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Generalized Einstein gravitational theory with vacuum vectorial field

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Abstract
In this paper a new generalized Einstein gravitational theory with four physical vacuum potentials and the Weyl connection is proposed. In the examined case of dust-like matter, differential equations of the second order for the vacuum potentials consistent with the proposed gravitational equations are obtained. A nonsingular cosmological solution for the homogeneous and isotropic physical vacuum and a vacuum cosmological influence on particles are found. Astronomical applications of this solution are discussed.

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1. Introduction

As is well known, in 1918 Weyl proposed a generalization of the Einstein gravitational theory in which the connection Γ^i_{jk} is defined as follows [1]:

$$\Gamma^i_{jk} = \tfrac{1}{2}g^{im}(\partial_j g_{mk} + \partial_k g_{jm} - \partial_m g_{jk}) + \tfrac{1}{2}(\lambda^i g_{jk} - \lambda_j \delta^i_k - \lambda_k \delta^i_j), \qquad \Gamma^i_{jk} = \Gamma^i_{kj}, \tag{1.1}$$

where $\partial_i \equiv \partial/\partial x^i$, x^i are four-dimensional coordinates, g_{ik} are components of the metric tensor, $\delta^i_i = 1$ and $\delta^i_k = 0$, $i \neq k$, and λ^i are components of a four-dimensional vector.

The Weyl formula (1.1) for the connection Γ^i_{jk} gives the following expressions for the covariant derivatives ∇_i of the metric tensor:

$$\nabla_i g_{jk} = \lambda_i g_{jk}, \qquad \nabla_i g^{jk} = -\lambda_i g^{jk}. \tag{1.2}$$

The Weyl connection (1.1) is invariant under the gauge transformation

$$g_{ik} \to \exp(\phi) g_{ik}, \qquad \lambda_i \to \lambda_i + \partial_i \phi, \tag{1.3}$$

where ϕ is an arbitrary differentiable function of the coordinates x^i. This property of the connection (1.1) plays a major role in the Weyl conception.

In the Weyl geometry the four-dimensional interval ds is defined as the invariant differential expression $(g_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k)^{1/2}$ that gives the kinematic equation for beams of light [1]

$$g_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k = 0. \tag{1.4}$$

Then the gauge transformations (1.3) consistent with equation (1.4) give physically permissible components g_{ik} and λ_i which do not change the connection Γ^i_{jk}.

Weyl regarded the components λ_i of formula (1.1) as electromagnetic field potentials and his aim was to find a unified theory of gravitational and electromagnetic fields. But despite all its attractions, this theory was not supported by Einstein and other famous physicists, since it did not give satisfactory results.

That is why our objective is to investigate a gauge-invariant generalization of the Einstein gravitational theory based on the Weyl geometry in which the components λ_i are not regarded as electromagnetic field potentials. In order to have a theory consistent with the well-known experimental data, we will interpret the components λ_i as potentials of the physical vacuum giving only small corrections to the Einstein gravitational equations.

Though we will use the Weyl geometry to generalize the Einstein gravitational equations, to do this we will choose a different way from that chosen by Weyl.

We will adhere to the following two principles.

(1) Weyl's principle: The sought generalization of the Einstein gravitational equations should be covariant and gauge-invariant.

(2) Extra principle: In the case of the standard gauge for the metric components g_{ik}, this generalization should be reducible to the Einstein equations with an additional energy–momentum tensor of the vacuum vectorial field with the potentials λ_i.

It should be noted that the theory proposed by Weyl is based on a gauge-invariant Lagrangian of second order in the curvature [1]. Weyl's Lagrangian and a number of other gauge-invariant Lagrangians of second order in the curvature give gravitational equations of fourth order in the derivatives of the metric [2], in contrast with the Einstein gravitational equations of the second order.

Therefore, such generalizations of the Einstein theory do not satisfy the second principle stated above and the sought gravitational equations should be of second order in the derivatives of the metric.

Consider now the following covariant generalization of the Einstein gravitational equations which is based on the Weyl connection (1.1) and different from the Weyl equations:

$$\begin{aligned} &R_{ik} + R_{ki} - g_{ik}R = (16\pi f/c^4)T_{ik}, \\ &R_{ik} = \partial_l \Gamma^l_{ik} - \partial_k \Gamma^l_{il} + \Gamma^l_{ik}\Gamma^m_{lm} - \Gamma^m_{il}\Gamma^l_{km}, \qquad R = g^{ik}R_{ik}, \end{aligned} \tag{1.5}$$

where R_{ik} is the Ricci tensor, the connection Γ^i_{jk} is defined by formula (1.1), T_{ik} is the energy–momentum tensor of matter and classical fields, λ^i is a 4-vector of vacuum potentials and f is the gravitational constant. In the first equation of (1.5) we have taken into account that the energy–momentum tensor T_{ik} has to be symmetric: $T_{ik} = T_{ki}$ [3].

Let us show that the gravitational equations (1.5) satisfy the two principles stated above.

First, since the connection Γ^i_{jk} is invariant under the gauge transformations (1.3), the tensors R_{ik} and $g_{ik}R = g_{ik}g^{mn}R_{mn}$ contained in the first equation of (1.5) are also gauge-invariant.

Let us now define gauge-invariant components T_{ik}. Such an energy–momentum tensor T_{ik} can be easily defined by the following standard way [3]: in local inertial reference frames T_{00} is the energy density of a matter, $-cT_{0\alpha}$ are its energy flow densities, $T_{\alpha\beta}$ are its momentum flow densities, $\alpha, \beta = 1, 2, 3$ and $T_{ik} = T_{ki}$. In other coordinate systems T_{ik} are transformed as tensor components.

Then the proposed covariant gravitational equations (1.5) are gauge-invariant and coincide with the Einstein gravitational equations when the vacuum potentials $\lambda_i = 0$.

As will be seen in section 4, when the standard gauge for the components g_{ik} is chosen, the vacuum potentials λ_i generally take very small values: $\sqrt{\lambda^i\lambda_i} \sim 1/A$, where A is the space curvature radius of the universe. However, we will see that the small vacuum potentials can significantly influence cosmological processes.

Consider the gravitational equations (1.5). From them we have

$$R_{ki} - R_{ik} = \partial_k\Gamma^l_{il} - \partial_i\Gamma^l_{kl} = 2(\partial_i\lambda_k - \partial_k\lambda_i). \tag{1.6}$$

Let us introduce the antisymmetric tensor

$$\Lambda_{ik} = \nabla_i\lambda_k - \nabla_k\lambda_i \equiv \partial_i\lambda_k - \partial_k\lambda_i. \tag{1.7}$$

The tensor Λ_{ik}, which is invariant under the gauge transformations (1.3), can be regarded as the tensor of vacuum field intensities.

From (1.6) and (1.7) we get

$$R_{ki} - R_{ik} = 2\Lambda_{ik}. \tag{1.8}$$

Let us put

$$\begin{gathered}
\Gamma^i_{jk} = \bar{\Gamma}^i_{jk} + \gamma^i_{jk}, \qquad \bar{\Gamma}^i_{jk} = \tfrac{1}{2}g^{im}(\partial_j g_{mk} + \partial_k g_{jm} - \partial_m g_{jk}),\\
\gamma^i_{jk} = \tfrac{1}{2}\left(\lambda^i g_{jk} - \lambda_j\delta^i_k - \lambda_k\delta^i_j\right), \qquad R_{ik} = \bar{R}_{ik} + W_{ik},\\
\bar{R}_{ik} = \partial_l\bar{\Gamma}^l_{ik} - \partial_k\bar{\Gamma}^l_{il} + \bar{\Gamma}^l_{ik}\bar{\Gamma}^m_{lm} - \bar{\Gamma}^m_{il}\bar{\Gamma}^l_{km}, \qquad \bar{R} = g^{ik}\bar{R}_{ik},\\
W_{ik} = \partial_l\gamma^l_{ik} - \partial_k\gamma^l_{il} + \bar{\Gamma}^l_{ik}\gamma^m_{lm} + \gamma^l_{ik}\bar{\Gamma}^m_{lm} - \bar{\Gamma}^m_{il}\gamma^l_{km} - \gamma^m_{il}\bar{\Gamma}^l_{km} + \gamma^l_{ik}\gamma^m_{lm} - \gamma^m_{il}\gamma^l_{km}.
\end{gathered} \tag{1.9}$$

Taking into account (1.9), equations (1.5) can be rewritten in the form

$$\bar{R}_{ik} - \frac{1}{2}g_{ik}\bar{R} = \frac{8\pi f}{c^4}(T_{ik} + \Theta_{ik}), \qquad \Theta_{ik} = -\frac{c^4}{8\pi f}\left(W_{ik} - \frac{1}{2}Wg_{ik} + \Lambda_{ik}\right),$$
$$W = g^{ik}W_{ik} \tag{1.10}$$

and from (1.10) we get the differential equation of energy–momentum conservation [3]

$$\bar{\nabla}_i(T^{ik} + \Theta^{ik}) \equiv \partial_i(T^{ik} + \Theta^{ik}) + \bar{\Gamma}^i_{mi}(T^{mk} + \Theta^{mk}) + \bar{\Gamma}^k_{im}(T^{im} + \Theta^{im}) = 0, \tag{1.11}$$

where $\bar{\nabla}_i$ denotes the covariant derivative defined by means of the Christoffel symbols$\bar{\Gamma}^i_{jk}$.

Let us choose the standard gauge for the components g_{ik}. This implies that in local inertial reference frames $g_{ik} = \bar{g}_{ik}$, where $\bar{g}_{ik}$ is the Minkowski metric tensor. Then from (1.11) we get

$$\partial_i(T^{ik} + \Theta^{ik}) = 0, \qquad g_{ik} = \bar{g}_{ik} \tag{1.12}$$

and hence the components Θ^{ik} can be regarded as an additional energy–momentum tensor corresponding to the vacuum vectorial field.

Therefore, the gravitational equations (1.5) can be reduced to the Einstein equations (1.10) with the additional energy–momentum tensor Θ^{ik} of the vacuum vectorial field.

Thus, the proposed equations (1.5) are in accord with the two principles stated above.

In section 2 we will investigate the gravitational equations (1.5), which contain the vacuum potentials λ_i and will apply the Bianchi identities [3] to obtain a generalized differential correlation for the energy–momentum tensor of matter T^{ik}.

In section 3, we will apply this correlation to obtain covariant kinematic equations for a free movement of a dust-like matter in a gravitational field. We will have four differential equations of the second order for the three functions $x^l(x^0)$, $l = 1, 2, 3$ describing the time dependence of spatial coordinates of material points of the dust-like matter. The condition of consistency of these equations will allow us to find four differential equations of second order for the four vacuum potentials λ_i in the examined case. These four differential equations and the ten gravitational equations (1.5) will give 14 differential equations of the second order for the 14 components λ_i and g_{ik}.

In section 4, we will consider cosmological problems within the framework of the proposed gravitational equations (1.5). We will find a nonsingular cosmological solution for the homogeneous and isotropic physical vacuum and its potentials λ_i and will determine the influence of the potentials on particles moving in the physical vacuum.

In section 5 we will discuss this solution and apply it to the interpretation of a number of astronomical phenomena.

2. Generalized differential correlation for the energy–momentum tensor T^{ik}

Consider the curvature tensor $R^i_{\ jkn}$ defined as follows [3]:

$$R^i_{\ jkn} = \partial_k \Gamma^i_{\ jn} - \partial_n \Gamma^i_{\ jk} + \Gamma^p_{\ jn}\Gamma^i_{\ pk} - \Gamma^p_{\ jk}\Gamma^i_{\ pn}, \tag{2.1}$$

where the connection $\Gamma^i_{\ jk}$ is determined by formula (1.1) and, since $\Gamma^i_{\ jk} = \Gamma^i_{\ kj}$, let us use the well-known Bianchi identities [3]

$$\nabla_m R^i_{\ jkn} + \nabla_n R^i_{\ jmk} + \nabla_k R^i_{\ jnm} = 0. \tag{2.2}$$

Identities (2.2) give the following equality:

$$g^{jk}\left(\nabla_m R^m_{\ jkn} + \nabla_n R^m_{\ jmk} + \nabla_k R^m_{\ jnm}\right) = 0. \tag{2.3}$$

Let us represent equality (2.3) in another form containing the Ricci tensor R_{ik} instead of the curvature tensor $R^i_{\ jkn}$, taking into account the well-known correlations [3]

$$R^m_{\ jmk} = R_{jk}, \qquad R^m_{\ jkn} = -R^m_{\ jnk}. \tag{2.4}$$

Such a representation of (2.3) will be obtained later on. For this purpose, first consider the following tensor:

$$R_{ijkn} \equiv g_{im} R^m_{\ jkn} = g_{im}\left(\partial_k \Gamma^m_{\ jn} - \partial_n \Gamma^m_{\ jk} + \Gamma^p_{\ jn}\Gamma^m_{\ pk} - \Gamma^p_{\ jk}\Gamma^m_{\ pn}\right). \tag{2.5}$$

The Weyl connection satisfies the correlation $\Gamma^i_{\ jk} = \Gamma^i_{\ kj}$. Therefore, as is well known [3], in the vicinity of any point x^i we can choose a local coordinate system in which at that point $\Gamma^i_{\ jk} = 0$. In this coordinate system at the considered point expression (2.5) acquires the form

$$\Gamma^m_{\ jn} = 0, \qquad R_{ijkn} = g_{im}\left(\partial_k \Gamma^m_{\ jn} - \partial_n \Gamma^m_{\ jk}\right) = \partial_k\left(g_{im}\Gamma^m_{\ jn}\right) - \partial_n\left(g_{im}\Gamma^m_{\ jk}\right). \tag{2.6}$$

Formulae (1.1) and (2.6) give

$$R_{ijkn} = \tfrac{1}{2}\partial_k(\partial_j g_{in} - \partial_i g_{jn} + \lambda_i g_{jn} - \lambda_j g_{in} - \lambda_n g_{ij}) - \tfrac{1}{2}\partial_n(\partial_j g_{ik} - \partial_i g_{jk} + \lambda_i g_{jk} - \lambda_j g_{ik} - \lambda_k g_{ij}), \qquad \Gamma^m_{\ jn} = 0. \tag{2.7}$$

In the considered local coordinate system from (2.7) we derive

$$R_{ijkn} = -R_{ijnk}, \tag{2.8}$$

$$R_{ijkn} + R_{jikn} = \nabla_n(\lambda_k g_{ij}) - \nabla_k(\lambda_n g_{ij}) = g_{ij}(\nabla_n\lambda_k - \nabla_k\lambda_n) = g_{ij}\Lambda_{nk}, \tag{2.9}$$

where we have taken into account formulae (1.2) and (1.7).

Since the left- and right-hand sides of equalities (2.8) and (2.9) are tensors, these equalities are true in arbitrary coordinate systems.

Using correlations (2.8), (2.9) and (2.4), we obtain

$$g^{jk}R_{ijnk} = -g^{jk}R_{ijkn} = -g^{jk}(-R_{jikn} + g_{ij}\Lambda_{nk}) = g^{jk}g_{jm}R^m_{\ ikn} + \delta^k_i\Lambda_{kn} = R^m_{\ imn} + \Lambda_{in} = R_{in} + \Lambda_{in}. \tag{2.10}$$

From (2.5) and (2.10) we find

$$g^{jk} R^m_{jnk} = g^{mi} g^{jk} R_{ijnk} = R^m{}_n + \Lambda^m{}_n. \tag{2.11}$$

Let us turn to equality (2.3) and represent it in the form

$$\nabla_m\big(g^{jk} R^m_{jkn}\big) + \nabla_n\big(g^{jk} R^m_{jmk}\big) + \nabla_k\big(g^{jk} R^m_{jnm}\big) - R^m_{jkn}\nabla_m g^{jk} - R^m_{jmk}\nabla_n g^{jk} - R^m_{jnm}\nabla_k g^{jk} = 0. \tag{2.12}$$

Using (1.2) and (2.4), from (2.12) we get

$$(\nabla_m + \lambda_m)\big(g^{jk} R^m_{jnk}\big) - (\nabla_n + \lambda_n) R + (\nabla_k + \lambda_k) R^k{}_n = 0, \qquad R = g^{jk} R_{jk}. \tag{2.13}$$

Taking into account correlation (2.11), from equality (2.13) we derive

$$(\nabla_m + \lambda_m)\left(R^m{}_n - \tfrac{1}{2}\delta^m_n R + \tfrac{1}{2}\Lambda^m{}_n\right) = 0. \tag{2.14}$$

Let us multiply (2.14) by g^{nk} and use the following formula, taking into account the second formula in (1.2):

$$g^{nk}\nabla_m Q^m{}_n = \nabla_m\big(g^{nk} Q^m{}_n\big) - Q^m{}_n \nabla_m g^{nk} = (\nabla_m + \lambda_m) Q^{mk}, \tag{2.15}$$

where

$$Q^m{}_n = R^m{}_n - \tfrac{1}{2}\delta^m_n R + \tfrac{1}{2}\Lambda^m{}_n.$$

Then we obtain

$$(\nabla_m + 2\lambda_m) Q^{mk} = 0, \qquad Q^{mk} = R^{mk} - \tfrac{1}{2} g^{mk} R + \tfrac{1}{2}\Lambda^{mk}. \tag{2.16}$$

Using (1.2), we find

$$\nabla^m \lambda^k = g^{mp}\nabla_p(g^{kn}\lambda_n) = g^{mp} g^{kn}(\nabla_p\lambda_n - \lambda_p\lambda_n) = g^{mp} g^{kn}\nabla_p\lambda_n - \lambda^m\lambda^k. \tag{2.17}$$

From (2.17) and (1.7) we get

$$\nabla^m\lambda^k - \nabla^k\lambda^m = g^{mp} g^{kn}(\nabla_p\lambda_n - \nabla_n\lambda_p) = g^{mp} g^{kn}\Lambda_{pn} = \Lambda^{mk}. \tag{2.18}$$

Therefore, (2.16) can be represented as

$$(\nabla_m + 2\lambda_m)\left[R^{mk} - \tfrac{1}{2} g^{mk} R + \tfrac{1}{2}(\nabla^m\lambda^k - \nabla^k\lambda^m)\right] = 0. \tag{2.19}$$

From (1.5) and (1.8) we find

$$R^{mk} - \tfrac{1}{2} g^{mk} R = (8\pi f/c^4) T^{mk} - \Lambda^{mk}. \tag{2.20}$$

Hence, equations (2.18)–(2.20) give the differential correlation for the energy–momentum tensor T^{ik}

$$(\nabla_m + 2\lambda_m)[(16\pi f/c^4) T^{mk} + \nabla^k\lambda^m - \nabla^m\lambda^k] = 0. \tag{2.21}$$

In the following section we will consider the case of dust-like matter and will apply correlation (2.21) to obtain differential equations for the vacuum potentials λ_i consistent with the gravitational equations (1.5).

3. Generalized kinematic equations for dust-like matter and differential equations for the vacuum potentials λ_i

Consider dust-like matter moving in a gravitational field. Let us choose the standard gauge for the metric tensor g_{ik}. Then, as is well known, the energy–momentum tensor T^{ik} for the dust-like matter under consideration takes the form [3]

$$T^{ik} = c^2\rho_0\, \mathrm{d}x^i/\mathrm{d}s\, \mathrm{d}x^k/\mathrm{d}s, \qquad \mathrm{d}s^2 = g_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k = (\mathrm{d}\bar{x}^0)^2 - (\mathrm{d}\bar{x}^1)^2 - (\mathrm{d}\bar{x}^2)^2 - (\mathrm{d}\bar{x}^3)^2, \tag{3.1}$$

where x^i are arbitrary coordinates of a material point of the matter, $\bar{x}^i$ are its coordinates in a local inertial reference frame and ρ_0 is the density of the rest mass of the matter in a comoving local inertial frame.

In a local inertial frame with $x^i = \bar{x}^i$ and $g_{ik} = \bar{g}_{ik}$ we have the well-known differential equation of rest mass conservation [3]

$$\partial_m(\rho_0\, \mathrm{d}x^m/\mathrm{d}s) = 0, \qquad x^i = \bar{x}^i, \qquad g_{ik} = \bar{g}_{ik}, \tag{3.2}$$

where $\bar{g}_{ik}$ denotes the Minkowski metric tensor.

In the considered local inertial frame from (1.1) and (3.2) we find

$$\begin{aligned}&\nabla_m(\rho_0\, \mathrm{d}x^m/\mathrm{d}s) = \partial_m(\rho_0\, \mathrm{d}x^m/\mathrm{d}s) + \Gamma^l_{ml}\rho_0\, \mathrm{d}x^m/\mathrm{d}s = -2\lambda_m\rho_0\, \mathrm{d}x^m/\mathrm{d}s,\\ &x^i = \bar{x}^i, \qquad g_{ik} = \bar{g}_{ik}.\end{aligned} \tag{3.3}$$

From (3.3) we get the following covariant equation of rest mass conservation:

$$(\nabla_m + 2\lambda_m)(\rho_0\, \mathrm{d}x^m/\mathrm{d}s) = 0. \tag{3.4}$$

Using (2.18), let us rewrite the differential correlation (2.21) for the energy–momentum tensor T^{ik} in the form

$$(\nabla_m + 2\lambda_m)\left(T^{km} + \frac{c^4}{16\pi f}\Lambda^{km}\right) = 0. \tag{3.5}$$

As follows from (3.1) and (3.4), we have

$$\begin{aligned}(\nabla_m + 2\lambda_m)T^{km} &= c^2\, \mathrm{d}x^k/\mathrm{d}s(\nabla_m + 2\lambda_m)(\rho_0\, \mathrm{d}x^m/\mathrm{d}s) + c^2\rho_0\, \mathrm{d}x^m/\mathrm{d}s\nabla_m(\mathrm{d}x^k/\mathrm{d}s)\\ &= c^2\rho_0\, \mathrm{d}x^m/\mathrm{d}s\left[\partial_m(\mathrm{d}x^k/\mathrm{d}s) + \Gamma^k_{mn}\, \mathrm{d}x^n/\mathrm{d}s\right]\\ &= c^2\rho_0\left(\mathrm{d}^2x^k/\mathrm{d}s^2 + \Gamma^k_{mn}\, \mathrm{d}x^m/\mathrm{d}s\, \mathrm{d}x^n/\mathrm{d}s\right).\end{aligned} \tag{3.6}$$

Therefore, equations (3.5) and (3.6) give

$$\rho_0\left(\frac{\mathrm{d}^2x^k}{\mathrm{d}s^2} + \Gamma^k_{mn}\frac{\mathrm{d}x^m}{\mathrm{d}s}\frac{\mathrm{d}x^n}{\mathrm{d}s}\right) + \frac{c^2}{16\pi f}(\nabla_m + 2\lambda_m)\Lambda^{km} = 0, \tag{3.7}$$

where the components $\mathrm{d}x^k/\mathrm{d}s$ satisfy the evident equality

$$\frac{\mathrm{d}x^k}{\mathrm{d}s}\frac{\mathrm{d}x_k}{\mathrm{d}s} = 1. \tag{3.8}$$

Equations (3.7) and (3.8) present five differential equations for only four functions $x^k(s)$ (they can be easily reduced to four equations for the three functions $x^l(x^0)$, $l = 1, 2, 3$). That is why we will consider conditions for the potentials λ_i providing the consistency of these equations.

For this purpose, taking into account formula (1.9) for Γ^i_{jk}, let us represent (3.7) as

$$\rho_0\left[\frac{\mathrm{d}^2x^k}{\mathrm{d}s^2} + \left(\bar{\Gamma}^k_{mn} + \gamma^k_{mn}\right)\frac{\mathrm{d}x^m}{\mathrm{d}s}\frac{\mathrm{d}x^n}{\mathrm{d}s}\right] + \frac{c^2}{16\pi f}(\nabla_m + 2\lambda_m)\Lambda^{km} = 0 \tag{3.9}$$

and note the identity (see equation (14) in [4])

$$\frac{\mathrm{d}x_k}{\mathrm{d}s}\left(\frac{\mathrm{d}^2x^k}{\mathrm{d}s^2} + \bar{\Gamma}^k_{mn}\frac{\mathrm{d}x^m}{\mathrm{d}s}\frac{\mathrm{d}x^n}{\mathrm{d}s}\right) = 0. \tag{3.10}$$

In a local inertial reference frame with $x^k = \bar{x}^k$ and $g_{ik} = \bar{g}_{ik}$ formula (3.10) is evident, since, taking into account equality (3.8), in this frame we have

$$\bar{\Gamma}^k_{mn} = 0, \qquad \frac{\mathrm{d}x_k}{\mathrm{d}s}\frac{\mathrm{d}^2x^k}{\mathrm{d}s^2} = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\mathrm{d}x_k}{\mathrm{d}s}\frac{\mathrm{d}x^k}{\mathrm{d}s}\right) \equiv 0, \qquad x^k = \bar{x}^k, \qquad g_{ik} = \bar{g}_{ik}, \tag{3.11}$$

where $\bar{g}_{ik}$ is the Minkowski metric tensor.

As is well known, the left-hand side of (3.10) is a scalar [3]. Therefore, formula (3.10) is also true in arbitrary coordinate systems.

Let us multiply equation (3.9) by $\mathrm{d}x_k/\mathrm{d}s$ and use identity (3.10) and formula (1.9) for γ^k_{mn}. Then we obtain

$$\frac{\mathrm{d}x_k}{\mathrm{d}s}\left[\rho_0(\lambda^k g_{mn} - \lambda_m\delta^k_n - \lambda_n\delta^k_m)\frac{\mathrm{d}x^m}{\mathrm{d}s}\frac{\mathrm{d}x^n}{\mathrm{d}s} + \frac{c^2}{8\pi f}(\nabla_m + 2\lambda_m)\Lambda^{km}\right] = 0. \tag{3.12}$$

From (3.12) we derive

$$\mathrm{d}x_k/\mathrm{d}s[(\nabla_m + 2\lambda_m)\Lambda^{km} - (8\pi f/c^2)\rho_0\lambda^k\ ] = 0. \tag{3.13}$$

Since $\mathrm{d}x_k/\mathrm{d}s$ is arbitrary, equation (3.13) gives

$$(\nabla_m + 2\lambda_m)\Lambda^{km} = (8\pi f/c^2)\rho_0\lambda^k. \tag{3.14}$$

From equations (1.5), (2.18), and (3.14) we get the following 14 differential equations of the second order for the 10 metric components g_{ik} and 4 vacuum potentials λ_i:

$$R_{ik} + R_{ki} - g_{ik}R = (16\pi f/c^4)T_{ik}, \qquad (\nabla_m + 2\lambda_m)(\nabla^k\lambda^m - \nabla^m\lambda^k) = (8\pi f/c^2)\rho_0\lambda^k. \tag{3.15}$$

The four differential equations of second order obtained for the vacuum potentials λ_i present the sought conditions providing the consistency of equations (3.7) and (3.8).

From equations (3.7) and (3.14) we easily get

$$\mathrm{d}^2x^k/\mathrm{d}s^2 + \Gamma^k_{mn}\,\mathrm{d}x^m/\mathrm{d}s\,\mathrm{d}x^n/\mathrm{d}s + \tfrac{1}{2}\lambda^k = 0. \tag{3.16}$$

The obtained equations (3.16) describe the kinematics of a dust-like matter in a gravitational field and equations (3.14) describe the vacuum potentials λ_i in the case of a dust-like matter with the density ρ_0 moving in a gravitational field.

It should be noted that

$$\mathrm{d}x_k/\mathrm{d}s\left(\mathrm{d}^2x^k/\mathrm{d}s^2 + \Gamma^k_{mn}\,\mathrm{d}x^m/\mathrm{d}s\,\mathrm{d}x^n/\mathrm{d}s + \tfrac{1}{2}\lambda^k\right) \equiv 0. \tag{3.17}$$

Identity (3.17) can be easily verified by using identity (3.10) and formula (1.9) for Γ^i_{jk}.

Therefore, the four differential equations (3.16) are not independent: the first equation ($k = 0$) is a consequence of the other three equations ($k = 1, 2, 3$). Hence, it suffices to solve the differential equations (3.16) only for $k = 1, 2, 3$.

4. Cosmological effects of the generalized Einstein gravitational theory with four vacuum potentials

Examine the spacetime geometry in a big spatial region of the physical vacuum situated sufficiently far from massive bodies and assume that this vacuum region is homogeneous and isotropic in comoving local inertial frames of reference. Let us choose an extended reference frame consisting of a big set of such local inertial frames covering the examined vacuum region.

From equations (1.5), (1.7), (1.8) and (3.14) we get the vacuum gravitational equations

$$\begin{aligned}
&R = -(8\pi f/c^4)\bar{T}, \qquad \bar{T} \equiv g^{ik}\bar{T}_{ik}, \qquad R_{ik} = (8\pi f/c^4)\left(\bar{T}_{ik} - \tfrac{1}{2}g_{ik}\bar{T}\right) - \Lambda_{ik},\\
&\Lambda_{ik} = \partial_i\lambda_k - \partial_k\lambda_i, \qquad (\nabla_m + 2\lambda_m)\Lambda^{km} = (8\pi f/c^2)\bar{\rho}_0\lambda^k,\\
&R_{ik} = \partial_l\Gamma^l_{ik} - \partial_k\Gamma^l_{il} + \Gamma^l_{ik}\Gamma^m_{lm} - \Gamma^m_{il}\Gamma^l_{km},\\
&\Gamma^i_{jk} = \tfrac{1}{2}g^{im}(\partial_j g_{mk} + \partial_k g_{jm} - \partial_m g_{jk}) + \tfrac{1}{2}(\lambda^i g_{jk} - \lambda_j\delta^i_k - \lambda_k\delta^i_j),
\end{aligned} \tag{4.1}$$

where $\bar{T}_{ik}$ and $\bar{\rho}_0$ are the energy–momentum tensor and rest mass density of the physical vacuum, respectively.

Let us describe the homogeneous and isotropic space under consideration by the Robertson–Walker metric. This metric can be represented as [5]

$$ds^2 = (dx^0)^2 - A^2[dr^2/(1-Kr^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)], \qquad A = A(x^0), \tag{4.2}$$

where A is the scale factor, x^0/c is the time, r, θ, φ are some spherical coordinates and the parameter K takes the values $-1, 1, 0$ corresponding to the cases of negative space curvature, positive space curvature and to the flat case, respectively.

The vacuum potentials λ_i have to be independent of rotations and translations of rectangular space axes in the considered homogeneous and isotropic space. Therefore, in this space for the vacuum potentials λ_i in the considered coordinate system we get

$$\lambda_1 = \lambda_2 = \lambda_3 = 0, \qquad \lambda_0 = \lambda_0(x^0). \tag{4.3}$$

Let us choose the dimensionless time coordinate η which is called 'conformal time' in [5] and defined as follows (see page 35 in [5]):

$$d\eta = dx^0/A(x^0). \tag{4.4}$$

Then from (4.2) and (4.4) we find

$$ds^2 = A^2[d\eta^2 - dr^2/(1-Kr^2) - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)], \qquad A = A(\eta). \tag{4.5}$$

Let λ_i^* denote the vacuum potentials in the coordinate system $(\eta, r, \theta, \varphi)$. Then, since λ_i and λ_i^* are components of a covariant vector in the coordinate systems $(x^0, r, \theta, \varphi)$ and $(\eta, r, \theta, \varphi)$, respectively, from (4.3) and (4.4) we find

$$\lambda_1^* = \lambda_2^* = \lambda_3^* = 0, \qquad \lambda_0^* = \lambda_0\, dx^0/d\eta = A\lambda_0, \qquad \lambda_0^* = \lambda_0^*(\eta). \tag{4.6}$$

In the coordinate system $(\eta, r, \theta, \varphi)$ from (4.1), (4.5), and (4.6) we obtain the following non-zero components Γ^i_{jk} and R^i_k:

$$\begin{aligned}
&\Gamma^0_{00} = \Gamma^1_{01} = \Gamma^2_{02} = \Gamma^3_{03} = \alpha, \qquad \alpha = \dot{A}/A - \lambda_0^*/2, \qquad \dot{A} \equiv dA/d\eta,\\
&\Gamma^0_{11} = \alpha/(1-Kr^2), \qquad \Gamma^0_{22} = \alpha r^2, \qquad \Gamma^0_{33} = \alpha r^2 \sin^2\theta,\\
&\Gamma^1_{11} = Kr/(1-Kr^2), \qquad \Gamma^1_{22} = -r(1-Kr^2), \qquad \Gamma^1_{33} = -r(1-Kr^2)\sin^2\theta,\\
&\Gamma^2_{12} = \Gamma^3_{13} = 1/r, \qquad \Gamma^2_{33} = -\sin\theta\cos\theta, \qquad \Gamma^3_{23} = \mathrm{ctg}\theta,
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
&\Lambda^i_k = 0, \qquad R^0_0 = -3A^{-2}\dot{\alpha},\\
&R^1_1 = R^2_2 = R^3_3 = -A^{-2}(2K + \dot{\alpha} + 2\alpha^2), \qquad \dot{\alpha} \equiv d\alpha/d\eta.
\end{aligned} \tag{4.8}$$

From the gravitational equations (4.1) and formulae (4.8) we find

$$\begin{aligned}
&3A^{-2}\dot{\alpha} = -(4\pi f/c^4)(\bar{T}^0_0 - 3\bar{T}^1_1), \qquad \bar{T}^1_1 = \bar{T}^2_2 = \bar{T}^3_3,\\
&A^{-2}(2K + \dot{\alpha} + 2\alpha^2) = (4\pi f/c^4)(\bar{T}^0_0 + \bar{T}^1_1).
\end{aligned} \tag{4.9}$$

Consider now the energy–momentum tensor $\bar{T}^i_k$ of the vacuum. Assuming that the vacuum is very rarefied, we can regard it as a system of noninteracting particles. Then to describe the vacuum energy–momentum tensor $\bar{T}^i_k$ we can apply formula (3.1) which gives

$$\bar{T}^i_k = c^2\bar{\rho}_0\, dx^i/ds\, dx_k/ds, \tag{4.10}$$

where $\bar{\rho}_0$ is the vacuum rest mass density.

It should be stressed that we do not consider the case of a non-zero cosmological constant [5] which could give an additional term in formula (4.10).

From (4.3), (1.7) and (3.14) we find

$$\Lambda^{ik} = 0, \qquad \bar{\rho}_0 = 0. \tag{4.11}$$

As follows from (4.10) and (4.11), the rest mass of vacuum particles is zero and they move at the speed of light.

From (4.10) and (4.11), we get

$$\bar{T} = \bar{T}^m_m = c^2 \bar{\rho}_0 = 0 \tag{4.12}$$

and from (4.9) and (4.12), we find

$$\bar{T}^1_1 = \bar{T}^2_2 = \bar{T}^3_3, \qquad \bar{T}^1_1 = -\bar{T}^0_0/3. \tag{4.13}$$

Substituting (4.13) into equations (4.9), we have

$$\dot{\alpha} = -\frac{8\pi f A^2}{3c^4}\bar{T}^0_0, \qquad \dot{\alpha} + 2(\alpha^2 + K) = \frac{8\pi f A^2}{3c^4}\bar{T}^0_0. \tag{4.14}$$

The summation of equations (4.14) gives

$$\dot{\alpha} + \alpha^2 + K = 0, \qquad K = \pm 1, 0, \qquad \dot{\alpha} \equiv \partial\alpha/\partial\eta. \tag{4.15}$$

From equation (4.15) we easily find the following two solutions in the case $K = -1$, one solution in the case $K = 1$ and one solution in the case $K = 0$:

$$\begin{aligned} &(1a) K = -1, \alpha = \mathrm{th}(\eta + \eta_0), \qquad (1b) K = -1, \alpha = \mathrm{cth}(\eta + \eta_0), \qquad \eta_0 = \mathrm{const}, \\ &(2) K = 1, \alpha = -\mathrm{tg}(\eta + \eta_0), \qquad (3) K = 0, \alpha = 1/(\eta + \eta_0). \end{aligned} \tag{4.16}$$

Let us represent the scale factor A in the form

$$A = A_0 \exp(\chi/2), \qquad \chi = \chi(\eta), \qquad A_0 = \mathrm{const} > 0. \tag{4.17}$$

Then from the expression for α in (4.7) we get

$$\alpha = \dot{A}/A - \lambda^*_0/2 = (\dot{\chi} - \lambda^*_0)/2. \tag{4.18}$$

As is seen from (4.16) and (4.14), in cases (1b) and (3) $\alpha = \infty$ and $\bar{T}_{00} = \infty$ when $\eta = -\eta_0$ and in case (2) $\alpha = \infty$ and $\bar{T}_{00} = \infty$ when $\eta = \pi\left(n + \frac{1}{2}\right) - \eta_0$, $n = 0, \pm 1, \pm 2, \ldots$. That is why we regard these cases as inadmissible.

Consider the only admissible case (1a). Then from (4.14), (4.16) and (4.18) we find

$$\alpha = \mathrm{th}(\eta + \eta_0), \qquad \bar{T}^0_0 = -\frac{3c^4}{8\pi f A^2}\mathrm{ch}^{-2}(\eta + \eta_0), \qquad \lambda^*_0 = -2\mathrm{th}(\eta + \eta_0) + \dot{\chi}, \tag{4.19}$$

where $\chi = \chi(\eta)$ is an arbitrary differentiable function.

When $\chi(\eta)$ is finite, from (4.19) and (4.17) we get that the component $\bar{T}^0_0$ of the vacuum energy–momentum tensor is negative and bounded.

Since in the considered case (1a) $K = -1$, from (4.5), (4.6), (4.17) and (4.19) we derive

$$\begin{aligned} &\mathrm{d}s^2 = (A_0)^2 \exp(\chi)[\mathrm{d}\eta^2 - \mathrm{d}r^2/(1 + r^2) - r^2(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\varphi^2)], \\ &A = A_0 \exp(\chi/2), \lambda^*_0 = -2\mathrm{th}(\eta + \eta_0) + \dot{\chi}, \qquad \lambda^*_1 = \lambda^*_2 = \lambda^*_3 = 0, \qquad K = -1. \end{aligned} \tag{4.20}$$

When the function $\chi(\eta)$ is differentiable and finite, expressions (4.20) give a nonsingular solution of the gravitational equations (4.1).

Let us now apply the gauge transformation (1.3) with $\phi = -\chi(\eta)$ to the metric and vacuum potentials (4.20). Then they reduce to the following expressions:

$$\begin{aligned} &\mathrm{d}s^2 = (A_0)^2[\mathrm{d}\eta^2 - \mathrm{d}r^2/(1 + r^2) - r^2(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\varphi^2)], \\ &A = A_0, \qquad \lambda^*_0 = -2\mathrm{th}(\eta + \eta_0), \qquad \lambda^*_1 = \lambda^*_2 = \lambda^*_3 = 0, \end{aligned} \tag{4.21}$$

where λ^*_i are the vacuum potentials in the coordinate system $(\eta, r, \theta, \varphi)$.

Choosing the time coordinate $x^0 = A_0\eta$ instead of η and using (4.6), we can represent (4.21) in the form

$$ds^2 = (dx^0)^2 - d\bar{r}^2/(1+\bar{r}^2/A_0^2) - \bar{r}^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \quad \bar{r} = A_0 r, \tag{4.22}$$

$$\lambda_0 = -(2/A_0)\,\mathrm{th}[(x^0+\beta)/A_0], \qquad x^0 = A_0\eta, \quad \beta = A_0\eta_0, \quad \lambda_1 = \lambda_2 = \lambda_3 = 0, \tag{4.23}$$

where λ_i are the vacuum potentials in the coordinate system $(x^0, r, \theta, \varphi)$ and $A_0 = \mathrm{const}$.

As stated at the beginning of this section, the extended reference frame under consideration consists of a big set of local inertial frames in which the examined vacuum region is homogeneous and isotropic.

Let us take this into account and note that in a small spatial region: $\bar{r} \ll A_0$ metric (4.22) coincides with the Minkowski metric, after choosing rectangular spatial coordinates. Then we find that metric (4.22), having no time-dependent multiplier, corresponds to the standard gauge used in the Einstein theory.

As to metric (4.20) with an arbitrary time-dependent multiplier $\exp(\chi)$, it is derived from metric (4.22) by using a gauge different from the standard one.

Thus, choosing the standard gauge, we come to metric (4.22) which corresponds to the Lobachevsky geometry of the physical vacuum with the constant curvature radius A_0.

Consider now a free movement of a material point in the physical vacuum along an abscissa x^1 of the Lobachevsky space. For this purpose let us use the kinematic equations (3.16) which, as well as formula (4.22), corresponds to the standard gauge of the metric (note that in section 3 the standard gauge (3.1) is used).

At an arbitrary moment x^0 let us choose a local rectangular coordinate system x^1, x^2, x^3 near the material point. In this local coordinate system the Lobachevsky geometry coincides with the Euclidean geometry and in the considered case of one-dimensional movement from (4.22) and (1.1) we find

$$ds^2 = \bar{g}_{ik}\, dx^i\, dx^k, \qquad \Gamma^i_{jk} = \tfrac{1}{2}(\lambda^i \bar{g}_{jk} - \lambda_j \delta^i_k - \lambda_k \delta^i_j), \qquad x^2 = x^3 = 0, \tag{4.24}$$

where $\bar{g}_{ik}$ is the Minkowski metric tensor.

Putting $k = 1, 2, 3$ in equations (3.16) and taking into account formulae (4.23) and (4.24), we get the following equations of a free movement of a material point along the axis x^1:

$$\begin{aligned} &d^2x^1/ds^2 - \lambda_0\, dx^0/ds\, dx^1/ds = 0, \qquad x^2 = x^3 = 0, \\ &ds^2 = (dx^0)^2 - (dx^1)^2, \qquad \lambda_0 = -(2/A_0)\,\mathrm{th}[(x^0+\beta)/A_0]. \end{aligned} \tag{4.25}$$

As stated above, from identity (3.17) we find that in the system of the four equations (3.16) the first equation ($k = 0$) is a consequence of the other three equations ($k = 1, 2, 3$). Therefore, in the considered case the first equation of (3.16) ($k = 0$) is a consequence of equations (4.25).

Let us introduce the time $\tau = (x^0+\beta)/c$. Then from (4.25) and (4.23) we derive

$$d\bar{V}/d\tau = c\lambda_0\bar{V}, \qquad \bar{V} \equiv V(1 - V^2/c^2)^{-1/2}, \qquad V \equiv dx^1/d\tau, \tag{4.26}$$

$$\begin{aligned} &\lambda_0 = -(2/A_0)\,\mathrm{th}(c\tau/A_0), \qquad \tau = (x^0+\beta)/c = (\eta+\eta_0)A_0/c, \\ &A_0 = \mathrm{const}, \qquad \beta = \mathrm{const}. \end{aligned} \tag{4.27}$$

As follows from (4.19) and (4.27), taking into account the equality for A in (4.21): $A = A_0$, the zero time $\tau = 0$ corresponds to the maximum absolute value of the vacuum energy density.

From (4.26) and (4.27) we easily get

$$\bar{V} \equiv V(1 - V^2/c^2)^{-1/2} = a_0 \exp\left(-(2c/A_0)\int \mathrm{th}(c\tau/A_0)\, d\tau\right) = b_0\, \mathrm{ch}^{-2}(c\tau/A_0), \tag{4.28}$$

where $a_0 = \mathrm{const}$, $b_0 = \mathrm{const}$.

Formula (4.28) gives

$$(1 - V^2/c^2)^{-1} = 1 + (b_0/c)^2 \operatorname{ch}^{-4}(c\tau/A_0), \qquad V \equiv \mathrm{d}x^1/\mathrm{d}\tau. \tag{4.29}$$

From (4.29) we find

$$E^2 = (m_0c^2)^2 + d_0 \operatorname{ch}^{-4}(c\tau/A_0), \qquad E = m_0c^2(1 - V^2/c^2)^{-1/2}, \quad d_0 = \text{const} \geqslant 0, \tag{4.30}$$

where E and m_0 are the energy and rest mass of the considered material point, respectively.

When $\tau \gg A_0/c$ and $m_0 > 0$ formula (4.30) gives

$$\begin{aligned} &E_{\text{kin}}(\tau) = \frac{8d_0}{m_0c^2} \exp(-4c\tau/A_0), \quad \tau \gg A_0/c, \\ &A_0 > 0, \quad m_0 > 0, \quad E_{\text{kin}} \equiv E - m_0c^2 = E_{\text{kin}}(\tau), \end{aligned} \tag{4.31}$$

where E_{kin} is the kinetic energy of the material point.

From (4.30) and (4.31) we have

$$\begin{aligned} &E_{\text{kin}}(0)/E_{\text{kin}}(\tau) = D_0 \exp(4c\tau/A_0), \qquad \tau \gg A_0/c, \qquad A_0 > 0, \qquad m_0 > 0, \\ &D_0 = (\sqrt{1 + \delta_0} - 1)/(8\delta_0), \qquad \delta_0 = d_0/(m_0c^2)^2. \end{aligned} \tag{4.32}$$

Let us apply formula (4.30) to a photon. Then we get

$$E = \sqrt{d_0} \operatorname{ch}^{-2}(c\tau/A_0), \qquad m_0 = 0. \tag{4.33}$$

When $\tau \gg A_0/c$, from (4.33) and (4.27) we derive

$$\begin{aligned} &E = 4\sqrt{d_0} \exp(-2c\tau/A_0), \qquad \nu = \nu_0 \exp(-2l/A_0), \\ &\lambda_0 = -2/A_0, \quad A_0 > 0, \qquad \tau \gg A_0/c, \end{aligned} \tag{4.34}$$

where $\nu = \nu(l)$ is the photon frequency, $\nu_0 = \nu(0)$ and l is the distance gone by the photon.

When $l \ll A_0$, from (4.34) we get the formula of the redshift z of the photon frequency

$$z = (\nu_0 - \nu)/\nu = 2l/A_0, \quad l \ll A_0, \quad \tau \gg A_0/c, \tag{4.35}$$

where A_0 presents the space curvature radius of the universe.

As is well known [5], the redshift z in the spectrum of a galaxy is proportional to the distance l to it, when $z \ll 1$. At the same time, formula (4.35) also signifies that the values z and l are proportional and, besides, it describes a cosmological law, sinceA_0 is the radius of the universe.

Therefore, formula (4.35) could give a new explanation of the redshift in the spectra of galaxies without presuming their dispersion: the redshift could be interpreted as the result of a vacuum field influence.

It is interesting to note that formula (4.34) for the function $\nu(l)$ was earlier obtained in [6] by investigating a photon wavefunction covariant in the Lobachevsky space.

Let us now apply equations (4.25) in the non-relativistic case. Then for a free movement in the vacuum of a material point along the axis x^1 from (4.25) and (4.27) we get

$$\begin{aligned} &\mathrm{d}^2x^1/\mathrm{d}\tau^2 = c\lambda_0 \,\mathrm{d}x^1/\mathrm{d}\tau, \qquad x^2 = x^3 = 0, \qquad |\mathrm{d}x^1/\mathrm{d}\tau| \ll c, \\ &\tau = (x^0 + \beta)/c, \qquad \lambda_0 = -(2/A_0) \operatorname{th}(c\tau/A_0). \end{aligned} \tag{4.36}$$

From formulae (4.36) we find that the following small force $\mathbf{F}_{\text{vac}}$ acts on the considered material point which is caused by the influence of the vacuum vectorial field:

$$\mathbf{F}_{\text{vac}} = \lambda_0 c m_0 \mathbf{V}, \qquad \lambda_0 = -(2/A_0) \operatorname{th}(c\tau/A_0), \tag{4.37}$$

where m_0 and $\mathbf{V}$ are the mass at rest and the three-dimensional vector of the velocity of the material point, respectively, and A_0 is the space curvature radius of the universe.

When the time $\tau < 0$, this small force accelerates the material point and when $\tau > 0$ the force decelerates it.

It should be stressed that the vector $\mathbf{V}$ presents the velocity of the material point relative to the considered reference frame in which the physical vacuum is homogeneous and isotropic.

From (4.37) we get the following asymptotic values of the force $\mathbf{F}_{\mathrm{vac}}$:

$$\mathbf{F}_{\mathrm{vac}} = (2c/A_0)m_0\mathbf{V}, \qquad \tau \to -\infty, \tag{4.38}$$

$$\mathbf{F}_{\mathrm{vac}} = -(2c/A_0)m_0\mathbf{V}, \qquad \tau \to +\infty, \quad A_0 = \mathrm{const} > 0. \tag{4.39}$$

As follows from (4.37), the asymptotic formulae (4.38) and (4.39) correspond to the stationary values $\lambda_0 = 2/A_0$ and $\lambda_0 = -2/A_0$ of the vacuum potential λ_0, respectively.

The following should be noted. As is shown in [6], formulae (4.38) and (4.39) can be obtained by accepting the following axiom of a free translation of a solid body in the Lobachevsky space: the points of the body should move in straight lines and the body line segments should remain straight [6].

5. Conclusions

In this paper we have proposed the generalized gravitational equations (1.5) with the Weyl connection depending on the metric tensor and four vacuum potentials. The field equations (1.5) are covariant, gauge-invariant and of second order in the derivatives of the metric. From them, using the Bianchi identities, we derived the differential correlation (2.21) for the energy–momentum tensor of matter T^{ik}. In the case of dust-like matter, from correlation (2.21) and identity (3.10) we found the four differential equations (3.14) of second order for the vacuum potentials λ_i consistent with the gravitational equations (1.5). The differential correlation (2.21) and equations (3.14) allowed us to find the differential equations (3.16) describing the kinematics of dust-like matter in a gravitational field.

It is worth noting that the gravitational differential equations (1.5), which are of second order, can be reduced to the Einstein equations (1.10) with an additional energy–momentum tensor corresponding to the vacuum vectorial field. As is seen from equation (1.12), the proposed gravitational equations (1.5) give the conservation law for the resulting energy–momentum tensor of matter including the energy–momentum components of the vacuum vectorial field.

The proposed gravitational equations (1.5) were applied to cosmological problems and their nonsingular cosmological solution for the homogeneous and isotropic physical vacuum was found. This solution is described by metric (4.20) containing an arbitrary time-dependent function χ. This function depends on the choice of a gauge for the metric tensor and the choice of the standard gauge gives metric (4.22) corresponding to the Lobachevsky space geometry with the constant curvature radius $A = A_0$.

As follows from formulae (4.19) and (4.23), in this solution the vacuum energy density $\bar{T}_0^0$ and the vacuum potential λ_0 depend on the time.

Hence, the obtained cosmological solution describes a nonstationary universe.

The kinematic equations (3.16) and formula (4.23) for the vacuum potential λ_0 above were applied to the movement of a photon. As a result, we came to formula (4.35) for the redshift of a photon frequency and associated it with the redshift in the spectra of galaxies.

It should be stressed that formula (4.35) is valid when the cosmological time $\tau \gg A_0/c$. That is why we assume that the cosmological time $\tau = \tau_0$ corresponding to the present epoch satisfies the correlation $\tau_0 \gg A_0/c$.

It is worth noting that, as follows from (4.19), (4.27), and the equality for A in (4.21): $A = A_0$, the vacuum energy density $\bar{T}_0^0$ and vacuum potential λ_0 are finite at any time and hence the cosmological solution obtained is applicable to arbitrary values of the time coordinate τ: $-\infty < \tau < +\infty$. In this solution the absolute value of the vacuum energy density reaches its maximum at the cosmological time $\tau = 0$ and tends to zero as the time $\tau \to \pm\infty$.

From (4.27) we find that the vacuum potential $\lambda_0 = \lambda_0(\tau)$ is a decreasing function and

$$\lambda_0(-\infty) = 2/A_0, \quad \lambda_0(0) = 0, \quad \lambda_0(+\infty) = -2/A_0, \quad A_0 > 0.$$

Thus, the proposed cosmological model has no singularity and is applicable to both positive and negative values of the cosmological time.

It should be noted that in our model the physical vacuum influences the energies of particles, but their rest masses do not change (see equation (3.2) of rest mass conservation). This is an essential difference between our model and the Hoyle–Narlikar cosmology which is also applicable to negative values of cosmological time. In the Hoyle–Narlikar model, in contrast with our model, the rest masses of electrons and nucleons change with time (see page 416 in [7]).

Let us now examine the thermal history of the universe. As follows from formula (4.30), due to an influence of the vacuum field, the kinetic energy $E_{\mathrm{kin}} \equiv E - m_0c^2$ of a free particle increases when the cosmological time $\tau < 0$ and reaches its maximum at the time $\tau = 0$. When $\tau > 0$ the kinetic energy E_{kin} decreases and tends to zero as $\tau \to +\infty$.

Consider time $\tau = \tau_0$ corresponding to the present epoch. As has been assumed above, $\tau_0 \gg A_0/c$. From formula (4.31) we find that the kinetic energy $E_{\mathrm{kin}}(\tau)$ of a free particle with a non-zero rest mass decreases exponentially when $\tau \gg A_0/c$ and from formula (4.32) we find that at the time $\tau_0 \gg A_0/c$, $E_{\mathrm{kin}}(0) \gg E_{\mathrm{kin}}(\tau_0)$.

Therefore, at the moment $\tau = 0$ the kinetic energies of particles inside cosmic bodies must have taken very large values and hence the temperatures of the cosmic bodies must have been very high, as compared with those at the present epoch. That is why the epoch near time $\tau = 0$ could be identified with the stage of the early universe of the standard cosmology [5].

Thus, we come to the conclusion that the early universe, for which time τ is near zero, was very hot, similar to the standard cosmology. Hence, the well-known theoretical results of the standard cosmology for the early universe concerning the primordial nucleosynthesis and cosmic microwave background radiation [5] could be valid in our conception.

As follows from formula (4.34), in our model the temperature of the cosmic microwave background radiation decreases exponentially when time $\tau \gg A_0/c$.

Let us now apply the proposed cosmological model to unsolved cosmological problems of the evolution and structure of spiral galaxies.

First, as follows from formula (4.37), when $\tau > 0$ small decelerating forces act on the stars of a galaxy. That is why the stars rotating about the centre of a galaxy have to move in spiral orbits slowly approaching the galaxy centre.

Therefore, formula (4.37) may explain the visible spiral structure of many galaxies [8].

Second, old stars approaching the galaxy centre for a sufficiently long time could be near the centre. Hence, the proposed model allows one to explain the well-known fact that the galaxy central condensation is mostly composed of old stars (population II stars), whereas the galaxy spiral arms contain a large number of young stars (population I stars) (see page 296 in [8]).

Besides, since in our model the stars of the spiral arms of a galaxy approach its centre, the earlier the spiral arms of a galaxy were formed in the past, the closer they are at present to the galaxy centre.

It should be noted that this conclusion corresponds with observational data. Namely, in passing from subclass **c** of the spiral galaxies to subclass **b** and then to **a**, we observe an increasing percentage of old stars and, at the same time, a decreasing spreading of the spiral arms (see page 303 in [8]).

Let us now compare the proposed gravitational equations with vacuum potentials with the Einstein equations containing a time-dependent cosmological constant Λ (see page 267 in [5]). Instead of the standard cosmological term Λ, equations (4.9) contain our terms $(\lambda_0^*/A)\,\mathrm{d}A/\mathrm{d}\eta$, $(\lambda_0^*)^2$, and $\mathrm{d}\lambda_0^*/\mathrm{d}\eta$, taking into account the expression for α in (4.7).

Thus, there are essential differences between the proposed gravitational equations and the Einstein equations with a time-dependent cosmological constant.

References

[1] Weyl H 1918 *Sitzungsber. d. Berl. Acad.* 465

[2] Alonso J C, Barbero F, Julve J and Tiemblo A 1994 *Class. Quantum Grav.* **11** 865

[3] Landau L D and Lifshitz E M 1971 *The Classical Theory of Fields* (Oxford: Pergamon)

[4] Rabinowitch A S 1998 *Int. J. Theor. Phys.* **37** 1477

[5] Kolb E W and Turner M S 1990 *The Early Universe* (New York: Addison-Wesley)

[6] Rabinowitch A S 1991 *Int. J. Theor. Phys.* **30** 521

[7] Hoyle F and Narlikar J 1980 *The Physics—Astronomy Frontier* (San Francisco: Freeman)

[8] Oster L 1973 *Modern Astronomy* (San Francisco: Holden-Day)