I. Course description

RS “Symmetric functions ”

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1. Prerequisites include familiarity with the basic notions of algebra and analysis.
2. The course is electory.
3. This course will be devoted to the theory of symmetric functions. The main goal will be to study Schur functions, Hall polynomials and Macdonald polynomials. The course will be based on the book ”Symmetric functions and Hall polynomials” by I.G.Macdonald.

II. The objectives and goals of the RS “Symmetric functions” are as follows:
1. introducing the audience to the theory of symmetric functions;
2. explaining the key notions of said theory, such as Schur functions, complete symmetric functions, orbit sums, etc.

III. After mastering the course, the student is expected to:
1. understand such fundamental theorems as Cauchy identities;
2. know the transition tables from Hall-Littlewood functions to Schur functions.

IV. Plan:
1. Partitions.
2. Symmetric functions ring.
3. Schur functions.
4. Orthogonality.
5. Polynomial functors.
6. Hall algebra.
7. Hall polynomials.
8. Hall-Littlewood functions.
9. Green functions.
10. Macdonald polynomials.

V. Reading lists:
1. Required
   1) I.G.Macdonald, “Symmetric functions and Hall polynomials”, OUP (1995)
   2) W.Fulton, “Young tableaux with applications to representation theory and geometry”, CUP (1977)
2. Optional
   1) A.Postnikov https://math.mit.edu/~apost/courses/18.218/

VI. Current control grade equals the percentage of the number of solved problems (including bonus problems) to the total number of problems given throughout the semester. The exam consists of a written 4-hour test, containing 8 problems. For a 100% result it suffices to solve at least 6 of 8 problems.

The total grade for the course is computed via the following formula:

$$\mathrm{Max}(150, E+H)/15$$

where E equals the mark for the written exam and H is the percentage of number of solved problems to the total number of the problems.

VII. Guidelines for Knowledge Assessment:
1. Sample problems which will be used for knowledge assessment:
   1) Prove the hook formula for the number of standard tableaux of a given shape.
   2) Find the image under the Robinson-Schensted-Knuth correspondence of a given permutation in S_8.
   3) Find the Kostka polynomials for partitions of 5.
2. Sample questions that can be used for the exam:
   1) Prove the orthogonality relations of the Hall-Littlewood functions.

2) Prove the positivity of Kostka polynomials.
3) Prove the Littlewood-Richardson rule.
4) Prove the Cauchy identity.
5) Prove the Frobenius determinant formula.

VIII. The students are given home tasks, containing routine exercises, which assist in understanding theoretical material, and research problems, which require more effort to solve and motivate the students to study extra materials. The solutions are either submitted in written form to the lecturer and his assisstants or can be sent via email. Some of the more difficult topics are made into talks, which then are given by students.