Syllabus for Abstract Mathematics

A course for the undergraduate students on specialization Mathematics and Economics

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Course description

Abstract Mathematics is a two-semester course for the third year students studying at ICEF specializing in Mathematics and Economics. It is based on the Introduction to Abstract Mathematics course of the University of London (UoL) with further expansions into selected topics from algebra, real analysis and topology.

The emphasis of the course is on the theory rather than on the method. One central topic of the course is formal mathematical reasoning. We will practise in formulating precise mathematical statements and proving them rigorously. These skills are essential for the current specialization, although they often remain in shadows in other math courses where the focus is on solving problems through calculation.

The second central topic of the course is the abstract mathematical structures from algebra (groups, fields, etc.), analysis, topology (topological spaces, manifolds) and functional analysis. We will develop some of these theories roughly to the extent of standard 1^{st} and 2^{nd}-year courses of the mathematical departments.

Upon completion of this course the students will have to take the University of London (UoL) exam at the end of the fourth semester of their studies at ICEF.

Learning objectives

Having taken this course you should

- have a knowledge of main mathematical concepts in discrete mathematics, algebra, real analysis and topology;
- be able to use formal notations correctly and in connection with precise statements in English;
- be able to formulate statements of the key theorems and present their proofs;
- be able to find and formulate proofs of problems based on those theorems.

Teaching Methods

The course program consists of:

- lectures,
- classes,
- regular self-study based on class problem sets, regular homework assignment problem sets and extra problem sets.

Assessment and grade determination

There are the following forms of control:

- written home assignments posted and turned in every week;
- written exam at the end of module 2.
- Mid-term tests in module 1 and 3.
- University of London exam by the end of module 4 on Abstract Mathematics MA103.

Home assignments in module 1 + midterm test at the end of module 1 = 10%
Home assignments in module 2 = 5%
Exam at the end of module 2 = 20%
Home assignments in module 3 + midterm test at the end of module 3 = 15%
Home assignments in module 4 = 5%
UOL external exam = 45%

Reading

Recommended by UoL Abstract Mathematics syllabus:

1. Biggs, Norman L. *Discrete mathematic,* 2nd edition. (Oxford University Press, 2002).
2. Eccles, P.J., *An Introduction to Mathematical Reasoning: numbers, sets and functions*. (Cambridge University Press, 1997).
3. Binmore, K.G., *Mathematical Analysis: A Straightforward Approach.* (Cambridge University Press, 1982).
4. Bryant, V., *Yet Another Introduction to Analysis* (Cambridge University Press, 1982).
5. Halmosh, P.R., *Finite Dimentional Vector Spaces*. (Springer, 1996).
6. Rudin, W., *Principles of Mathematical Analysis*, 3rd edition, (McGraw-Hill, 1976).

Additional reading:

1. Vinberg E.B., *A Course in Algebra.* (Factorial Press, 2001).
2. Kolmogorov A.N., Fomin S.V., *Elements of the Theory of Functions and Functional Analysis* (any Russian or English edition).
3. Warner, Frank W., *Foundations of Differentiable Manifolds and Lie Groups*. (Springer, 1983).
4. Vassiliev V.A., *Introduction to Topology*. (MCCME 2014).

Course outline

1. Group Theory
   a. Definition and basic properties of groups
   b. Subgroups, quotient groups and homomorphisms, cosets & Lagrange’s theorem.
   c. Homomorphisms, group isomorphism theorems
   d. Automorphisms and semi-direct products
   e. Group actions

f. Sylow theorems
g. Classification of Abelian groups*
h. Introduction to the representation theory of finite groups.*

2. Rings and Fields
    a. Divisibility of integers
    b. Congruence and modular arithmetic.
    c. Definition and basic properties of rings
    d. Ideals, ring homomorphisms
    e. The Chinese Remainder Theorem
    f. Definition and basic properties of fields
    g. Complex numbers
    h. Finite fields
    i. Rings of polynomials
    j. Real numbers as a complete ordered field
    k. Field Theory and Galois theory*.
3. Analysis and elements of Topology
    a. The Archimedian property of real numbers
    b. Equivalent definitions of completeness in an archimedian field
    c. Metric spaces
    d. Norm and normed spaces
    e. Topological spaces and operations with them
    f. Homotopy groups, homotopy equivalence.*
    g. Coverings, cell spaces (CW-complexes).*
    h. Differentiable manifolds, diffeomorphisms*
    i. Tangent vectors and differentials, differential forms*.
4. Modules and Vector spaces
5. Elements of mathematical logic
6. Lie Groups*

Note: topics marked with * will be only touched at the introductory level in the additional homework assignments. They are not included in any of the internal exams.