Topological conjugacy of Ω-stable flows on surfaces

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The most traditional method to study dynamics of flows with finite number is dividing the carrying manifold into regions with the same behaviour trajectories in, that are the *cells*. Such way rises up to the classical work of A. Andronov and L. Pontryagin [1] of 1937, where they introduced the idea of *roughness* and proved the roughness criterion for flows given by a vector field on the plane. After that E. Leontovich and A. Mayer [2,3] got the topological classification for more general flow class by means of the *Leontovich-Mayer scheme* also by dividing the plane into cells. In 1971 in [4] M. Peixoto generalised the Leontovich-Mayer scheme for structural stable flows on arbitrary surface. In 1976 D. Neumann and T. O'Brien [5] considered the so-called regular flows on arbitrary surfaces including as a particular case the class considered by Peixoto and introduced the full topological invariant the orbital complex. In 1998 A. Oshemkov and V. Sharko [6] introduced the new invariant for Morse flows, called the three-colour graph and got the full topological classification for Morse-Smale flows on surfaces.

The non-wandering set of Morse-Smale flows, that is the same, on surfaces consists of finite number of critical points and closed trajectories, all of them are hyperbolic; also such flows does not have trajectories connecting saddle points. Violation of the last condition leas to the Ω-stable flows on surfaces which are not structural stable.

In 1978 J. Palis [7] considered a neighbourhood of two saddle points connected by a separatrix and constructed a conjugating homeomorphism for two such neighbourhoods with the same moduli of stability and proved the topological conjugacy criterion by means of moguli of stability. In our work we consider the class of Ω-stable flows with finite number of moduli of stability and get conjugacy classification by means of directed graph equipped with colour graph and moduli of stability.

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