Advances in the enumeration of regular graphs and not-so-regular graphs

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Abstract

Problems of counting graphs with given degree sequence arise naturally in network analysis where the only data available are collective properties of network nodes. Determining exact values for the counts of interest is usually a difficult task, Even for such a fundamental question as the number of regular graphs there is no computationally efficient exact formula available in the literature when the degree $d > 4$. Asymptotic enumeration of regular graphs has been the subject of extensive study since Read solved the case $d = 3$ in his 1958 PhD thesis. Until recently the best result for sublinear d in the number of vertices n was obtained by McKay and Wormald in 1991, who proved an asymptotic formula for $d = o(n^{1/2})$. Even earlier, McKay and Wormald used complex-analytic methods in 1990 to show that the same asymptotic formula holds for $d \geq n/\log n$. The gap between these two domains remained open for more than a quarter-century, until Liebenau and Wormald closed it with an elegant argument in 2017.

The aforementioned results considered not only regular graphs, but graphs having some some limited amount of irregularity in the vertex degrees. They are general enough to cover typical degree sequences that appear in the standard random graph models with high probability. In the case where the vertex degrees are linear, an even wider range was achieved by Barvinok and Hartigan in 2013.

In this talk we describe an extension of the complex-analytic approach that works for average degree at least n^c for any $c > 0$, and allows a large amount of variation of degrees as well as specification of a possibly large forbidden subgraph. It is based on a bound of Isaev on the truncation error for the cumulant generating function of a complex random variable. In the case of regular graphs, we show that the method leads to an asymptotic expansion and determine the first few coefficients.