I. Course description

<u>RS "Riemann Surfaces "</u>
<u>prof. S. Lvovski</u>

1. Prerequisites include familiarity with basic courses in complex analysis and topology.
2. The course is electory.
3. This course will be devoted to the theory of Riemann surfaces, with focus on compact Riemann surfaces. The course will be based on the books "Introduction to algebraic and abelian functions" by S.Lang and "Lectures on Riemann surfaces" by R.Gunning.

II. The objectives and goals of the RS "Riemann surfaces" are as follows:
1. introducing the audience to the notion of Riemann surface, meromorphic functions and forms, and line bundle;
2. explaining the key notions of the theory of compact Riemann surfaces, such as divisors, linear systems, and canonic class.

III. After mastering the course, the student is expected to:
1. understand such fundamental notions as divisors and line systems;
2. know how to use Riemann-Roch theorem for obtaining information about linear sustems and meromorphic mappings attached to them.

IV. Plan:
1. Definition of Riemann surfaces, meromorphic functions, meromorphic forms.
2. Construction of the compact Riemann surface associated to an algebraic equation.
3. Genus and Riemann-Hurwitz formula.
4. Riemann surface associated to a nodal algebraic curve..
5. Construction of differentials via Poincare residue.
6. Riemann-Roch theorem.
7. Line bundles, divisors, and linear systems.
8. Mappings associated to linear systems, and their properties..

V. Reading lists:
1. Required
   1) S. Lang, *Introduction to algebraic and abelian functions*, Springer, 1982.
   2) R.Gunning, *Lectures on Riemann surfaces*, Princeton University Press, 1967.
2. Optional
   1) Ph. Griffiths, J. Harris, *Principles of algebraic geometry*, Wiley-Interscience, 1978.

VI. Current control grade equals the percentage of the number of solved problems (including bonus problems) to the total number of problems given throughout the semester. The exam consists of a written 4-hour test, containing 8 problems. For a 100% result it suffices to solve at least 6 of 8 problems.
The total grade for the course is computed via the following formula:

Max(150, E+H)/15

where E equals the mark for the written exam and H is the percentage of number of solved problems to the total number of the problems.

VII. Guidelines for Knowledge Assessment:

1. Sample problems which will be used for knowledge assessment:
   1) Compute the genus of the compact Riemann surface associated to the equation y^3=x^4+1..
   2) Find out whether the smooth projective model of a plane quantic with one node is hyperelliptic.
   3) Find l(P+Q+R), where P, Q, and R are three collinear points on a smooth plane quantic..
2. A number of questions which can be used for the examination:
   1) Describe the divisors corresponding to the points of order 2 on the Jacobian of a hyperelliptic curve of genus g..
   2) Compute explicitly a basis of the space of holomorphic differentials on the compact Riemann surface corresponding to the equation x^4+y^4=1..
   3) Does there exist a non-constant holomorphic mapping from a compact Riemann surface of genus 7 to a compact Riemann surface of genus 5?.
   4) Find poles and zeroes of the meromorphioc function x on the compact Riemann surface corresponding to the equation y^3=x(x-1)(x-2)..

VIII. The students are given home tasks, containing routine exercises, which assist in understanding theoretical material, and research problems, which require more effort to solve and motivate the students to study extra materials. The solutions are either submitted in written form to the lecturer and his assisstants or can be sent via email. Some of the more difficult topics are made into talks, which then are given by students.