**State University Higher School of Economics**

International College of Economics and Finance

# Calculus Syllabus

## 1. Teachers

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## 2. Course description

This course is designed to introduce students to the basic ideas and methods of mathematical analysis and their application to mathematical modeling. This course helps lay the foundation for the entire block of quantitative disciplines that are studied at ICEF, and it also provides some of the analytical tools that are required by advanced courses in economics. This course provides students with experience in the methods and applications of calculus to a wide range of theoretical and practical situations. The course is taught in English.

### Prerequisites

Students are expected to have a firm grounding in elementary mathematics, algebra, trigonometry, and geometry on the coordinate plane, the properties and graphs of elementary functions at the level of Russian high school.

### Course objectives:

By the end of this course the students should:

- be able to analyze functions of one variable represented in a variety of ways: graphical, numerical, analytical, or verbal, and understand the relationships between these various representations;
- understand the concepts of the limit of an infinite sequence, the limit of a function at a point and the limit of a function as its argument approaches infinity;
- understand the meaning of the derivative in terms of a rate of change and local linear approximation, and be able to use derivatives to solve a variety of problems;
- understand the concept of infinite series and the idea of approximating a function by its Taylor series, use Taylor polynomials to approximate function values;
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and be able to use integrals to solve a variety of problems;
- understand the relationship between the derivative and the definite integral, as expressed by the Fundamental Theorem of Calculus;
- understand how the concept of definite integral extends to double and triple integrals, and be able to compute multiple integrals by reducing them to iterated integrals;
- be able to communicate mathematics in well-written sentences and to explain the solutions to problems;
- be able to model a written description of a simple economic or physical situation with a function, differential equation, or an integral;
- be able to use mathematical analysis to solve problems, interpret results, and verify conclusions;

- be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.

## 3. Topical Outline

| **No.** | **Topic titles** | **TOTAL (hours)** | **Contact hours** | | **Self-study** |
|---|---|---|---|---|---|
| | | | **Lectures** | **Classes** | |
| 1 | Introduction. Functions of one variable. | 8 | 2 | 2 | 4 |
| 2 | Sequences. Limit of a sequence. | 8 | 2 | 2 | 4 |
| 3 | Limit of a function. | 8 | 2 | 2 | 4 |
| 4 | Continuity. | 8 | 2 | 2 | 4 |
| 5 | The derivative. | 16 | 4 | 4 | 8 |
| 6 | Applications of the derivative. | 24 | 6 | 6 | 12 |
| 7 | Infinite series, power series, Taylor series. | 24 | 6 | 6 | 12 |
| 8 | Anti-derivatives and the indefinite integral. | 24 | 6 | 6 | 12 |
| 9 | The definite integral. | 24 | 6 | 6 | 12 |
| 10 | Applications of the definite integral. | 32 | 8 | 8 | 16 |
| 11 | Improper integrals. | 8 | 2 | 2 | 14 |
| 12 | Double integrals, triple integrals. | 16 | 4 | 4 | 8 |
| 13 | Introductions to differential equations. | 24 | 6 | 6 | 12 |
| 14 | Course review. | 16 | 4 | 4 | 8 |
| | Total: | **240** | **60** | **60** | **120** |

## 4. Assessment

The academic year 2018 – 2019 at ICEF is divided into 4 modules as follows:

- Fall Semester
  - Module I: from 3rd September 2018 till 21th October 2018;
  - Module II: from 29th October 2018 till 23rd December 2018;
- Spring Semester
  - Module III: from 14th January 2019 till 24th March 2019;
  - Module IV: from 1st April 2019 till 30th April 2019.

At the end of each module the students sit a written exam. Each exam is marked out of 100 points. The weighted sum of the exam marks combined with bonus points for homework and in-class quizzes gives the student’s mark out of 100 pints for each of the semesters. The weighted sum of the semester marks gives the student’s final mark out of 100 points for the whole course.

The exam approximate dates and weights are combined in the table below. The exact dates of the exams are decided during the year and announced in form of the cross-subject exam schedule.

| Semester | Module | Time period | Control name | Semester weight | Final weight |
|---|---|---|---|---|---|
| Fall | I | 22 – 28 October 2018 | Fall mock | 0.425 | 17% |
| Fall | II | 24 – 30 December 2018 | Winter exam | 0.575 | 23% |
| | | 14 – 30 January 2019 | Fall retake | | |
| Spring | III | 25 – 31 March 2019 | Spring mock | 0.317 | 19% |
| Spring | IV | May 2019 | Final exam | 0.683 | 41% |
| | | September 2019 | Final retake | | |

The exact formulas to be used for the semester and final year marks out of 100 points:

Final = 0.4 * Fall + 0.6 * Spring + Calculus++Bonus

Fall = 0.425 * Fall_Mock + 0.575 * Winter + a * Fall_Hwk + b * Fall_Class

Spring = 0.317 * Spring_Mock + 0.683 * Final + c * Spring_Hwk + d * Spring_Class

Here Fall_Hwk, Fall_Class, Spring_Hwk and Spring_Class are marks out of 100 points for the homework and in-class activities in the corresponding semesters. It is expected that the scores will be based on at least 5 homework assignments and at least 5 in-class quizzes in each module. The weights a, b, c and d are between 0 and 0.1, the exact values will be determined at the end of each semester.

Calculus++ is a standalone additional course ICEF offers as part of its advanced program. There will a separate exam for this course. The Calculus++Bonus is defined as (mark for the Calculus++ exam out of 10 points) – 4. Thus, the bonus value is 6 points at maximum.

## 5. Textbooks and problem books

Main textbooks.

- Stewart J. (editor), Calculus. Early Transcendentials. 6 edition. Thomson Brooks/Cole, 2008 (S). Any later edition is good as well.
- Краснов М. и др., Вся высшая математика, том 1, 2.
- Зорич В.П., Математический анализ (в 2-х) томах. Фазис, 1997-1998.

Alternative textbooks

- Dowling E.T. Introduction to Mathematical Economics. McGraw-Hill, 1980. (D)
- Красс М. С., Высшая математика для экономиста. М., 1998 (Кр).
- Кремер Н.Ш., Путко И.М., Фридман М.Н. Высшая математика для экономистов. М, 2000 (КПФ)
- Фихтенгольц Г.М. Курс дифференциального и интегрального исчисления (в трех томах). М., 1998 (Ф).

Main problem books.

- Lockshin J., Calculus: theory, examples, exercises. ICEF
- Демидович Б.М. Сборник задач и упражнений по математическому анализу. М., Наука, 1996.

Supplementary reading.
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- Simon C.P., Blume L. Mathematics for Economists. W.W.Norton & Company, 1994.
- Chiang A.C. Fundamental Methods of Mathematical Economics. McGraw-Hill, 1984
- Anthony M., Biggs N. Mathematics for Economics and Finance. CUP, 1996.

**6. Course outline:**

**1. Introduction**

The application of mathematics to describing phenomena. The role of mathematics and mathematical modeling in economics. Different forms of representation of functions. Elementary concepts: domain and range of a function, even and odd functions, periodic functions. Graphs of elementary functions. Shifts and distortions of graphs. Implicit functions. Examples of functions in economics: utility function, production function, cost function, demand and supply functions.
***(D. Ch. 7; K. – pp. 23-51; Кр. - pp. 11-14, 46-58, 88-91, 155-161; КПФ – pp. 125-137; Ф. - V.1, pp. 93-114; S. – pp. 10-72)***

**2. Sequences. Limit of a sequence**

Sequences: bounded and unbounded, infinitely small and infinitely large. Limit of a sequence. Limit theorems for sequences: arithmetic operations, sandwich theorem. Monotone sequences. Convergence of a monotone increasing sequence. The number ***e***.
***(Кр. - pp. 24-45; КПФ – pp. 141-142; Ф. - V.1, pp. 43-92; S. – pp. 674-686)***

**3. Limit of a function**

The limit of a function at infinity. Asymptotes of a function at infinity. The limit of a function at a point. Limit theorems for functions. Functions that tend to zero, functions that tend to infinity. First and Second Special Limits. Types of indeterminate forms. Finding limits. Left and right limits.
***(D. - Ch. 3.1; K. - pp. 71-91; Кр. - pp. 58-73; КПФ – pp. 143-160; Ф. - V.1, pp. 115-145; S. – pp. 88-118)***

**4. Continuity**

Definition of continuity of a function at point and on an interval. Continuity of elementary functions. Properties of continuous functions. Points of discontinuity. Classification of points of discontinuity. Vertical asymptotes.
***(D. – Ch. 3.2; K. – pp. 92-95; Кр. - pp. 74-87; КПФ – pp. 161-164; Ф. - V.1, pp. 146-185; S. – pp. 119-143)***

**5. The derivative**

Definition of the derivative. Tangent lines and normal lines. Geometric, physical and economic interpretations of the derivative. Right and left derivatives. Differentiability at a point. Differentiability and continuity. Differentiation. Rules of differentiation. Derivatives of elementary functions. Differentiation of inverse functions. Logarithmic differentiation. Differentiation of implicit functions. Existence of a differentiable implicit function. Definition and geometric interpretation of differentials. Approximate calculations using differentials. The second derivative. The economic meaning of the second derivative. Higher-order derivatives and differentials. Properties of differentiable functions: Rolle's theorem, the Mean Value theorem, Cauchy’s theorem, and their geometric interpretation.
***(D. - pp. 41-47; K. pp. 109-144, 163-166, 211-214, 216-218; Кр. - pp. 98-123; КПФ – pp. 176-198, 209-211; Ф. V.1, pp. 186-222, 231-245, S. – pp. 143-166, 172-261)***

**6. Applications of the derivative**

L’Hospital’s rule. Necessary and sufficient conditions for increasing/decreasing functions. Related rates. Concave and convex functions. Different ways of expressing concavity. Economic interpretation of concave and convex functions. Points of inflection. Local extrema. First-order necessary and sufficient conditions for a local extremum. Second-order necessary and sufficient conditions for a local extremum. Maximum and minimum values of a function on an interval. Geometric and economic applications of optimisation. Curve sketching.

***(D. – Ch. 4; K. – pp. 167-210; Кр. - pp. 124-132, 140-161; КПФ – pp.212-234, 240-241; Ф. – V.1, pp. 268-336, S. – pp. 270-333)***

**7. Number series, power series, and Taylor expansions**

Necessary condition for convergence of a series. Harmonic series and power series. The ratio test. Comparing series to test for convergence. Alternating series. Sufficient condition for convergence of an alternating series. Absolute convergence. Radius and interval of convergence of a power series. Abel's theorems. Taylor's formula. Taylor and Maclaurin series. Taylor and Maclaurin expansions for elementary functions. Application of Taylor series for analyzing the behavior of a function at a point and for conducting approximate calculations.
***(Кр. - pp. 133-139; КПФ – pp. 356-372, 379-390; Ф. - V.1, pp. 246-262; S. – Ch. 30.2, S. – pp. 687-747 )***

**8. Anti-derivatives and the indefinite integral**

Anti-derivatives. The indefinite integral and its properties. Table of indefinite integrals. Basic methods of integration: direct integration, substitution and integration by parts. Integration of rational functions.
***(D. –pp. 357-362; K. – pp. 234-257; Кр. - pp. 162-186; КПФ – pp. 251-270; Ф. – V.2, pp. 11-93, S. – pp. 340-345, 452-489 )***

**9. The definite integral**

Problems that require the definite integral. Definition of the definite integral using Riemann sums. Sufficient condition for the existence of the definite integral. Approximate calculation of definite integrals using rectangles and trapezoids. Simpson's rule. Properties of the definite integral. Differentiation of a definite integral with variable upper bound. The fundamental theorem of calculus. Substitution and integration by parts.
***(D. –pp. 373-375; K. – pp. 273-285; Кр. - pp. 187 - 210, 233-236; КПФ – pp. 283-296, 312-317; Ф. - V.2, pp. 94-168, S. – pp. 354 - 390)***

**10. Applications of the definite integral**

Applications of the definite integral in geometry, economics and physics. Area of a flat region, volume of a solid of revolution, volume of a solid with known cross-sections. Use of definite integrals to solve separable differential equations.
***(D. – p. 376; K. – pp. 286-315; Кр. - pp. 211-232; КПФ – pp. 298-306; Ф. - V.2, pp. 169-243, S. – pp. 414-447, 524-561)***

**11. Improper Integrals**

Integrals with infinite bounds. Improper integrals of the first kind. Integration of unbounded functions. Improper integrals of the second kind. Principle value. Convergence tests for improper integrals. Absolute and relative convergence of improper integrals.
***(Кр. - pp. 237-248; КПФ – pp. 307-311; Ф. - V.2, pp. 552-653, S. – pp.508-517)***

**12. The double and triple integrals**

Definition of double and triple integrals. Reduction of double integrals to iterated integrals. Changing the order of integration in iterated integrals. The geometric interpretation and main properties of double integrals. ***(Кр. – c. 390-405; КПФ: pp. 425---427; Ф.: V. 3, pp. 136---141, 154---167, S. – pp. 950- 973)***

**13. Differential equations and slope fields**

Definition of first order differential equations. General and particular solutions. Existence and uniqueness theorem. Isoclines and direction fields. Solution of separable differential

equations. Solution of homogeneous differential equations and first-order linear equations. Application of differential equations to physics and economics.
***(D. –pp. 392, 395-396; K. – pp. 316-323; Кр. - pp. 477-544; КПФ – pp. 325-336; Ф. - V.2, pp. 244-257, S. – pp. 566-607)***

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| Author of the original course program: | J. Lockshin, 2010. |
| Updated by | A. Akhmetshin, 2012, 2018 |