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Abstract

This article contributes to research dealing with the optimal dividend policy problem of a firm whose goal is to maximize the expected total discounted dividend payments before bankruptcy. We consider a model of a firm whose cash surplus exhibits regime switching, but unlike the existing literature, we exclude diffusion from our model. We assume firm's cash surplus follows the telegraph process, which leads to the problem of singular stochastic control. Surprisingly, this problem turns out to be more complicated than the ones arising in the models involving diffusion. We solve this problem using the method of variational inequalities and show that the optimal dividend policy can be of three significantly different types depending on the parameters of the model.

JEL classification: C61, G35.

Keywords: optimal dividend policy, regime switching, telegraph process.

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1 Introduction and the formulation of the model

The optimal dividend problem was first discussed in [9]. The key idea of this work was that the goal of a firm is to maximize the expected present value of the flow of dividends before bankruptcy. In the simplest discrete framework it was shown that the optimal dividend strategy is of a threshold type — the surplus above a certain level should be paid as dividends. If the capital is less than this level, the company should not pay any dividends. The renewed interest in the optimal dividend problems was stimulated by the articles [25], [16] and [1], which addressed the optimal dividend problems in continuous environment with firm's cash reserves following the Brownian motion with drift. Numerous works which followed after them considered the optimal dividend problems for more complicated dynamics of cash reserves based on the Brownian motion, see for example [4], [32], [10], [24], [29]. Another big strand of optimal dividend policy literature is based on the compound Poisson process and presented in, for example, [11], [2] and [14].

Our point of interest is an optimal dividend problem in the model where firm's cash reserves follow the telegraph process. Introduced in [15] and [19], the telegraph process was extensively analyzed, for example, in [23], [13] and [3]. Notable generalizations of the telegraph process include the telegraph process with random velocities introduced in [31] and with an alternating renewal process defining switching times in [34]. A jump-telegraph process with jumps occurring at the moments of switching is introduced in [26] and analyzed in [7], [20] and [6]. The telegraph process and its generalizations are widely used in finance as alternatives to the Brownian motion since it is free from the limitations of the Brownian motion such as infinite propagation velocities and independence of returns on separated time intervals. For example, in [22] the telegraph process is used in the context of stochastic volatility. In [5] the very basic model of evolution of stock prices based on the telegraph process is presented. In [8] a geometric telegraph process is used to describe the dynamics of the price of risky assets, and the analogue of the Black-Sholes equation is derived. In [26], [28] and [27], the jump-telegraph process is used to develop an arbitrage-free model of financial market. In [21] it is used in the option pricing model.

The models of optimal dividend policy with regime switching, such as [30], [35], [18], [33], [17] among others are closest to ours, but they all involve diffusion, which is absent in our model. For example, our model may be considered as just a special case of [30] with two states of the world, first one with a positive drift coefficient, and the second one with a negative drift coefficient with diffusion coefficients set to zero. But, as we shall see, the absence of diffusion significantly changes the mathematical properties of the problem and leads to substantially different results.

The contribution of this paper is twofold. From the point of view of mathematics, we analyse the limiting case of well-known models with diffusion, which cannot be solved directly by standard techniques used in diffusion-based models. Moreover, the solution we derived has significantly different structure depending on the parameters of the model, which is highly unusual for this class of optimal control problems. From the point of view of economics, we introduce a model with regime switching as the only source of risk, unlike the existing models with diffusion as the only source of risk. Indeed, in every paper cited above drift coefficient(s) are assumed to be positive and thus, setting diffusion coefficient(s) to zero, we obtain a model of a firm which never goes bankrupt. With two states of the world, the model is obviously highly stylized, but it can be interpreted as a description of a firm working in market conditions switching from favourable to unfavourable ones and vice versa.

We now formulate our model. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space of trajectories of changes of the state of the world and a filtration $\mathcal{F}(t)$ represents the information up to time t. We assume that cash reserves of a firm $X(t)$ follow the equation

$$X(t) = x + \int_0^t \mu_{\pi(u)} du - L(t),$$

where x is the initial level of reserves, $\pi(u) \in \{0, 1\}$ is the state of the world, $\mu_0 < 0$ and $\mu_1 > 0$ are the drift coefficients and $L(t) \in \mathcal{F}(t)$ is the total amount of dividends paid up to the time t, which is non-negative and non-decreasing and also assumed to be left-continuous with right limits. The switching between the states of the world is defined by the frequencies $\Lambda_0 > 0$ and $\Lambda_1 > 0$: if the state of the world is 0 then the probability of switching to the state 1 during a short period of time Δt is $\Lambda_0 \Delta t$ and similarly for the state 1. The goal of a firm is to maximize the expected total amount of dividends paid before bankruptcy time τ, which occurs when a firm's level of reserves becomes negative for the first time:

$$J(s, x, L(\cdot)) = \mathbb{E}\left[\int_0^\tau \mathrm{e}^{-ct} dL(t)|_{\pi(0)=s, X(0)=x}\right] \to \max_{L(\cdot)}, \tag{1}$$

where $c > 0$. We denote the admissible dividend policy, which maximizes $J(s, x, L(\cdot))$, by L^* and then introduce

$$P(s,x) = J(s,x,L^*(\cdot)).$$

2 Analysis of the model

2.1 Variational inequalities

In this subsection we derive variational inequalities which the solution of the optimal dividend problem must satisfy, following the standard technique described, for example, in [1]. Consider a small interval $[0,\delta]$. Fix some $\varepsilon > 0$ and consider an admissible policy $L^{s,x}(\cdot)$ such that for any $x > 0$ and $s \in \{0,1\}$

$$J(s,x,L^{s,x}(\cdot)) \geq P(s,x) - \varepsilon.$$

Let $W(t) = x + \mu_{\pi(t)}t$. Consider the following policy:

$$L_\varepsilon(t) = \begin{cases} 0, t < \delta, \\ L^{\pi(\delta),W(\delta)}(t-\delta), t \geq \delta. \end{cases}$$

This policy means that we pay no dividends before δ and then switch to suboptimal policy. We get

$$P(s,x) \geq \mathrm{e}^{-c\delta}\mathbb{E}\left[P(\pi(\delta),W(\delta)) - \varepsilon\right]. \tag{2}$$

By the definition of the telegraph process

$$\mathbb{E}P(\pi(\delta),W(\delta)) = (1-\Lambda_s\delta)P(s,x+\mu_s\delta) + \Lambda_s\delta P(1-s,x) + o(\delta).$$

Using it and the arbitrariness of ε, (2) may be rewritten as

$$P(s,x) \geq (1-c\delta)[(1-\Lambda_s\delta)P(s,x+\mu_s\delta) + \Lambda_s\delta P(1-s,x)] + o(\delta).$$

Assuming $P(s,x)$ is continuously differentiable and using Taylor expansion, we get

$$P(s,x) \geq (1-c\delta)[(1-\Lambda_s\delta)(P(s,x) + \mu_s\delta\frac{\partial}{\partial x}P(s,x)) + \Lambda_s\delta P(1-s,x)] + o(\delta).$$

Simplifying this expression and tending δ to zero we get the first variational inequality:

$$\mu_s \frac{\partial}{\partial x} P(s,x) - (\Lambda_s + c)P(s,x) + \Lambda_s P(1-s,x) \le 0, s \in \{0,1\}. \tag{3}$$

To obtain another one, we fix x, $\delta > 0$. Consider the policy $L_\varepsilon(t) = \delta + L^{x-\delta}(t)$, which prescribes to pay δ instantaneously and then use the policy $L^{x-\delta}$. We get

$$P(s,x) \ge \delta + P(s,x-\delta) + \varepsilon.$$

Again using Taylor expansion and arbitrariness of ε we get the second variational inequality

$$\frac{\partial}{\partial x} P(s,x) \ge 1. \tag{4}$$

Now combining (3), (4) and the obvious boundary condition $P(0,0) = 0$, we get (see [12]) the following

Theorem 1. Let the function P be continuously differentiable. Then it satisfies the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} &\max\{\mu_s \frac{\partial}{\partial x} P(s,x) - (\Lambda_s + c)P(s,x) + \Lambda_s P(1-s,x), 1 - \frac{\partial}{\partial x} P(s,x)\} = 0, \\ &P(0,0) = 0. \end{aligned} \tag{5}$$

We now solve this equation and show that the dividend strategy associated with the solution is indeed optimal.

2.2 Solution of the Hamilton-Jacobi-Bellman equation

Standard arguments (see e.g. [30]) verify that $P(s,\cdot)$ is concave. This implies that there exists $m_s, s \in \{0,1\}$ such that $P(s,x) > 1$ for $x < m_s$ and $P(s,x) = 1$ for $x \ge m_s$, so the associated dividend policy is of barrier type. We now analyze several cases.

Case 1. $m_0 \ge m_1 > 0$. Denote $m = m_1, M = m_0$. In this case $\mathbb{R}_+$ is divided into three domains. In the lower domain $[0,m]$ it follows from (3) that function P follows equations

$$\mu_s \frac{\partial}{\partial x} P(s,x) - (\Lambda_s + c)P(s,x) + \Lambda_s P(1-s,x) = 0 \tag{6}$$

for $s \in \{0,1\}$ with the boundary condition $P(0,0) = 0$. This is a linear system of equations, so it can be solved by substitution, but we prefer to use Laplace transform in order to derive the solution in terms of m. Applying Laplace transformation to (6), we get

$$\xi\,\mathcal{L}\left(s,\xi\right)-P\left(s,0\right)=-\frac{\Lambda_s\mathcal{L}\left(1-s,\xi\right)}{\mu_s}+\frac{\mathcal{L}\left(s,\xi\right)\Lambda_s}{\mu_s}+\frac{\mathcal{L}\left(s,\xi\right)c}{\mu_s},$$

where $\mathcal{L}(s,\xi)$ is the Laplace transformation of $P(s,x)$. This leads to

$$\begin{aligned}\mathcal{L}\left(0,\xi\right)&=-\frac{P\left(1,0\right)\Lambda_0\mu_1}{\xi^2\mu_0\mu_1-c\xi\,\mu_0-c\xi\,\mu_1-\xi\,\Lambda_0\mu_1-\xi\,\Lambda_1\mu_0+c^2+c\Lambda_0+c\Lambda_1},\\ \mathcal{L}\left(1,\xi\right)&=-\frac{-P\left(1,0\right)\xi\,\mu_0\mu_1+P\left(1,0\right)c\mu_1+P\left(1,0\right)\Lambda_0\mu_1}{\xi^2\mu_0\mu_1-c\xi\,\mu_0-c\xi\,\mu_1-\xi\,\Lambda_0\mu_1-\xi\,\Lambda_1\mu_0+c^2+c\Lambda_0+c\Lambda_1}.\end{aligned}\tag{7}$$

Consider the denominator as the square polynomial on ξ. It can be easily shown that its roots are always real. Also note that by Vieta's theorem the roots of this polinomial always have different signs. Denoting the negative root by a and the positive root by b, we obtain

$$\begin{aligned}\mathcal{L}\left(0,\xi\right)&=-\frac{P\left(1,0\right)\Lambda_0}{\mu_0\left(\xi-a\right)\left(\xi-b\right)},\\ \mathcal{L}\left(1,\xi\right)&=-\frac{-P\left(1,0\right)\xi\,\mu_0\mu_1+P\left(1,0\right)c\mu_1+P\left(1,0\right)\Lambda_0\mu_1}{\mu_0\mu_1\left(\xi-a\right)\left(\xi-b\right)},\end{aligned}$$

Inverting Laplace transformations, we get

$$\begin{aligned}P\left(0,x\right)&=\frac{\Lambda_0P\left(1,0\right)\left(-\mathrm{e}^{ax}+\mathrm{e}^{bx}\right)}{\mu_0\left(a-b\right)},\\ P\left(1,x\right)&=\frac{P\left(1,0\right)\left(\mathrm{e}^{ax}\left(a\mu_0-c-\Lambda_0\right)+\left(-b\mu_0+c+\Lambda_0\right)\mathrm{e}^{bx}\right)}{\mu_0\left(a-b\right)}\end{aligned}\tag{8}$$

for $x\in[0,m]$. The threshold level m is defined by the condition $\frac{\partial}{\partial x}P(1,x)=1$, which can be rewritten as

$$\frac{P\left(1,0\right)\left(a\mathrm{e}^{am}\left(a\mu_0-c-\Lambda_0\right)+\left(-b\mu_0+c+\Lambda_0\right)b\mathrm{e}^{bm}\right)}{\left(a-b\right)\mu_0}=1.\tag{9}$$

Substituting (9) into (8), we get

$$\begin{aligned}P\left(0,x\right)&=\frac{\Lambda_0\left(\mathrm{e}^{bx}-\mathrm{e}^{ax}\right)}{a\mathrm{e}^{am}\left(a\mu_0-c-\Lambda_0\right)+\left(-b\mu_0+c+\Lambda_0\right)b\mathrm{e}^{bm}},x\in[0,m],\\ P\left(1,x\right)&=\frac{\mathrm{e}^{ax}\left(a\mu_0-c-\Lambda_0\right)+\left(-b\mu_0+c+\Lambda_0\right)\mathrm{e}^{bx}}{a\mathrm{e}^{am}\left(a\mu_0-c-\Lambda_0\right)+\left(-b\mu_0+c+\Lambda_0\right)b\mathrm{e}^{bm}},x\in[0,m].\end{aligned}\tag{10}$$

We now consider the middle domain $[m,M]$. In this domain the function $P(1,\cdot)$ follows the equation $\frac{\partial}{\partial x}P(1,x)=1$. Integrating it and using obvious boundary condition, we obtain

$$P(1,x) = x - m + P(1,m), x \in [m,M]. \tag{11}$$

Function $P(0,\cdot)$ follows (6). Substituting (11) into (6) and solving the differential equation, we get

$$P\left(0,x\right) = \frac{\left((x - m + P(1,m))(c+\Lambda_0) + \mu_0\right)\Lambda_0}{\left(\Lambda_0+c\right)^2} + C\,\mathrm{e}^{\frac{(\Lambda_0+c)x}{\mu_0}} \tag{12}$$

for $x \in [m,M]$. Since $P(0,\cdot)$ is assumed to be continuously differentiable, we impose two conditions: $P(0,m_-) = P(0,m_+)$ and $\frac{\partial}{\partial x}P(0,m_-) = \frac{\partial}{\partial x}P(0,m_+)$, but they turn out to be identical:

$$C = -\frac{\Lambda_0\left(P\left(1,0\right)\left(\Lambda_0+c\right)\left(a\mathrm{e}^{am} - b\mathrm{e}^{bm}\right) + \mu_0\left(a-b\right)\right)}{\left(a-b\right)\left(\Lambda_0+c\right)^2}\mathrm{e}^{-\frac{(\Lambda_0+c)m}{\mu_0}}. \tag{13}$$

Substituting (13) into (12) and also substituting $P(1,m)$ found from (10), we get

$$\begin{gathered} P\left(0,x\right) = \frac{\Lambda_0}{\Lambda_0+c}\left(\frac{A\mathrm{e}^{am} - B\mathrm{e}^{bm}}{aA\mathrm{e}^{am} - bB\mathrm{e}^{bm}} - cm + cx - \Lambda_0 m + \Lambda_0 x + \mu_0\right) + \\ \frac{\Lambda_0\mu_0^2\left(a^2\mathrm{e}^{am} - b^2\mathrm{e}^{bm}\right)}{(\Lambda_0+c)^2\left(aA\mathrm{e}^{am} + bB\mathrm{e}^{bm}\right) + \left(aA\mathrm{e}^{am} + bB\mathrm{e}^{bm}\right)}\mathrm{e}^{\frac{\Lambda_0(x-m)+c(x-m)}{\mu_0}} \end{gathered} \tag{14}$$

for $x \in [m,M]$, where $B = -b\mu_0 + c + \Lambda_0$, $A = -a\mu_0 + c + \Lambda_0$.

Unlike similar problems involving diffusion, our problem cannot be solved using only equalities which follow from the HJB equation, so we have to impose several conditions which follow from variational inequalities.

Condition 1.1. $\frac{\partial}{\partial x}P(0,x) \geq 1$ for $x \in [0,m]$. To guarantee the fulfillment of this inequality, we can demand $\frac{\partial}{\partial x}P(0,x)|_{x=m} \geq 1$ and $\frac{\partial^2}{\partial x^2}P(0,x) \leq 0$ for $x \in [0,m]$.

First inequality may be rewritten as

$$\frac{\Lambda_0\left(b\mathrm{e}^{bm} - a\mathrm{e}^{am}\right)}{a\left(a\mu_0 - c - \Lambda_0\right)\mathrm{e}^{am} + \left(-b\mu_0 + c + \Lambda_0\right)b\mathrm{e}^{bm}} \geq 1. \tag{15}$$

After some simplification, the denominator ofthe fraction on the left side may be represented as $\mu_0\left(c+\Lambda_1\right)\left(a\mathrm{e}^{am} - b\mathrm{e}^{bm}\right) + c\left(\Lambda_0 + c + \Lambda_1\right)\left(\mathrm{e}^{bm} - \mathrm{e}^{am}\right)$ and hence is always positive. Inequality (15) is thus equivalent to

$$\begin{gathered} \left(-bc\mu_0 - b\Lambda_0\mu_1 - b\Lambda_1\mu_0 + c^2 + c\Lambda_0 + c\Lambda_1\right)\mathrm{e}^{-am+bm} + \\ ac\mu_0 + a\Lambda_0\mu_1 + a\Lambda_1\mu_0 - c^2 - c\Lambda_0 - c\Lambda_1 \leq 0, \end{gathered}$$

which in turn may be rewritten as

$$\mathrm{e}^{-am+bm} \leq \frac{a^2\mu_0 - ac}{b^2\mu_0 - bc}. \tag{16}$$

Now consider the inequality $\frac{\partial^2}{\partial x^2}P(0,x) \leq 0$ for $x \in [0,m]$. It can be rewritten as

$$\frac{\Lambda_0\left(-a^2\mathrm{e}^{ax} + b^2\mathrm{e}^{bx}\right)}{a\mathrm{e}^{am}\left(a\mu_0 - c - \Lambda_0\right) + \left(-b\mu_0 + c + \Lambda_0\right)b\mathrm{e}^{bm}} \leq 0.$$

Since the denominator is always positive, this is equivalent to

$$\mathrm{e}^{(b-a)x} \leq \frac{a^2}{b^2}.$$

But this condition holds automatically if (16) holds. Indeed,

$$\mathrm{e}^{(b-a)x} \leq \mathrm{e}^{(b-a)m} \leq \frac{a^2\mu_0 - ac}{b^2\mu_0 - bc} \leq \frac{a^2}{b^2}.$$

In order to prove the last inequality, note that

$$\frac{a^2}{b^2} - \frac{a^2\mu_0 - ac}{b^2\mu_0 - bc} = \frac{ac\left(b-a\right)}{b^2\left(b\mu_0 - c\right)} > 0.$$

Condition 1.2. $\frac{\partial}{\partial x}P(1,x) \geq 1$ for $x \in [0,m]$. This inequality may be rewritten as

$$\frac{a\mathrm{e}^{ax}\left(a\mu_0 - c - \Lambda_0\right) + \left(-b\mu_0 + c + \Lambda_0\right)b\mathrm{e}^{bx}}{a\mathrm{e}^{am}\left(a\mu_0 - c - \Lambda_0\right) + \left(-b\mu_0 + c + \Lambda_0\right)b\mathrm{e}^{bm}} \geq 1, x \in [0,m].$$

which can be guaranteed if the function in the numerator has a negative derivative for any $x \in [0,m]$. This condition after some simplification may be rewritten as

$$\mathrm{e}^{(b-a)x} \leq \frac{a^2\left(a\mu_0 - c - \Lambda_0\right)}{b^2\left(b\mu_0 - c - \Lambda_0\right)}, x \in [0,m]. \tag{17}$$

Condition 1.3. Inequality (3) in the middle domain for $s=1$ has the following form:

$$\mu_1 - \left(c + \Lambda_1\right)\left(x + P\left(1,m\right) - m\right) + \Lambda_1 P\left(0,x\right) \leq 0.$$

It holds as equality for $x = m$. To guarantee that it holds for $x \in [m, m']$ for some $m' > m$ we impose the following condition

$$\frac{\partial}{\partial x}P\left(0,x\right)|_{x=m} \leq \frac{c + \Lambda_1}{\Lambda_1}.$$

Substituting (14) into this inequality, we get

$$0 \leq \frac{b^2(b\mu_0 - c - \Lambda_0)\mathrm{e}^{bm} - a^2(a\mu_0 - c - \Lambda_0)\mathrm{e}^{am}}{\Lambda_1\left(a^2\mathrm{e}^{am}\mu_0 - ac\mathrm{e}^{am} - a\mathrm{e}^{am}\Lambda_0 - b^2\mathrm{e}^{bm}\mu_0 + b\mathrm{e}^{bm}c + b\mathrm{e}^{bm}\Lambda_0\right)}.$$

The denominator can be rewritten as

$$\frac{\Lambda_1}{\mu_1}\left(\mu_0(c+\Lambda_1)\left(a\mathrm{e}^{am} - b\mathrm{e}^{bm}\right) + c\left(c+\Lambda_0+\Lambda_1\right)\left(\mathrm{e}^{bm} - \mathrm{e}^{am}\right)\right)$$

and hence is always positive. The condition that the numerator is non-negative can be rewritten as

$$\mathrm{e}^{(b-a)m} \geq \frac{a^2\left(a\mu_0 - c - \Lambda_0\right)}{b^2\left(b\mu_0 - c - \Lambda_0\right)}. \tag{18}$$

Hence, the only possibility of both (17) and (18) to be satisfied is

$$\mathrm{e}^{(b-a)m} = \frac{a^2\left(a\mu_0 - c - \Lambda_0\right)}{b^2\left(b\mu_0 - c - \Lambda_0\right)}. \tag{19}$$

This equation defines the optimal lower threshold level m. In order to prove this, we need, first, to show that it satisfies Condition 1.1. To do so, note that

$$\frac{a^2\left(a\mu_0 - c - \Lambda_0\right)}{b^2\left(b\mu_0 - c - \Lambda_0\right)} - \frac{a^2\mu_0 - ac}{b^2\mu_0 - bc} = \frac{ac\mu_0\Lambda_0\left(a-b\right)}{\mu_1 b^2\left(b\mu_0 - c - \Lambda_0\right)\left(b\mu_0 - c\right)} < 0.$$

Hence, m indeed satisfies Condition 1.1. Second, we should check that m is positive, which is obviously equivalent to the fact that the expression on the right side of (19) is greater than 1. This condition, after some simplification, leads to

$$\frac{\left(c^2\mu_0 + 2\,c\Lambda_1\mu_0 + \Lambda_0\Lambda_1\mu_1 + {\Lambda_1}^2\mu_0\right)\sqrt{\Omega}}{\mu_0\mu_1^3} < 0,$$

where Ω is the discriminant of the demoninator in (7). Hence we arrive at the condition of positivity of m:

$$(c+\Lambda_1)^2\mu_0 + \Lambda_0\Lambda_1\mu_1 > 0. \tag{20}$$

And third, we should check that variational inequalities (3) hold for the upper domain $[M, +\infty)$, but this is rather trivial.

In order to find upper threshold M, we apply condition $\frac{\partial P}{\partial x}(0,x)|_{x=M} = 1$. After some rather lengthy computations this leads to

$$M = m + \frac{\mu_0}{\Lambda_0 + c} \ln\left(\frac{\Lambda_1}{\Lambda_0 + \Lambda_1 + c}\right). \tag{21}$$

Note that $M > m$ and hence (20) also guarantees that M is positive. Hence, if (20) holds, there exists the unique solution for the optimal dividend problem in Case 1, and this solution is defined by threshold levels (19) and (21). In the region lower than m, the firm should not pay any dividends in both states of the world. In the region between m and M, the firm should immediately pay an excess above m as dividends if the state of the world is 1 and do not pay anything if the state of world is 0. This may look a bit counter-intuitive — in the state 0 firm loses money and then the state of the world switches to 1, it pays the excess above m. Why not pay before switching? The answer is that in the case of paying before switching, the firm suffers losses after that, because the state of the world is 0, and in the case of paying at switching, it finds itself on the threshold in the state of the world 1 and makes more money. Finally, if the firm has more money than M, in both states of the world it should immediately pay the excess above M as dividends (and then also the excess above m if the state of the world is 1). Also note that if (20) doesn't hold, there are no solutions in this case.

Case 2. $m_1 > m_0 > 0$. Denote $m = m_0, M = m_1$. Again, we should consider three domains. In the lower domain $[0, m]$, as in the previous case, function P follows (6), which leads to (8), but the boundary condition is now $\frac{\partial}{\partial x}P(0, x) = 1$, which can be rewritten as

$$\frac{\Lambda_0 P\left(1, 0\right)\left(-a\mathrm{e}^{am} + b\mathrm{e}^{bm}\right)}{\mu_0\left(a - b\right)} = 1.$$

Substituting it to (8), we get

$$P\left(0, x\right) = \frac{\mathrm{e}^{bx} - \mathrm{e}^{ax}}{-a\mathrm{e}^{am} + b\mathrm{e}^{bm}}, P\left(1, x\right) = \frac{\mathrm{e}^{ax}\left(a\mu_0 - c - \Lambda_0\right) + \left(-b\mu_0 + c + \Lambda_0\right)\mathrm{e}^{bx}}{\Lambda_0\left(-a\mathrm{e}^{am} + b\mathrm{e}^{bm}\right)}.$$

For the middle domain, similarly to the Case 1,

$$P(0, x) = x - m + P(0, m), x \in [m, M]. \tag{22}$$

Function $P(1, \cdot)$ follows (6). Substituting (22) into (6) and solving the differential equation, we get

$$
\begin{aligned}
P\left(1,x\right) = C\mathrm{e}^{\frac{\left(c+\Lambda_1\right)x}{\mu_1}} + \\
\frac{\left(cP\left(0,m\right)+\Lambda_1 P\left(0,m\right)-cm+cx-\Lambda_1 m+\Lambda_1 x+\mu_1\right)\Lambda_1}{\left(c+\Lambda_1\right)^2}.
\end{aligned}
\tag{23}
$$

Again, we impose two conditions: $P(1,m_-) = P(1,m_+)$ and $\frac{\partial}{\partial x}P(1,m_-) = \frac{\partial}{\partial x}P(1,m_+)$ but they turn out to be identical:

$$
C = -\frac{\Lambda_1\mu_1}{\left(c+\Lambda_1\right)^2}\mathrm{e}^{-\frac{\left(c+\Lambda_1\right)m}{\mu_1}} + \frac{P\left(1,0\right)\left(\tilde{B}\mathrm{e}^{bm}-\tilde{A}\mathrm{e}^{am}\right)}{\left(a-b\right)\mu_0\left(c+\Lambda_1\right)}\mathrm{e}^{-\frac{\left(c+\Lambda_1\right)m}{\mu_1}},
$$

where $\tilde{B} = -bc\mu_0 - b\Lambda_1\mu_0 + c^2 + c\Lambda_0 + c\Lambda_1$ and $\tilde{A} = -ac\mu_0 - a\Lambda_1\mu_0 + c^2 + c\Lambda_0 + c\Lambda_1$. Substituting it to (23), we get

$$
\begin{aligned}
P\left(1,x\right) = \frac{\Lambda_1}{\left(c+\Lambda_1\right)^2}\left(\frac{\left(c+\Lambda_1\right)\left(\mathrm{e}^{bm}-\mathrm{e}^{am}\right)}{-a\mathrm{e}^{am}+b\mathrm{e}^{bm}} - cm + cx - \Lambda_1 m + \Lambda_1 x + \mu_1\right) + \\
\frac{\mu_1\left(-b\tilde{B}\mathrm{e}^{bm}+a\tilde{A}\mathrm{e}^{am}\right)}{\left(c+\Lambda_1\right)^2\Lambda_0\left(a\mathrm{e}^{am}-b\mathrm{e}^{bm}\right)}\mathrm{e}^{\frac{\left(c+\Lambda_1\right)\left(x-m\right)}{\mu_1}}.
\end{aligned}
\tag{24}
$$

Similarly to Case 1, we consider several conditions which follow from variational inequalities, but in this Case they turn out to be inconsistent.

Condition 2.1. $P'(1,x) \geq 1$ for $x = m$. After some simplifications this condition can be rewritten as

$$
\mathrm{e}^{-am+bm} \geq \frac{a\left(a\mu_0-c\right)}{b\left(b\mu_0-c\right)}.
\tag{25}
$$

Condition 2.2. Concavity of $P(1,x)$ in the middle domain. Taking the second derivative of (24) and demanding it to be less or equal zero, after some simplifications we get

$$
\mathrm{e}^{-am+bm} \leq \frac{a^2\left(a\mu_0-c-\Lambda_0\right)}{b^2\left(b\mu_0-c-\Lambda_0\right)}.
\tag{26}
$$

Now we note that (25) and (26) cannot both hold, because

$$
\frac{a\left(a\mu_0-c\right)}{b\left(b\mu_0-c\right)} - \frac{a^2\left(a\mu_0-c-\Lambda_0\right)}{b^2\left(b\mu_0-c-\Lambda_0\right)} = \frac{(a-b)c\mu_0\Lambda_0}{b^2\left(b\mu_0-c-\Lambda_0\right)\left(b\mu_0-c\right)\mu_1} > 0.
$$

Hence, there are no solutions to the HJB equation in this case for any values of parameters.

Case 3. $m_1 = 0, m_0 > 0$. Denote $m = m_0$ and consider domain $x \in [0, m]$. Note that in this case we can derive a lower boundary condition for $P(1, x)$. Indeed,

$$P(1,0) = \mathbb{E}\int_0^{\tau} \mu_1 \mathrm{e}^{-ct} dt = \frac{\mu_1}{c}\left(1 - \mathbb{E}\mathrm{e}^{-c\tau}\right) =$$
$$\frac{\mu_1}{c}\left(1 - \int_0^{\infty} \mathrm{e}^{-cu}\Lambda_1 \mathrm{e}^{-\Lambda_1 u} du\right) = \frac{\mu_1}{\Lambda_1 + c}.$$

So in this domain

$$P\left(1, x\right) = x + \frac{\mu_1}{\Lambda_1 + c}.$$

Function $P(0, x)$ can be found from (7), which, together with the boundary condition $P(0, 0) = 0$, gives

$$P\left(0,x\right) = -\frac{\Lambda_0\left(c\mu_0 + c\mu_1 + \Lambda_0\mu_1 + \Lambda_1\mu_0\right)}{\left(\Lambda_1 + c\right)\left(\Lambda_0 + c\right)^2}\mathrm{e}^{\frac{(\Lambda_0+c)x}{\mu_0}} +$$
$$\frac{\Lambda_0 c^2 x + {\Lambda_0}^2 cx + \Lambda_0 cx\Lambda_1 + {\Lambda_0}^2 x\Lambda_1 + \Lambda_0 c\mu_0 + \Lambda_0 c\mu_1 + {\Lambda_0}^2\mu_1 + \Lambda_0\Lambda_1\mu_0}{\left(\Lambda_1 + c\right)\left(\Lambda_0 + c\right)^2}.$$

Equalizing the derivative of this function to 1, we find the condition defining the threshold m:

$$\mathrm{e}^{\frac{m(\Lambda_0+c)}{\mu_0}} = -\frac{\mu_0 c\left(\Lambda_1 + c\right)}{\Lambda_0\left(c\mu_0 + c\mu_1 + \Lambda_0\mu_1 + \Lambda_1\mu_0\right)}. \tag{27}$$

Since the exponent is negative, the expression on the right side must be from 0 to 1. First condition leads to

$$c\mu_0 + c\mu_1 + \Lambda_0\mu_1 + \Lambda_1\mu_0 > 0, \tag{28}$$

and the second one, taken together with (28), leads to

$$c\mu_0 + \Lambda_0\mu_1 + \Lambda_1\mu_0 > 0, \tag{29}$$

but (29) obviously implies (28).

Condition 3.1. $\frac{\partial}{\partial x}P(0, x) \geq 1$ for $x \in [0, m]$. This condition can be rewritten as

$$1 \leq \frac{\Lambda_0\left(\Lambda_1 + c\right)\mu_0 - \Lambda_0\left(c\mu_0 + c\mu_1 + \Lambda_0\mu_1 + \Lambda_1\mu_0\right)\mathrm{e}^{\frac{(\Lambda_0+c)x}{\mu_0}}}{\left(\Lambda_0 + c\right)\left(\Lambda_1 + c\right)\mu_0}.$$

Since the denominator is negative, this is equivalent to

$$-c\mu_0 - \Lambda_1\mu_0 - \Lambda_0\left(c\mu_0 + c\mu_1 + \Lambda_0\mu_1 + \Lambda_1\mu_0\right)\mathrm{e}^{\frac{(\Lambda_0+c)x}{\mu_0}} \le 0.$$

If it holds for $x = 0$, it also holds for greater values of x. And for $x = 0$ it can be rewritten as

$$\left(\Lambda_0 + c\right)\left(c\mu_0 + \Lambda_0\mu_1 + \Lambda_1\mu_0\right) \ge 0,$$

which is true due to (29).

Condition 3.2. Inequality (3) for the lower domain and $s = 0$. It has the following form:

$$P\left(0, x\right) \le \frac{cx + \Lambda_1 x}{\Lambda_1}.$$

Similarly to the previous cases and using an easily varifiable fact that $P(0, x)$ is concave, we can guarantee that this ineqality holds if the following condition holds:

$$\frac{\partial}{\partial x} P\left(0, x\right)|_{x=0} \le \frac{c + \Lambda_1}{\Lambda_1}.$$

This condition can be rewritten as

$$\frac{c^2\mu_0 + 2\,c\Lambda_1\mu_0 + \Lambda_0\Lambda_1\mu_1 + {\Lambda_1}^2\mu_0}{\Lambda_1\mu_0\left(\Lambda_1 + c\right)} \ge 0,$$

which implies

$$c^2\mu_0 + 2\,c\Lambda_1\mu_0 + \Lambda_0\Lambda_1\mu_1 + {\Lambda_1}^2\mu_0 \le 0. \tag{30}$$

Hence, if the parameters of the model are such that (29) and (30) hold, the solution of the HJB equation is defined by the positive threshold for the state of the world 0, thich is defined by (27), and the zero threshold for the state of the world 1.

Case 4. $m_1 > 0, m_0 = 0$. Denote $m = m_1$. In this case $P(0, x) = x$ and $P(1, x)$ can be found from (6) for $s = 1$. Solving it, we get

$$P\left(1, x\right) = \frac{\Lambda_1\left(cx + \Lambda_1 x + \mu_1\right)}{c^2 + 2\,c\Lambda_1 + {\Lambda_1}^2} + C\,\mathrm{e}^{\frac{x(\Lambda_1+c)}{\mu_1}}.$$

Equalizing the derivative of this function to 1 and finding C, we get

$$P\left(1,x\right)=\frac{1}{c^2+2\,c\Lambda_1+{\Lambda_1}^2}\left(c\mu_1\mathrm{e}^{-\frac{(\Lambda_1+c)(-x+m)}{\mu_1}}+cx\Lambda_1+x{\Lambda_1}^2+\Lambda_1\mu_1\right).$$

But this function is convex, which is impossible for the solution of our HJB equation. So we conclude that there are no solutions in this case.

Case 5. $m_0=m_1=0$.

In this case $P(0,x)=x, P(1,x)=x+\frac{\mu_1}{\Lambda_1+c}$. Inequality (3) now has the form $cx\geq 0$ for $s=1$, which is always true, and

$$x\geq\frac{c\mu_0+\Lambda_0\mu_1+\Lambda_1\mu_0}{c\left(\Lambda_1+c\right)}.$$

For this inequality to hold for every positive x, we require

$$c\mu_0+\Lambda_0\mu_1+\Lambda_1\mu_0<0. \tag{31}$$

Hence, if (31) holds, then the solution of the HJB equation is defined by a strategy, which prescribes to immediately pay all cash reserves as dividends in both states of the world. As we can see, the set of all possible values of the parameters of the model is divided into three subsets with significantly different optimal dividend policies.

Now we unite all the above results in the following

Theorem 2. 1. Let the parameters of the model be such that (20) is satisfied. Then thresholds m and M, defined by (19) and (21) respectively, are positive and the solution of the HJB equation (5) is given by

$$P(0,x)=\begin{cases}\frac{\Lambda_0\left(\mathrm{e}^{bx}-\mathrm{e}^{ax}\right)}{-a\mathrm{e}^{am}A+Bb\mathrm{e}^{bm}}, x\leq m,\\ \frac{\Lambda_0}{\Lambda_0+c}\left(\frac{A\mathrm{e}^{am}-B\mathrm{e}^{bm}}{aA\mathrm{e}^{am}-bB\mathrm{e}^{bm}}-cm+cx-\Lambda_0 m+\Lambda_0 x+\mu_0\right)+\\ +\frac{\Lambda_0\mu_0^2\left(a^2\mathrm{e}^{am}-b^2\mathrm{e}^{bm}\right)}{(\Lambda_0+c)^2\left(aA\mathrm{e}^{am}+bB\mathrm{e}^{bm}\right)+\left(aA\mathrm{e}^{am}+bB\mathrm{e}^{bm}\right)}\mathrm{e}^{\frac{(\Lambda_0+c)(x-m)}{\mu_0}}, x\in(m,M],\\ x-M+\frac{\Lambda_0}{\Lambda_0+c}\left(\frac{A\mathrm{e}^{am}-B\mathrm{e}^{bm}}{aA\mathrm{e}^{am}-bB\mathrm{e}^{bm}}-cm+cM-\Lambda_0 m+\Lambda_0 M+\mu_0\right)+\\ +\frac{\Lambda_0\mu_0^2\left(a^2\mathrm{e}^{am}-b^2\mathrm{e}^{bm}\right)}{(\Lambda_0+c)^2\left(aA\mathrm{e}^{am}+bB\mathrm{e}^{bm}\right)+\left(aA\mathrm{e}^{am}+bB\mathrm{e}^{bm}\right)}\mathrm{e}^{\frac{(\Lambda_0+c)(M-m)}{\mu_0}}, x>M,\end{cases}$$

$$P(1,x)=\begin{cases}\frac{-A\mathrm{e}^{ax}+B\mathrm{e}^{bx}}{-aA\mathrm{e}^{am}+bB\mathrm{e}^{bm}}, x\in[0,m],\\ x-m+\frac{-A\mathrm{e}^{am}+B\mathrm{e}^{bm}}{-aA\mathrm{e}^{am}+bB\mathrm{e}^{bm}}, x>m,\end{cases}$$

where $B=-b\mu_0+c+\Lambda_0$, $A=-a\mu_0+c+\Lambda_0$ and the associated dividend policy is

$$L^*(t) = (x-m)^+ \mathbb{1}_{\{s=1\}} + (x-M)^+ \mathbb{1}_{\{s=0\}} + \int_0^\tau \mu_1 \mathbb{1}_{\{\pi(u)=1, X(u)=m\}} du.$$

2. Let the parameters of the model be such that (29) and (30) hold. Then threshold m, defined by (27), is positive and the solution of HJB equation (5) is given by

$$P(0,x) = \begin{cases} \frac{c^2 x\Lambda_0 + cx\Lambda_0{}^2 + cx\Lambda_0\Lambda_1 + x\Lambda_0{}^2\Lambda_1 + c\Lambda_0\mu_0 + c\Lambda_0\mu_1 + \Lambda_0{}^2\mu_1 + \Lambda_0\Lambda_1\mu_0}{(\Lambda_1+c)(\Lambda_0+c)^2} - \\ \frac{\Lambda_0(c\mu_0 + c\mu_1 + \Lambda_0\mu_1 + \Lambda_1\mu_0)}{(\Lambda_1+c)(\Lambda_0+c)^2} \mathrm{e}^{\frac{(\Lambda_0+c)x}{\mu_0}}, x \in [0,m], \\ x - m + \frac{\Lambda_0 mc^2 + cm\Lambda_0{}^2 + cm\Lambda_0\Lambda_1 + m\Lambda_0{}^2\Lambda_1 + c\Lambda_0\mu_0 + c\Lambda_0\mu_1 + \Lambda_0{}^2\mu_1 + \Lambda_0\Lambda_1\mu_0}{(\Lambda_1+c)(\Lambda_0+c)^2} - \\ \frac{\Lambda_0(c\mu_0 + c\mu_1 + \Lambda_0\mu_1 + \Lambda_1\mu_0)}{(\Lambda_1+c)(\Lambda_0+c)^2} \mathrm{e}^{\frac{m(\Lambda_0+c)}{\mu_0}}, x > m, \end{cases}$$

$$P(1,x) = x + \frac{\mu_1}{\Lambda_1 + c}, x \geq 0$$

and the associate dividend policy is

$$L^*(t) = x\mathbb{1}_{\{s=1\}} + (x-m)^+ \mathbb{1}_{\{s=0\}} + \int_0^\tau \mu_1 \mathbb{1}_{\{\pi(u)=1, X(u)=0\}} du.$$

3. Let the parameters of the model be such that (31) holds. Then the solution of the HJB equation (5) is given by

$$P(0,x) = x, P(1,x) = x + \frac{\mu_1}{\Lambda_1 + c}, x \in [0, +\infty)$$

and the associate dividend policy is given by

$$L^*(t) = x + \int_0^\tau \mu_1 \mathbb{1}_{\{\pi(u)=1, X(u)=0\}} du.$$

We now prove that the solution of HJB equation described in the Theorem 2 is the solution of the optimal control problem (1).

Theorem 3. Let G be a solution of the HJB equation (5). Then G is the objective function of the problem (1) and the associated dividend policy is optimal.

Proof. Let $L(\cdot)$ be some admissible control. Denote the set of its discontinuities by Φ, and let $L^d(t) = \sum_{u\in\Phi, s\leq t}(L(u_+) - L(u))$ and $L^c(t) = L(t) - L^d(t)$ be the discontinuous and continuous parts of L. Also denote $f(t,s,x) = \mathrm{e}^{-ct}G(s,x)$. We have

$$
\begin{aligned}
&\mathbb{E}_{s,x} d_t f(t,\pi(t),X(t)) = \\
&\left[\mu_{\pi(t)}\frac{\partial}{\partial x} f(t,\pi(t),X(t)) + \frac{\partial}{\partial t} f(t,\pi(t),X(t))\right] dt - \frac{\partial}{\partial x} f(t,\pi(t),X(t)) dL^c(t) + \\
&[f(t,\pi(t),X(t_+)) - f(t,\pi(t),X(t))] I_{t\in\Phi} + \\
&[-\Lambda_{\pi(t)} f(t,\pi(t),X(t)) + \Lambda_{\pi(t)} f(t,1-\pi(t),X(t))] dt = \\
&\mathrm{e}^{-ct}[(\mu_{\pi(t)} - c - \Lambda_{\pi(t)}) G(\pi(t),X(t)) + \Lambda_{\pi(t)} G(1-\pi(t),X(t))] dt - \\
&- \mathrm{e}^{-ct}\frac{\partial}{\partial x} G(\pi(t),X(t)) dL^c(t) + \mathrm{e}^{-ct}[G(\pi(t),X(t_+)) - G(\pi(t),X(t))] I_{t\in\Phi}.
\end{aligned}
$$

Integrating it, we have

$$
\begin{aligned}
&\mathrm{e}^{-c(t\wedge\tau)} G(\pi(t\wedge\tau), X(t\wedge\tau)) = G(s,x) + \int_0^{t\wedge\tau} \mathrm{e}^{-cy} R(y) dy - \\
&\int_0^{t\wedge\tau} \mathrm{e}^{-cy}\frac{\partial}{\partial x} G(\pi(y),X(y)) dL^c(y) + \\
&+ \sum_{0\le y\le t\wedge\tau, y\in\Phi} \mathrm{e}^{-cy}\left(G(\pi(y),X(y_+)) - G(\pi(y),X(y))\right),
\end{aligned}
$$

where $R(y) = \mu_{\pi(y)} G(\pi(y),X(y)) - cG(\pi(y),X(y)) - \Lambda_{\pi(y)} G(\pi(y),X(y)) + \Lambda_{\pi(y)} G(1-\pi(y),X(y))$. Taking conditional expectations, we have

$$
\begin{aligned}
&\mathbb{E}_{s,x}\left[\mathrm{e}^{-c(t\wedge\tau)} G(\pi(t\wedge\tau), X(t\wedge\tau))\right] \\
&= G(s,x) + \mathbb{E}_{s,x}\left[\int_0^{t\wedge\tau} \mathrm{e}^{-cy} R(y) dy\right] - \mathbb{E}_{s,x}\left[\int_0^{t\wedge\tau} \mathrm{e}^{-cy}\frac{\partial}{\partial x} G(\pi(t),X(t)) dL^c(t)\right] + \\
&\mathbb{E}_{s,x}\left[\sum_{0\le y\le t\wedge\tau, y\in\Phi} \mathrm{e}^{-cy}\left(G(\pi(t),X(t_+)) - G(\pi(t),X(t))\right)\right].
\end{aligned}
$$

Inequality (3) guarantees that the integrand of the first integral is non-positive, and (4) guarantees, that for any $t \in \Phi$ $G(\pi(t),X(t_+)) - G(\pi(t),X(t)) \le X(t_+) - X(t) = L(t_+) - L(t)$. It also follows from (4) that $\mathrm{e}^{-cy}\frac{\partial}{\partial x} G(\pi(t),X(t)) \ge \mathrm{e}^{-cy}$. Hence,

$$
\mathbb{E}_{s,x}\left[\mathrm{e}^{-c(t\wedge\tau)} G(\pi(t\wedge\tau), X(t\wedge\tau))\right] \le G(s,x) - \mathbb{E}_{s,x}\left[\int_0^{t\wedge\tau} \mathrm{e}^{-cy} dL(y)\right].
$$

Note that for the dividend policy L^G, which is associated with the solution of the HJB equation, this inequality becomes equality. Indeed, $R(y) = 0$ a.e. for this strategy, hence the first integral is zero. Continuous flow of dividends corresponds to $X(t) = m$ and $s = 1$, and we know that $\frac{\partial}{\partial x} G(1, X(t))|_{X(t)=m} = 1$. Finally, in the points of discontinuity $G(\pi(t), X(t_+)) - G(\pi(t), X(t)) = X(t_+) - X(t)$. Hence, taking the limit $t \to +\infty$, we get

$$G(s,x) \geq \mathbb{E}_{s,x} \left[\int_0^\tau \mathrm{e}^{-cy} dL(y) \right]$$

for the arbitrary dividend policy with the equality for the strategy associated with the solution of the HJB equation (5).

3 Conclusion

It is shown that the optimal dividend policy in the model of firm's cash surplus following the telegraph process is of a threshold type, which is in line with results for models with diffusion and the regime switching. However, we had to perform rather tricky analysis of variational inequalities to find these thresholds. Further research may involve generalization of our results for the arbitrary number of regimes and the analysis of links between our model and the models with diffusion.

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