

# CURRICULUM VITAE OF ALEXANDER S. TIKHOMIROV

**NAME:** TIKHOMIROV Alexander Sergeevich

**DATE AND PLACE OF BIRTH:**

May 22, 1950, Yaroslavl, RUSSIA

**PERMANENT ADDRESS:**

Department of Mathematics  
National Research University  
Higher School of Economics  
7 Vavilova str.  
117312 Moscow, RUSSIA

**HOME ADDRESS:**

Tolbuhina str. 26, apt.58,  
Yaroslavl 1500014, RUSSIA

**EDUCATION:**

**1965-1967:** pupil of Moscow specialized physico-mathematical boarding school No. 18 ("Kolmogorov's internat")

**1967-1972:** student of Department of Applied Mathematics and Control at Moscow Physiko-Technical Institute.

**1973-1975:** graduate student at Yaroslavl State Pedagogical University.  
Research adviser: Professor Z.A.Skopets.

**PhD** May 25, **1975** (Yaroslavl, YSPU)

PhD thesis: *"Questions of line geometry in the 4-dimensional projective space and their interpretation on the grassmannian variety"*

**Doctor of Phys.-Math. Sci.** April 6, **1986:** (St. Petersburg State Univ.)

Doctoral thesis: *"Intermediate Jacobian of Fano threefolds of index two"*

## EXPERIENCE

**1975-1982:** assistant professor at Yaroslavl State Pedagogical University (YSPU)

**1982-1987:** associate professor at YSPU

**1984-2013:** chairman of the chair of algebra at YSPU

**1991-2000:** chairman of the PhD Council on Mathematical Logic, Algebra and Number Theory at USPU

**1987-2014:** professor at YSPU

Since **2014:** professor at National Research University Higher School of Economics (NRU HSE), Moscow

Since **1981** the following my graduate students defended Ph.D.: D.Logachev (1981), V.Alexeyev (1985), V.Shenderovky (1988), D.Kuznetsov (1989), T.Troshina (1993), N.Tyurin (1996), N.Timofeeva (1999), D.Artamkin (2004), M.Sorokina (2006), T.Matytsina (2007), A.Uvarov (2012), M.Zavodchikov (2012), S.Ermakova (2015).

## RESEARCH FIELD:

**Algebraic Geometry, relations to Gauge Theory and Differential Geometry**

## PROFESSIONAL ACTIVITIES

- June-July **1989**: Univ. of Chicago, USA-USSR Symposium on Algebraic Geometry
- December **1990**-April **1991**: Univ. of Erlangen-Nurnberg, Visiting Researcher, DFG
- March **1991**: Max Planck Institute for Math., Bonn, Workshop on Algebraic Geometry
- February-April **1992**: Univ. of Chicago, Visiting Professor
- August **1992**: YSPU, Yaroslavl, International Conference on Algebraic Geometry
- December **1992**: Univ. of Kaiserslautern, Visiting Researcher, DFG
- January-March **1994**: Univ. of Michigan, Visiting Professor
- June-July **1994**: Univ. of Kaiserslautern, Visiting Researcher, DFG
- August **1994**: YSPU, Yaroslavl, International Conference on Algebraic Geometry
- January-March **1995**: Univ. of Michigan, Visiting Professor
- June **1995**: A.Poincare Institute of Math., Paris, Workshop on Vector Bundles
- February-May **1996**: Johns Hopkins Univ., Baltimore, Visiting Professor
- April **1996**: Johns Hopkins Univ., Conference on Birational Geometry
- June-July **1996**: University of Bielefeld, Visiting Researcher, SFB
- June **1997**: Univ. of Lille, Conference on Algebraic Geometry
- October **1998**-March **1999**: Max Planck Institute for Math., Bonn, Visiting Researcher
- June **1999**: Univ. of Lille, Conference "Singularities in Algebraic Geometry", Invited Lecturer
- June-July **1999**: Math. Institute Oberwolfach, Visiting Researcher
- April-July **2000**: Univ. of Kaiserslautern, Visiting Professor
- September-November **2000**: Univ. of Lille, Visiting Researcher, CNRS
- December **2000**-February **2001**: Max Planck Institute for Math., Bonn, Visiting Researcher
- June **2001**: Steklov Math. Institute, Moscow, Conference on Monodromy in Topology and Algebraic Geometry
- June-July **2001**: Math. Institute Oberwolfach, Visiting Researcher
- October-December **2002**: Max Planck Institute for Math., Bonn, Visiting Researcher
- June **2003**: Univ. of Lille, Conference on Singularities in Algebraic Geometry, Invited Speaker
- May-July, September **2004**: Univ. of Kaiserslautern, Visiting Professor
- November **2004**-January **2005**: Univ. of Lille, Visiting Researcher, CNRS
- February-April **2005**: Max Planck Institute for Math., Bonn, Visiting Researcher
- May **2005**: Univ. of Kaiserslautern, Visiting Professor
- June **2005**: Univ. of Lille, Conference on Geometry of Moduli Spaces
- March-April **2006**: Univ. of Kaiserslautern, Visiting Professor
- June **2005**: Univ. of Lille, Visiting Professor
- February 20, **2007**: Max Planck Institute for Math., talk at Seminar on Algebra, Geometry and Physics
- February-March **2007**: Jacobs Univ. of Bremen, Visiting Professor
- March **2007**: Univ. of Kaiserslautern, Visiting Professor

- December **2007**: Univ. of Lille, Visiting Professor
- April **2008**: Jacobs Univ. of Bremen, Visiting Professor
- January-March **2008**: Max Planck Institute for Math., Bonn, Visiting Researcher
- May **2008**: Steklov Math. Institute, Moscow, and YSPU, Yaroslavl, Conference and Summer School "Algebraic Geometry", co-organizer
- June **2008**: Univ. of Lille, Visiting Professor
- February **2009**: Jacobs Univ. of Bremen, Visiting Professor
- March **2009**: Univ. of Kaiserslautern, Visiting Professor
- May **2009**: Steklov Math. Institute, Moscow, and YSPU, Yaroslavl, Conference and Summer School "Algebraic Geometry and Complex Analysis", Co-organizer
- June **2009**: Univ of Lille, Conference "Holomorphically Symplectic Varieties and Moduli Spaces", Invited Speaker
- February 23, **2010**: Max Planck Institute for Math., talk at Seminar on Algebra, Geometry and Physics
- March **2010**: Univ. of Kaiserslautern, Visiting Professor
- April-May **2010**: SISSA, Trieste, Visiting Researcher
- May **2010**: A.Poincare Institute of Math., Paris, Conference "Derived Categories, Holomorphic Symplectic Geometry, Birational Geometry, Deformation Theory"
- June-July **2010**: Univ. of Lille, Visiting Professor
- June-July **2010**: Math. Institute Oberwolfach, Visiting Researcher
- November **2010**: Freie Univ. Berlin, Conference "Instantons and Rationality of Moduli Spaces", Invited Lecturer
- December **2010**: Univ. of Edinburgh, Conference "Birational Geometry", Invited Speaker
- January-March **2011**: Max Planck Institute for Math., Bonn, Visiting Researcher
- March **2011**: Steklov Math. Institute and HSE, Moscow, Conference "Instantons in Complex Geometry", Co-organizer
- April **2011**: Jacobs Univ. of Bremen, Visiting Professor
- May **2011**: Steklov Math. Institute, Moscow, and YSPU, Yaroslavl, Conference and Summer School "Algebraic Geometry and Complex Analysis", Co-organizer
- July **2011**: HSE, Moscow, and YSPU, Yaroslavl, Summer School "Algebra and Geometry", Co-organizer
- October **2011**: CIRM, Luminy, Conference "Automorphic Forms and Moduli Spaces"
- November **2011**: Jacobs Univ. of Bremen, Visiting Professor
- December **2011**: S.Stoilov Institute of Mathematics, Bucharest, Visiting Professor
- January-March **2012**: SISSA, Trieste, Visiting Professor
- May-July **2012**: Univ. of Lille, Visiting CNRS Researcher
- May **2012**: Univ. of Nantes, Conference "Geometry of Kähler manifolds and Symplectic Geometry", Invited Speaker
- June **2012**: Univ. of Grenoble "Holomorphic Symplectic Manifolds and Moduli Spaces", Invited Speaker
- September-November **2012**: HSE, Moscow, Visiting Professor

July **2012**: HSE, Moscow, and YSPU, Yaroslavl, Summer School "Algebra and Geometry", Co-organizer

December **2012**: Max Planck Institute for Math., Bonn, talk at Seminar on Algebra, Geometry and Physics

December **2012**: Jacobs Univ. of Bremen, Visiting Professor

May **2011**: Steklov Math. Institute, Moscow, and YSPU, Yaroslavl, Conference and Summer School "Algebraic Geometry and Complex Analysis", Co-organizer

June **2013**: SISSA, Trieste, Workshop "Vector Bundles on Algebraic curves: Hilbert Schemes, Sheaves and Representations", Invited Speaker

July **2013**: HSE, Moscow, and YSPU, Yaroslavl, Summer School "Algebra and Geometry", Co-organizer

September-November **2013**: Jacobs Univ. of Bremen, Visiting Professor

January-March **2014**: Max Planck Institute for Math., Bonn, Visiting Researcher

March **2014**: Univ of Lille, Conference "Moduli spaces of irreducible symplectic varieties, cubics and Enriques surfaces", Invited Speaker

May-June **2014**: SISSA, Trieste, Visiting Professor

July **2014**: HSE, Moscow, and YSPU, Yaroslavl, Summer School "Algebra and Geometry", Co-organizer

September **2014**: Univ. of Lille, Visiting Professor

September 16, **2014**: Univ. of Lille, Talk "Moduli of symplectic instantons" at Seminar on Algebraic Geometry

November 20, **2014**: Steklov Math. Institute, Moscow, Talk "Geometry of moduli spaces of mathematical and symplectic instantons" at V.A.Iskovskikh Seminar on Algebraic Geometry

November 20, **2014**: Steklov Math. Institute, Moscow, Talk "New divisors in the boundary of the instanton moduli space" at I.R.Shafarevich Seminar on Algebraic Geometry

March 20, **2015**: NRU HSE, Moscow, Talk "On the Barth-Van de Ven-Tyurin-Sato theorem" at Bogomolov Laboratory on Algebraic Geometry

April-June **2015**: Univ. of Campinas, SP, Brazil, Visiting Professor

April 6-10, **2015**: IMPA, Rio de Janeiro, Workshop "Moduli Spaces and Enumerative Geometry"

April 22, **2015**: Univ. of Campinas, Talk "On the Barth-Van de Ven-Tyurin-Sato Theorem" at Algebraic Geometry Seminar

May 18-22, **2015**: IMPA, Rio de Janeiro, Workshop "Higher Dimensional Algebraic Geometry"

May 23, **2015**: Univ. of Campinas, Talk "Geometry of the moduli space of symplectic instantons" at Algebraic Geometry Seminar

June 17, **2015**: Univ. of Campinas, Talk "New divisors in the boundary of the instanton moduli space" at Algebraic Geometry Seminar

June 22-24, **2015**: Univ. of Campinas, Workshop "Poisson geometry and related topics"

August 17-21, **2015**: Guarujá - São Paulo, Brazil, Conference "Interactions between Geometry and Physics", Invited Speaker, Talk "New components of the moduli space of rank two semistable sheaves on projective space"

October 26-30, **2015**: CIRM, Luminy, France, Workshop "Moduli Spaces in Geometry", Invited Speaker, Talk (joint with M.Jardim and D.Markushevich) "Torsion free sheaves with zero dimensional singularities".

March 15-April 14 **2016**: Jacobs Univ. of Bremen, Visiting Professor

March 29, **2016**: Max Planck Institute for Math., Bonn, talk at Seminar on Algebra, Geometry and Physics

May-June **2016**: SISSA, Trieste, Visiting Professor. Lecture course at SISSA "Geometric Invariant Theory"

October 26 **2016**: Steklov Math. Institute, Moscow, Annual Memorial Conference of A.N.Tyurin, Invited Speaker, talk "Infinite series of rational components of the moduli space of stable rank 2 vector bundles on projective space"

January-February **2017**: Max Planck Institute for Math., Bonn, Visiting Researcher

January 24, **2017**: Max Planck Institute for Math., Bonn, talk at Seminar on Algebra, Geometry and Physics

January 26 **2017**: Univ. of Kaiserslautern, talk "On the moduli spaces of stable rank two coherent sheaves on projective space" at Seminar on Algebra, Geometry und Computer Algebra,

February 7, **2017**: Univ. of Lille, talk "Infinite series of rational moduli components of stable rank two vector bundles on projective space" at Algebraic Geometry Seminar,

August 25-30, **2017**: Northern (Arctic) Federal University, Korjashma branch, VI School-conference on algebraic geometry and complex analysis for young mathematicians of Russia, Invited Speaker, talk "New constructions of stable rank two vector bundles on projective space"

July 26-31, **2018**: University of Campinas (Brazil), Satellite conference of the ICM 2018 "Moduli Spaces in Algebraic Geometry and Applications", Invited Speaker, talk "Geometry and geometry of the moduli spaces of semi-stable rank 2 sheaves on projective space"

## LIST OF MAIN PUBLICATIONS OF ALEXANDER S. TIKHOMIROV

1. A.S.Tikhomirov. *The four-dimensional net of quadrics and the monad*. Izvestiya AN SSSR. Ser. mat. v. **44**, No. 1 (1980), 219–232.
2. A.S.Tikhomirov. *The geometry of the Fano surface of the double space  $P^3$  ramified in a quartic*. Izvestiya AN SSSR. Ser. mat. (1980) v. **44**, No. 2, 415–442.
3. A.S.Tikhomirov. *The Fano surface of the double Veronese cone*. Izvestiya AN SSSR. Ser. mat. (1980) v. **45**, No. 5, 1121–1197.
4. A.S.Tikhomirov. *The family of sextics of genus 3 on double spaces  $P^3$  of index two and its Abel-Jacobi mapping*. VINITI AN SSSR, No. 401-85 Dep., 1984, 1–148.
5. A.S.Tikhomirov. *The Abel-Jacobi mapping of sextics of genus 3 on double spaces  $P^3$  of index two*. Doklady AN SSSR v. **286**, No. 4 (1986), 821–824.
6. A.S.Tikhomirov. *Standard bundles on a Hilbert scheme of points on a surface*. In: Algebraic Geometry and its Applications (Proceedings of 8th Alg. Geom. Conf., Yaroslavl 1992). Aspects of Math., Vieweg, Vol. E 26, 1994, 183–204.
7. A.S.Tikhomirov, T.L.Troshina. *Top Segre classes of a standard vector bundle  $E_D^4$  on a Hilbert scheme  $Hilb^4 S$  of a surface  $S$* . In: Algebraic Geometry and its Applications (Proceedings of 8th Alg. Geom. Conf., Yaroslavl 1992). Aspects of Math., Vieweg, Vol. E 26, 1994, 205–227.
8. A.S.Tikhomirov. *A smooth model for the punctual Hilbert scheme of a surface*. In: Proceedings of the Steklov Inst. of Math., Vol. 208, 1995, 318–334.
9. A.S.Tikhomirov, A.N.Tyurin. *Application of the geometric approximation procedure to computing the Donaldson's polynomials for  $\mathbb{C}P^2$* . Mathematica Göttingensis, Sonderforschungsbereichs "Geometrie und Analysis", Heft 12 (1994), 1–71.
10. A.S.Tikhomirov. *The variety of complete pairs of zero-dimensional subschemes of an algebraic surface*. Izvestiya: Mathematics **61**:6 (1997), 1265–1291.
11. D.Markushevich, A.S.Tikhomirov. *The Abel-Jacobi map of a moduli component of vector bundles on the cubic threefold*. J. Algebraic Geometry **10** (2001), 37–62.
12. A.S.Tikhomirov. *The main component of the moduli space of mathematical instanton vector bundles on  $P^3$* . Journal of Math. Sciences. Vol. 86 (1997), 3004-3087.
13. A.S.Tikhomirov. *Barth map of the moduli space of stable rank-2 vector bundles on  $P^2$* . MPIM Preprint 99-9 (1999), 1–21.
14. A.S.Tikhomirov, T.L.Troshina. *Birational and enumerative geometry of the variety of complete pairs of double points of algebraic surface*. Math. Zametki **65**, No. 3 (1999), 412–419.
15. J.Le Potier, A.S.Tikhomirov. *Sur le morphisme de Barth*. Ann. Scient. Ec. Norm. Sup., 4<sup>e</sup> serie, t. **34** (2001), 523–629.
16. D.Markushevich, A.S.Tikhomirov. *Symplectic structure on a moduli space of sheaves on the cubic fourfold*. Izvestiya RAN, Ser. Mat. **67**, No.1 (2003), 131–158.
17. I.Coanda, A.Tikhomirov, G.Trautmann. *Irreducibility and smoothness of the moduli space of mathematical 5-instantons over  $P^3$* . Intern. J. Math., **14**, No.1 (2003), 1–45.
18. A.S.Tikhomirov. *New component of the moduli space  $M(2; 0, 3)$  of stable vector bundles on the double space  $P^3$  of index two*. Acta Appl. Math. **75** (2003), 271–279.
19. D.Markushevich, A.S.Tikhomirov. *A parametrization of the theta divisor of the quartic double solid*. Intern. Math. Res. Notes **51** (2003), 2747–2778.
20. A.S.Tikhomirov. *On birational transformations of Hilbert schemes of an algebraic surface*. Mathem. Notes, **73**, No. 2 (2003), 259–270.

21. A.S.Tikhomirov, S.A.Tikhomirov. *On the variety of complete punctual flags of length 5 in dimension 2*. In: Proceedings of the Steklov Inst. of Math., Vol. 246, 2004, 263–269.
22. D.Markushevich, A.S. Tikhomirov. *New symplectic V-manifolds of dimension four via the relative compactified Prymian*. Intern. J. Math., Vol. **18**, No. 10 (2007), 1187–1224.
23. I.Penkov, A.S.Tikhomirov. *Rank-2 vector bundles on ind-Grassmannians*. Algebra, Arithmetic, and Geometry: in honor of Yu. I. Manin, vol. II, Progr. Math., vol. 270, Birkhäuser, Boston-Basel-Berlin 2009, pp. 555–572.
24. I.B.Penkov and A.S.Tikhomirov. *Triviality of vector bundles on twisted ind-Grassmannians*. Sbornik:Mathematics **202**, No.1 (2011), 61–99.
25. A.S.Tikhomirov. *Moduli of mathematical instanton vector bundles with odd  $c_2$  on projective space*. Izvestiya: Mathematics **76**:5 (2012), 991–1073.
26. U. Bruzzo, D. Markushevich, A. S. Tikhomirov. *Moduli of symplectic instanton vector bundles of higher rank on projective space  $P^3$* . Cent. Eur. J. Math., **10**, No.4 (2012), 1232–1245.
27. D. Markushevich, A. S. Tikhomirov, G. Trautmann. *Bubble tree compactification of moduli spaces of vector bundles on surfaces*. Cent. Eur. J. Math., **10**, No.4 (2012), 1331-1355.
28. A.S.Tikhomirov. *Moduli of mathematical instanton vector bundles with even  $c_2$  on projective space*. Izvestiya: Mathematics **77**:6 (2013), 1195–1223.
29. U.Bruzzo, D.Markushevich, A.Tikhomirov. *Uhlenbeck-Donaldson compactification for framed sheaves on projective surfaces*. Math. Zeitschrift **275**, Issue 3-4 (2013), 1073–1093.
30. I.Penkov and A.S.Tikhomirov. *Linear ind-grassmannians*. Pure and Applied Mathematics Quarterly, vol. **10**:2 (2014), 289–323.
31. I. Penkov, A. S. Tikhomirov. *On the Barth-Van de Ven-Tyurin-Sato theorem*. Sbornik: Mathematics. **206**:6 (2015), 814–848.
32. V.Guletskii and A.Tikhomirov. *Algebraic Cycles on Quadric Sections of Cubics in  $\mathbb{P}^4$  under the Action of Symplectomorphisms*. Proceedings of the Edinburgh Mathematical Society (Ser. 2), **59**:02 (2016), 377-392.
33. U.Bruzzo, D.G.Markushevich, A.S.Tikhomirov. *Symplectic instanton bundles on  $\mathbb{P}^3$  and t Hooft instantons*. Eur. J. Math. **2**:1 (2016), 73-86.
34. M.Jardim, D.Markushevich, and A.S.Tikhomirov. *Two infinite series of moduli spaces of rank 2 sheaves on  $\mathbb{P}^3$* . Annali di Mat. Pura ed Appl. **196**:4 (2017), 1573-1608.
35. M.Jardim, M.Maican, and A.Tikhomirov. *Moduli spaces of rank 2 instanton sheaves on the projective space*. Pacific J. Math. **291**:2 (2017), 399-424.
36. A.N.Ivanov, A.S.Tikhomirov. *Moduli component of the space of semistable rank 2 sheaves on  $\mathbb{P}^3$  with singularities of mixed dimension*. Doklady Mathematics. **96**:2 (2017), 506-509.
37. A.Kytmanov, A.Tikhomirov, S.Tikhomirov *Series of rational moduli components of stable rank 2 vector bundles on  $\mathbb{P}^3$* . MPIM Preprint No. 2017-16, pp. 1-31.
38. C.Almeida, M.Jardim, A.Tikhomirov, S.Tikhomirov. *New moduli components of rank 2 bundles on projective space*. MPIM Preprint No. 2017-14, pp. 1-36.
39. M.Jardim , D.Markushevich, A.S.Tikhomirov. *New divisors in the boundary of the instanton moduli space*. Moscow Mathematical Journal. **18**:1 , 117-148.
40. A.N.Ivanov, A.S.Tikhomirov. *Semistable rank 2 sheaves with singularities of mixed dimension on  $\mathbb{P}^3$* . Journal of Geometry and Physics. **129** (2018), 90-98.

## RESEARCH PROJECT OF ALEXANDER S. TIKHOMIROV

### 1. Moduli of stable vector bundles and instanton sheaves on projective space $\mathbb{P}^3$ .

We study the moduli space  $M(n)$  of rank 2 stable vector bundles on projective space  $\mathbb{P}^3$  with  $c_1 = 0$  and  $c_2 = n \geq 1$ . In 1988 L.Ein found a series  $X_1, \dots, X_{d(n)}$  of irreducible components of the space  $M(n)$ . The importance of these Ein's components is due to the fact that their number  $d(n)$  is unbounded as  $n$  grows - a new phenomenon which has no analogues in smaller dimensions. (Namely, all the known moduli spaces of vector bundles of given rank and fixed Chern classes on projective curves and surfaces are irreducible.) However, at present no results about the inner geometry of Ein's components (e.g. their dimensions, spectra, problem of rationality etc.) are known. We elaborate a precise construction of rationality of Ein's components of  $M(n)$ . As a by-product of our approach we will obtain formulas for dimensions and spectra of Ein's components. This is a joint project with A.Kytmanov and S.Tikhomirov.

Our next interest is a description of certain families of (semi)stable rank 2 non-locally free sheaves on  $\mathbb{P}^3$  belonging to the Gieseker-Maruyama moduli space  $\mathcal{M}(n)$  of semistable rank 2 sheaves with  $c_1 = c_3 = 0$  and  $c_2 = n \geq 1$ . Among these sheaves we distinguish the so-called instanton sheaves which are those sheaves  $E$  which satisfy the instanton conditions  $h^0(E(-1)) = h^1(E(-2)) = h^2(E(-2)) = h^3(E(-3)) = 0$ . (These are just the same conditions as those satisfied by usual rank 2 mathematical instanton vector bundles on  $\mathbb{P}^3$ .) Recently (2016) in our joint work with M.Jardim and D.Markushevich (arXiv:1604.01605v1 [math.AG]) we found a big connected family of irreducible instanton components of  $\mathcal{M}(n)$ , i.e. components having instanton sheaves as their generic points. However, a complete description of irreducible instanton components of  $\mathcal{M}(n)$  is not known even for small values of  $n$ . The only known is the smallest nontrivial case  $n = 2$  (Le Potier 1993). So the cases  $n = 3$  and 4 are now in order. Our aim is to give a complete list of instanton components of  $\mathcal{M}(3)$  and  $\mathcal{M}(4)$  and to prove the connectedness of their union joined with the closure in  $\mathcal{M}(n)$  of the moduli of mathematical instanton vector bundles. This is a joint project with M.Jardim and M.Maican.

### 2. Moduli of higher rank symplectic instantons on $\mathbb{P}^3$ .

We continue the study of higher rank symplectic instanton vector bundles on  $\mathbb{P}^3$ . By a symplectic  $(n, r)$ -instanton of rank  $2r$  and charge  $n$  on  $\mathbb{P}^3$  one understands an algebraic vector bundle  $E$  of rank  $2r$  on  $\mathbb{P}^3$  with Chern classes  $c_1(E) = c_3(E) = 0$ ,  $c_2(E) = n$ ,  $n \geq r \geq 1$ , endowed with a symplectic structure  $\varphi : E \xrightarrow{\sim} E^\vee$ ,  $\varphi^\vee = -\varphi$ , and satisfying the instanton vanishing conditions  $h^0(E) = h^1(E(-2)) = 0$ . Let  $I_{n,r}$  be the moduli space of symplectic  $(n, r)$ -instantons. In our joint papers with U.Bruzzo and D. Markushevich [CEJM, **10** (2012), 1232-1245], [EJM, First online: 17 November 2015. DOI 10.1007/s40879-015-0082-0, pp. 1-14] first results on the geometry of the moduli space  $I_{n,r}$  were obtained. We exploited the monad method (Atiyah-Drinfeld-Hitchin-Manin, Barth, Tyurin), which allows one to study instantons by means of hyperwebs of quadrics, and we showed that a component  $I_{n,r}^*$  of  $I_{n,r}$  that is singled out by a certain open condition (which rules out some "badly behaved" monads) is irreducible.

Recently we have found a modification of this monad approach which allows us to relate the question of irreducibility of  $I_{n,r}$  to that of  $I_{n+1,r-1}$ . In particular, this implies the irreducibility of the spaces  $I_{n,n-2}$  for all  $n \geq 3$ . Using this approach, we expect to achieve the irreducibility of the spaces  $I_{n,r}$  in a wide range  $\frac{3}{2}n \leq r \leq n$ ,  $n \geq 2$ . This is a joint project with D.Markushevich.

### 3. Ind-varieties of generalized flags as linear ind-flag varieties.

We study the geometry of ind-flag varieties. The most general notion including all typical examples of flag ind-varieties is that of the ind-variety of generalized flags. These ind-varieties were first defined by I.Dimitrov and I.Penkov in 2004 in the paper [IMRN **55** (2004), 2935-2953], where they were understood as homogeneous spaces for classical ind-groups. On the other hand, in our recent paper with I.Penkov [PAMQ, **10:2** (2014), 289-323] we had introduced the so-called linear ind-Grassmannians for classical linear ind-groups which provide a natural example of ind-flag varieties, and gave their complete classification in both ordinary and isotropic cases. The notion of a linear ind-Grassmannian, in fact, can be extended to that of a linear ind-variety of flags.



Our first task is to give a classification of linear ind-varieties of flags in ordinary and isotropic cases via their explicit description in terms of standard extensions of flags. We then expect to prove a general hypothesis that these linear ind-varieties of flags are the same as the above mentioned ind-varieties of generalized flags. This will give us the expected relation between the ind-geometry and the representation theory of classical linear ind-groups. This is a joint project with I.Penkov.