis, differential geometry and topology instead of complicated algebraic computations.
As an additional bonus, the methods of Hodge theory can be used to study non-algebraic objects, such as general Kahler manifolds and more complicated geometric objects.

Learning Objectives

The objective is working knowledge of complex algebraic geometry and the ability to solve problems related to this subject.

Learning Outcomes

After learning the course, the student should be able to solve basic problems of complex algebraic geometry using Hodge theory and complex analysis.

Course Plan

0. Elliptic equations on manifolds and Hodge theory.
1. Kahler manifolds and algebraic manifolds 2. Hodge theory on Riemannian and Kahler manifolds 3. Poincare-Dolbeault-Grothendieck lemma and its applications 4. Line bundles, Chern connection and its curvature, dd^c-lemma and its applicattions 5. Kodaira-Nakano vanishing theorem 6. Kodaira embedding theorem

Reading List:
Required - none, the course is self-contained.
A. S. Mishchenko "Vector bundles and their applications"
should be useful for bakground.

Optional

Lectures on Kahler geometru, Andrei Moroianu
http://www.math.polytechnique.fr/~moroianu/tex/kg.pdf

Complex analytic and differential geometry, J.-P. Demailly http://www-fourier.ujf-grenoble.fr/~demailly/manuscripts/agbook.pdf

Lectures on Kahler manifolds, W. Ballmann http://people.mpim-bonn.mpg.de/hwbllmnn/notes.html

C. Voisin, ``Hodge theory".

D. Huybrechts, ``Complex Geometry - An Introduction"

A. Besse, ``Einstein manifolds".

Grading System

The grade is based on on solving the exercises during the course and the final exams (also problem-based).

Guidelines for Knowledge Assessment
Sample exam problems

Prove that $CP^4 \times CP^4 \times CP^4$ does not admit a Kahler structure with orientation which is opposite to the standard.

Prove that the group of integer upper triangular integer matrices 4 by 4 cannot be a fundamental group of a compact Kahler manifold.

Prove that a connected sum of several copies of CP^2 does not admit a Kahler structure.

Let M be a compact, non-projective Kahler manifold with $H^{2,0}(M)$ 1-dimensional, and p an involution acting freely on M. Prove that p acts trivially on $H^{2,0}(M)$.

Methods of Instruction
Lectures using beamer and blackboard, and exercise sheets (graded through personal examination by the lector and his support crew).
All course materials (slides of the lectures, exercise sheets, tests, test results and so on) will be available online on the page http://bogomolov-lab.ru/KURSY/CM-2018/