Mathematics for Economics and Finance

2018, fall semester

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Class teachers: K. Bukin, B. Demeshev, A.Zasorin

Course description

The objective of the course is to equip the students with some of the theoretical foundations of the modern mathematics and what is more important with the analytical methods of solving problems posed by the micro and macro analysis.

Prerequisites

Undergraduate level mathematics that includes: Calculus (both single and multi-dimensional), Linear Algebra, Probability theory and Mathematical Statistics, Ordinary Differential Equations.

Teaching objectives

The course has been designed to convey to the students how mathematics can be used in the modern micro and macro economic analysis.

Emphasis is placed on the model-building techniques, methods of solution and economic interpretations.

Topics studied comprise the following: differential equations, dynamic programming, optimal control theory and stochastic processes.

Upon completion an individual will:

- have the ability to solve differential equations and systems of differential equations,
- have acquired the knowledge of the methods of the optimal control theory and dynamic programming and its applicability for solving problems in economics,
- have developed skills in working with the Brownian and Wiener stochastic processes and have the idea how Ito’s integral is applied.

Teaching methods

Lectures and problem-solving sessions (classwork), intensive self-study, working on home assignments, on-line course “Mathematics for Economists” offered by Coursera

Assessment

There will be two exams 80 minutes each and the third exam will last for 180 minutes. The very first of them is scheduled on late September. It follows the completion of the intensive math refreshment.

The rest of examinations are tentatively scheduled on mid-November and late December, respectively.

Students absent from Examinations receive unsatisfactory mark unless the absence is excused documentary. The written documentation presented for a missed Examination must be presented to the Student’s Office no later than three days following return to class.

The test will be given back with a score on it. Students can also check with a lecturer about their scores anytime during the semester.

Homework Assignments

Homework will be assigned every second week (six home assignments in total). Homework will be collected, marked and returned to the students.

Attendance Policy

Attendance is strongly encouraged. Attendance on examinations is mandatory. The absence on an examination will be excused if the reasons, such as illness or a similar force majeure are documented in writing.

Grade determination

Final marks will be determined by weighting work on Examinations as follows:

- Examination that follows math refreshment — 20% of final mark
- Midterm Examination — 25% of final mark
- Final Examination — 40% of final mark
- Homework — 15% of final mark

Main reading

- E. Roy Weintraub, Mathematics for economists, 6th edition, Cambridge University Press, 1993
- Carl P. Simon, Lawrence Blume, Mathematics for economists, W.W. Norton company Inc., 1994 or latest edition

- Newbold Paul, Carlson William L., Thorne Betty, Statistics for business and economics, 5th edition, Pearson Education, Inc., Upper Saddle River, NJ, 2003

- Kamien, M.I., Schwartz, N.L. Dynamic optimization: the calculus of variations and optimal control in economics and management, 2nd ed. New York: North-Holland, 1991.

- Rangarajan K. Sundaram. A first course in optimization theory, Cambridge University Press, 1996, 11th printing in 2007

- Brzezniak, Zastawniak, (2006), Basic Stochastic Processes, Spinger

- Jeffrey S. Rosenthal. (2007), A first look at rigorous probability, World Scientific Publishing

- Cvitannic and Zapatero, Introduction to the Economics and Mathematics of Financial Markets, MIT Press, 2004 — Chapters 3 and 16.

- Coursera course “Mathematics for Economists” can be found at https://www.coursera.org/learn/mathematics-for-economists/home/

Additional reading

- Shreve S., (2004), Stochastic Calculus for Finance I, II,. Springer-Verlag

- Fima C. Klebaner, (2006), Introduction to stochastic calculus with applications, Imperial College Press

- Munk, Claus, Financial Asset Pricing Theory, mimeo, available at http://www.sam. — Chapters 2 and Appendix.

- Neftci, Salih N., An Introduction to the Mathematics of Financial Derivatives, 2nd edition, San Diego Academic Press, 2000, Chapters 3,5,6,9,10.

Since for many students enrolled to the program such math subjects as calculus and linear algebra were the topics studied on the undergraduate level a good while ago, and taking into account that the cohort of students was drawn from the various institutions thus them having different mathematical background it was suggested to teach students a refresher course whose purpose was to warm up their math aptitude, show their weaknesses (if any). That refresher would tune enrolled students up for a high level mathematics.

Course outline

Refresher course

1. Multidimensional calculus, basics of optimization (taught by Prof. M. Levin)

1. Euclidean spaces: basic notions and definitions
   - vector
   - distance
   - open and closed sets
   - neighborhood of a point, limiting points, boundary points
   - bounded sets, compact sets
2. Functions and their generalizations
   - vector-functions
   - limit of a function
   - continuity
3. Multidimensional calculus
   - Total differential and more
     - partial derivative
     - relation between partial and total derivatives
     - implicit function theorem
     - higher-order derivatives and differentials
     - Young’s theorem, Hessian
4. Optimization in many variables. Unconstrained optimization at first followed by constrained optimization
   - concept of extrema
   - Necessary conditions of extrema
   - Bordered Hessians

2. Linear Algebra (taught by Associate Prof. K. Bukin)

1. Basic notions, definitions and propositions
   - operations on matrices
   - matrix multiplication
   - inverse matrix, its properties

- rank of a matrix
- linear spaces and subspaces, their properties
- Gauss method of solving linear systems
- systems of linear equations
- eigenvalues and eigenvectors (definition, relation to the matrix rank, case of a symmetric matrix)
- quadratic forms: sign-definiteness of for
- kernel and image of a linear operator
- Eucleadean spaces
- orthogonalization by Gramm-Schmidt's method
- quadratic form sign-definiteness criterion (by eigenvalues)
- Sylvester's criterion
- reduction of a matrix to a diagonal form
- Projectors as operators in vector spaces
- Normal Jordan form of a matrix and its applications
- Complex vector spaces
- Notion of a pseudoinverse matrix

3. Convex analysis and Kuhn-Tucker theorem (taught by Prof. M. Levin)

1. Outset of a non-linear programming problem

- Convexity
  - convexity of a set
  - convex and concave functions, their properties
- separability theorem, separating hyperplane
- saddle point
- necessary and sufficient conditions of concave functions
- strict convexity of a function

2. Unconstrained optimization in many variables

- Taylor's expansion in a single variable case
- Jacobi's matrix
- Jacobi's matrix for a composite function

- sufficient conditions for extrema

3. Constrained optimization
    - Lagrange's classic problem
    - optimization of a quadratic form on a unit sphere
    - directional derivative
4. Constrained optimization with inequality constraints
    - problem setting, function requirements
    - necessary and sufficient conditions for extrema
    - problem modification for the nonnegative variables
    - differential characteristics of Kuhn-Tucker conditions
    - the meaning of Lagrange multiplier

4. Theory of probability and statistics (taught by A. Zasorin)

1. random variable, sample space
2. cumulative distribution function and its density
3. uniform distribution
4. normal distribution, reduction of the Gaussian variable to variable
5. standard expectation $E[X]$, $E[f(X)]$
6. initial and central moments
7. joint distributions of the random variables
8. conditional distributions
9. iterated expectations formula
10. limiting densities
11. covariation and correlation
12. standard normal vector and its properties
13. marginal and conditional normal distributions
14. quadratic forms in a standard normal vector
15. X^2 distribution and its properties
16. Student's distribution and its properties
17. Fisher's distribution and its properties
18. point estimation of parameters
19. unbiasness and efficiency of estimators
20. elements of large-sample distribution theory
21. convergence in probability and convergence in distribution

22. asymptotic distribution
23. interval estimation
24. hypothesis testing
25. errors of the first and second type
26. critical region of the test, decision rule

Main course

5. Differential Equations (taught by K. Bukin)

1. First-Order and higher-order Ordinary Differential Equations
   (a) Types of equations
   (b) Linear, first-order differential equations with constant and variable coefficients
   (c) Higher-order differential equations< linear with the constant coefficients

2. Systems of Linear Ordinary Differential Equations
   (a) Phase Diagrams.
   (b) Analytical Solutions of Linear, Homogeneous Systems.
   (c) The Relation between the Graphical and Analytical Solutions.
   (d) Stability.
   (e) Analytical Solutions of Linear, Nonhomogeneous Systems.
   (f) Linearization of Nonlinear Systems

6. Dynamic Optimization in Continuous Time (taught by K. Bukin and M. Levin)

1. The Typical Problem.
2. Derivation of the First-Order Conditions.
3. Transversality Conditions.
4. The Behavior of the Hamiltonian over Time.
5. Sufficient Conditions.
6. Infinite Horizons. Example: The Neoclassical Growth Model.
7. Transversality Conditions in Infinite-Horizon Problems.
8. Summary of the Procedure to Find the First-Order Conditions.

9. Present-Value and Current-Value Hamiltonians. Multiple Variables.

7. Finite-Horizon Dynamic Programming (taught by K. Bukin and M. Levin)

1. Examples of the Dynamic Programming Problems
2. Histories, Strategies and the Value function
3. Existence of an Optimal Strategy
4. The Bellman Equation
5. Stationary Strategies
6. Example: the Optimal Growth Strategy

7. **Uncertainty, information, and stochastic calculus (taught by B. Demeshev)**

1. Probability essentials
   - Sigma-algebras
   - Basic properties of sigma-algebras
   - Borel sigma algebras
   - Measurable functions
   - Probability as measure
   - Expectation
2. Conditional expectation
   - Definition
   - Calculation of conditional expectation
   - Properties of conditional expectations
3. Discrete-time stochastic processes
   - Filtration
   - Adapted process
   - Predictable process
   - Markov process
   - Markov chains
   - Examples
4. Martingales

- Definitions of martingales
- Properties
- Examples
- Random walk

5. Continuous-time stochastic process

    - Arithmetic and geometric Brownian motion
    - martingales in continuous time
    - multi-dimensional processes

6. Ito calculus

    - Stochastic integral
    - Ito’s lemma
    - SDE

7. Change of measure

    - Girsanov theorem
    - solution of Black-Scholes model via Girsanov theorem

8. Introduction to Matlab

    - Basic matrix operations
    - functions
    - scripts
    - graphs
    - flow control

Distribution of hours

#	Topic	Total hours	Contact hours: Lectures	Contact hours: Seminars	Self study
	Refresher course				
1.	Multidimensional calculus, optimization	30	8	6	16
2.	Linear Algebra	28	8	4	16
3.	Convex analysis and Kuhn- Tucker theorem	22	6	4	12
4.	Theory of probability and statistics	28	8	8	12
	Main course				
5.	Differential Equations.	30	6	8	16
6.	Dynamic Optimization in Continuous Time	34	10	8	16
7.	Finite-Horizon Dynamic Programming	32	8	8	16
8.	Uncertainty, information, and stochastic Calculus	34	10	8	16
	Total:	238	64	54	120