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ESTIMATING THE VOLUME OF INVESTMENT NEEDED FOR DEVELOPING A REGIONAL FREIGHT TRANSPORTATION INFRASTRUCTURE

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A decision-support tool for estimating the volume of investment needed for developing/modernizing a regional freight transportation infrastructure to rationalize the use of the region’s land is proposed. The estimation is done by solving the problems of choosing a) the places for building new freight transport hubs and the hub capacities, b) the capacities of the transport hubs that are to be modernized, and c) the types of access road to all the hubs and their optimal capacities, along with the optimal distribution of cargo flows among all the transport modes expected to function in the region. These problems are mathematically formulated as two mixed programming problems and a minimax one that is reducible to a mixed programming problem with all integer variables being Boolean. The proposed tool includes a) a mathematical model with a linear structure of constraints underlying the formulation of the above-mentioned three mixed programming problems on its basis, b) standard software packages for solving mixed programming problems with linear constraints, and c) standard software packages for processing data and graphically depicting problem solutions. Examples of solving these three mixed programming problems with the data taken from open sources demonstrate the effectiveness of the proposed tool.

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Keywords: Bilinear functions of vector arguments; investment in developing a regional freight transportation infrastructure; minimax problems with linear constraints; mixed programming problems; public-private partnership; transport hubs and access roads to them

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Introduction

Large-scale financial projects, including those in the field of transportation, are quite common in the economy of every country. Both modernizing the existing transportation infrastructure and developing a new one can serve as examples of such projects. The implementation of these projects requires a large volume of investment, which federal and regional authorities cannot usually provide in full. If this is the case, forming certain partnerships with the private sector, for instance, public-private partnerships or concession agreements to implement the projects may become an effective strategic decision that the authorities can make. (Here, it seems natural to assume that both legal and financial conditions that the authorities offer to their potential private partners are acceptable to the latter.) This is a general approach to financing any large-scale economic projects, including transportation ones. So to start negotiations with the private sector on the matter, the authorities are to estimate the investment volume they believe is needed for a particular project. However, the structure of negotiations with potential private investors much depends on what the authorities consider a part of the project. Also, this depends on how the project is expected to generate revenue in any particular planning period (or in several such periods). Both the negotiation structure and the revenue expected to be generated by the project are among the negotiation subjects.

In developing a regional freight transportation infrastructure, building a set of new transport hubs with access roads to them is one of the two key parts of a project that a regional administration may offer to finance to its potential partners from the private sector. The other key part is associated with modernizing the existing transportation fleet by acquiring a new one for at least some freight transportation modes. However, modernizing this fleet may or may not be part of negotiations with the private sector on investing in the regional freight transportation infrastructure for certain organizational or financial reasons.

Only the part of a project on developing a regional freight transportation infrastructure associated with building transport hubs and access roads to them is the subject of consideration in the present paper.

The importance of this part of the project is recognized by every country in the world. That is, as the country's economy develops, the cargo flows via particular regions of the country increase. At a certain point, the existing freight transportation infrastructure in these regions or even statewide may turn out to be insufficient to handle the increased cargo flows. In this case, evaluating the economic effectiveness of developing new transport hubs with access roads to them in either a particular region or even in every region of the country becomes inevitable.

This economic effectiveness much depends on a) the chosen locations and capacities of the new hubs, b) the chosen types and capacities of access roads to them, and c) a chosen redistribution of cargo flows among all the transport hubs. While the government of a country as a whole and/or the administration of a particular region of the country may recognize the importance of the project, all the efforts to make the project a reality may fail. This may happen even if the choices mentioned in a)-c) are recognized as important. That is, without securing the needed financing for the project, all the promises of the governments/regional administrations to the voters, particularly, on developing a regional freight transportation infrastructure may remain only promises. In order to avoid making unrealistic promises, as well as to make at least some of already made promises real, the governments/administrations need decision-support tools. These tools should help them a) estimate expenses associated with implementing the project, and b) negotiate with private investors both legal and financial conditions for their potential financial contributions. The latter is needed if the regional administration and the country's government cannot finance a particular project in full.

It is well known that applied mathematics helps a great deal in analyzing and solving logistics, operational, and managerial problems in transportation systems. However, investment problems associated with the logistics underlying a regional freight transportation infrastructure have their own specifics. These specifics complicate the use of general financial engineering approaches in solving these strategic management problems, which every regional administration faces. So the question is: Can operations researchers provide a decision-support tool that would reflect these specifics in analyzing the investment needs of regional administrations associated with developing regional transportation infrastructures?

This paper demonstrates that with respect to developing and/or modernizing regional freight transportation infrastructures, the answer is "yes," provided the analytical means needed to this end are properly chosen and correctly applied. Particularly, a mathematical model underlying a decision-support tool for financial negotiations with potential investors from the private sector is proposed. This model is a nonlinear generalization of the known facility location problem. It reflects the legal and financial capabilities of the regional administration to offer to the private sector its cooperation in the framework of, for instance, a potential public-private partnership.

On the basis of this (generalized) model, estimating the expenses associated with implementing the project can be done with the use of standard optimization software packages. Moreover, solutions to the corresponding optimization problems can quickly be obtained when a part of or even all the data reflecting the geography of a corresponding region can be known only approximately. This reflects the uncertainty conditions under which the above-mentioned expenses are estimated.

In addition to the Introduction, the paper contains eight more sections and three Appendices.

Section I contains a) the problem statement, b) the features of mathematical models to formalize this problem in two situations depending on the assumptions on what information is available to the regional authorities in formalizing this problem, and c) certain observations to bear in mind in solving optimization problems formulated on the basis of these models.

Section II presents a review of two groups of mathematical problems close in formulation to those under consideration in this paper, including a classification of these problems for one of the groups.

Section III provides mathematical formulations of the problems under consideration in two forms. The first form is that of an ordinary optimization problem, when all the coefficients in the goal (objective) function are considered to be fixed. The second form is that of a robust (minimax) optimization problem, when all the vectors of the coefficients in the goal function are considered as variables. In this case, the best economic strategy of the regional administration is searched for in the "worst-case scenario" of the uncertain input data value combinations.

Section IV presents the formulation of the Basic Assertion, which allows one to reduce the minimax problem of finding the best economic strategy of the regional administration in the "worst-case scenario" to a mixed programming problem with linear constraints.

Section V discusses the results of testing the proposed decision-support tool (for estimating the investment volume needed to develop a regional freight transportation infrastructure). This is done on several sets of the data needed to form the input information for the mathematical models underlying both the ordinary optimization problems and the minimax one. In the course of the testing, corresponding mixed programming problems were solved by the MATLAB software package, and solutions to these problems were compared.

Section VI provides a discussion of what should be considered a contribution to the research field, and to what extent the authors' paper could be considered as such. Also, it provides an assessment of the number of Boolean variables in real optimization problems of the considered kind. Further, it contains methodological recommendations for using the proposed decision-support tool by both regional and federal administrations in their negotiations with potential investors from the private sector on forming, for instance, a public-private partnership. Finally, this section discusses the requirements that the decision-support tool should meet to be helpful in solving problems associated with developing (modernizing) regional freight transportation infrastructures.

Section VII briefly summarizes the research results reflected in the paper.

Section VIII contains concluding remarks.

Appendix 1 offers the proof of the Basic Assertion from Section IV, Appendix 2 presents tables with numerical test results from Section V, and Appendix 3 illustrates some of them graphically.

I. The problem statement, features of mathematical models to formalize this problem, and those of optimization problems formulated on their basis

Usually, the geography of the region and the part of the country's transportation infrastructure already functioning there determine potential places in which new transport hubs could be built. If this is the case, the expected volumes of, for instance, yearly cargo flows via these new hubs help roughly estimate the desirable capacities of the new hubs. However, the capacities of both new transport hubs and access roads to them affect the distribution of the expected total cargo flows in the new freight transportation infrastructure as a whole, which is planned to be developed. So, in considering the development of a new regional fright transportation infrastructure, a decision-support tool for analyzing

a) how many new transport hubs should be built in the region,

b) where these new transport hubs should be located,

c) what capacities the new transport hubs and access roads to them should have,

d) what schemes for moving cargo via new and already functioning transport hubs and access roads to them could be viewed as optimal for the region and for the country as a whole,

e) what total expenses associated with building new transport hubs and access roads to them and with maintaining all the elements of the planned regional freight transportation infrastructure one should expect, and

g) what volume of the revenue the planned regional freight transportation infrastructure should generate in the form of taxes to allow the regional administration to offer this revenue as (at least a part of) its financial contribution to, for instance, a private-public partnership with potential investors

would be extremely helpful for both federal and regional administrations.

Such a tool together with appropriate software packages for processing available data to form the input information for the tool should allow regional administrations to analyze the effectiveness of the regional freight transportation infrastructures, both existing and those to be developed. This analysis is a part of a set of economic tasks that regional and federal administrations face.

The present paper proposes a mathematical model underlying a variant of a decision-support tool for determining

—optimal (from the regional administration view point) locations of new cargo transport hubs in a region to meet the expected demand for servicing cargo flows via thus modernized regional freight transportation infrastructure (on account of building both new transport hubs and access roads to them in these locations),

—total expenses associated with building new transport hubs in the chosen (optimal) locations and access roads to them, and

—the revenue expected to be generated by the functioning of thus modernized regional freight transportation infrastructure in any planning period being of interest to the regional administration

by solving two optimization problems. One of the problems is a mathematical programming problem with mixed variables, linear constraints, and a linear goal function, which can be solved by standard software packages for solving optimization problems. The other problem is a minimax problem with linear constraints, mixed variables, and a bilinear goal function of two vector arguments one of which has only integer coordinates. As is proven in Section IV, the second (minimax) problem can also be solved with the use of standard packages for solving optimization problems. For processing the available data to obtain the input information for the calculations and for graphically depicting the calculation results, other standard software packages can be used. The difference in the two problem formulations reflects the one that exists in two situations under study in this paper.

Situation 1. Depending on what information for a particular planning period is known to decision makers from the regional administration exactly, two cases are possible.

Case A. The total demand for cargo flows at each place of cargo origin and that at each cargo destination point are known numbers. Also, a) the total demand for cargo flows in the region, b) the cost values for building new transport hubs and new access roads to these new hubs, c) the cost values for transporting cargo to every point of cargo destination and from every place of cargo origin via each transport hub (both already existing and those to be built), and d) the maintenance cost values for both the transport hubs and access roads to them, are known numbers.

Case B. Only the areas to which volume values for a) the total demands for cargo flows at each place of cargo origin and at each cargo destination point belong, and b) the total demand for cargo flows in the region belongs are known. At the same time, all the above-mentioned cost values (for building new transport hubs and new access roads to these new hubs, for transporting cargo to every point of cargo destination and from every place of cargo origin via each transport hub (both known and to be built), and the maintenance cost values for both the transport hubs and access roads to them, are known values as they are in *Case A*.

Situation 2. Only the areas to which values of each vector of *all* the parameters listed in *Situation 1* belong are known to those who make decisions on how the existing regional freight transportation infrastructure should be developed and used.

In both cases of *Situation 1*, the regional administration intends to spend as little as possible for a) building both the new cargo transport hubs and access roads to them, and b) the maintenance for both the existing transport hubs and access roads to them and the new ones. This are the expenses that the regional administration would like private investors to cover. At the same time, the administration offers its financial contribution to the potential partnership. This contribution (or a part of it) comes in in the form of the expected cash flow volume generated by the taxes to be received by the regional budget. These taxes are to be paid by cargo owners and cargo carriers for using the transport hubs and access roads to them as elements of the (new) regional freight transportation infrastructure.

Thus, the total expenses reduced by the expected amount of revenue to be received in the form of the above-mentioned regional taxes are to be minimized in both cases. Corresponding optimization problems are mathematically formulated as mathematical programming ones with mixed variables and linear constraints. In these two optimization problems, both the expenses and the revenue are mathematically described by linear functions of mixed variables. Solutions to these problems determine a) an optimal location of new transport hubs to be built, along with their optimal capacities, b) an optimal set of access roads to these new transport hubs to be built, along with their types and capacities, and c) an optimal distribution of the cargo flows via both existing transport hubs and the new ones.

In *Situation 2*, the goal function in the (third) optimization problem is still the difference between the expenses and the expected amount of revenue to be received in the form of the above-mentioned regional taxes. However, in this situation, the goal function of the optimization problem is mathematically described by the maximum function of an algebraic sum of three bilinear functions of vector arguments in finite-dimensional spaces.

Components of the vector arguments of these bilinear functions are

a) the expenses associated with (the cost values for) building new transport hubs at each of the potential locations,

b) the expenses associated with (the cost values for) building access roads to new transport hubs at these locations,

c) the maintenance expenses associated with (the maintenance cost values for) both new transport hubs and access roads to them,

d) the volumes of transportation flows expected to be moved between all the transport hubs of the developed transportation network (both existing and those to be built) and all the places of cargo origin and cargo destination points,

e) Boolean variables determining whether a new transport hub of a particular capacity should be built at a particular location, and

g) Boolean variables determining whether a new access road of a particular type to a new transport hub should be built (or the existing access roads are sufficient to allow the transport hub to function in full capacity) and the capacity of each new access road to be chosen to be built.

Continuous variables mentioned in a)-d) belong to polyhedra described by compatible systems of linear equations and inequalities. It is natural to assume that each of the polyhedra to which each variable vector belongs is a subset of a parallelepiped in a corresponding Euclidean space. Boolean variables mentioned in e) and those mentioned in g) form two vectors, each belonging to a unit cube (different for each of these two vectors) in a corresponding Euclidean space.

Remark 1.

In using the proposed mathematical model and in solving both types of optimization problems formulated on the basis of this model (i.e., mathematical programming problems with linear constraints and mixed variables and the minimax one with linear constraints and mixed variables) in practical calculations with any real data, one should bear in mind three simple (though important) observations.

1. All the estimates that the regional administration makes in considering the development of the regional freight transportation infrastructure are usually made for a certain period of time. Usually, it makes them for a year, for several years, for a decade, etc., beginning from the time when all the new facilities are expected to start their cargo operations. Each of the new facilities that are planned to be built may require a different amount of time for completing the construction work. So to consider the operations in this infrastructure as having started, in all the calculations, the regional administration may need to solve either the above-mentioned mathematical programming problems with linear constraints and mixed variables, or the minimax one with linear constraints and mixed variables several times. At each of the times, the administration may consider facilities that will be built and will start their cargo operations before some (or all) of the others will as already existing ones. This is to be reflected in the corresponding version of the mathematical model underlying the mathematical formulation of the corresponding problem (via its system of constraints).

2. The revenue that is expected to be generated by the functioning of the regional freight transportation infrastructure will come in yearly beginning from the time of starting the cargo operations by each of the newly built facilities.

3. In running the calculations, one should check the comparability of the numbers determining the values of the parameters that are present in the chosen mathematical model. One should be sure that these numbers correspond to the same period of time (a year, several years, a decade, etc.), proceeding from the fact that the known capacities of both the (new and existing) transport hubs and access roads to them are usually yearly ones.

II. A review of scientific publications studying problems close to those under consideration in the present paper

Scientific publications that are close to the subject of this paper form two groups. The first group includes publications traditionally considered in studies associated with the hub location problem in various formulations. The second group includes publications dealing with public-private partnership investments in developing transportation infrastructures. Both groups are briefly reviewed in this section of the paper. For the first group of publications, the text to follow mostly only cites the papers in which brief or detailed reviews of the hub location problem studies are offered.

A review of publications on hub location problems.

A variety of formulations of the hub location problems can be structured, for instance, based upon several characteristics of the hubs and the places to be connected with them. One can view these characteristics as parameters of the corresponding mathematical models.

(1) *The type of the mathematical formulation of the problem.* In the framework of a "discrete" formulation, places for hub locations in a region are to be chosen within a set of a finite number of particular places in the region. In the framework of a "continuous" formulation of the problem, the hubs can be placed anywhere in the region.
(2) *The goal function type in the optimization problem.* Two major types of the goal functions are usually considered: the maximum cost of services for all the origin-destination pairs that is to be minimized (the minimax criterion), and the sum of all the costs that is to be minimized (the mini-sum criterion). In addition to the costs the goal function may include profits from providing services. Also, in some cases, non-financial objectives, reflecting the service level are among the criteria.
(3) *The available data on the number of hubs.* The number of hubs in a particular problem can be either an exogenous parameter or the one to be determined in the course of solving the problem.
(4) *The cost of placing the hubs.* Three types of the cost are considered in the hub location problems: the zero cost, the fixed cost, and the variable cost.
(5) *A connection type between the hubs and places connected with the hub.* There are two types of the connection between the hubs and such places: a single connection and a multiple connection. Under the single connection, each place (a sender or a recipient) may be associated with (or assigned to) the only service hub. Under the multiple connection, each place can be connected with (assigned to) several service hubs.
(6) *The cost of connecting the hubs to the customers (places).* As in the case of the cost of placing the hubs, three types of the connection cost are considered: the zero cost, the fixed cost, and the variable cost.
(7) *The existence of special conditions on the connections among the hubs (the types of subgraphs formed by sets of the hubs).* Among major assumptions on such conditions, the following four assump-

tions on a subgraph of the hubs — a) a complete graph, b) a star, c) a tree, and d) a line — dominate.

(8) *The existence of restrictions on the capacities of either the hubs or their connections with the places (or both).*

(9) *The existence of flows between particular origin-destination pairs in the sets of the hubs and those of the places connected to them.*

(10) *The existence of service level constraints.*

(11) *The existence of uncertainty in parameters of the network such as, for instance, costs and demands.*

(ReVelle and Swain 1970) published one of the first papers in which the problem of optimally locating service centers in a region was studied. The problem of locating hospitals, warehouses, factories, post offices, shops, and other facilities was formulated there as an integer programming one. Its objective function reflected the total distance from a particular point in the region to the service centers to be placed in the region, taking into account the importance of those points. One of the problem constraints was that on the number of service centers to be placed in the region. The problem formulated there has become known in applied mathematics as the p-median problem, and it received the name"p-median problem" due to its similarity with that of finding the median in a graph. ((In Hakimi 1964), in the median graph problem, the median is understood as a graph vertex that minimizes the weighted sum of the distances between this vertex and all other vertices of the graph.) (Daskin and Maas 2015) consider the p-median problem in which a location of the service centers that minimizes the average distance between the locations and the nearest of the service centers to be placed is searched. (Particular locations to be connected to the centered to be placed are weighted there by the importance of their locations.) (ReVelle and Swain 1970) and (Cornuejols and Nemhauser 1990) analyzed this problem in the case of no capacity limitations put on the service centers to be placed though capacitated versions of the problem are also known. (Garey and Johnson 1979) proved that, generally, all these problems are NP-hard.

Hub location problems have intensively been studied in the last several decades. Almost every recent publication, particularly, in the network analysis cites surveys on this subject in (Krarup and Pruzan 1983), (Campbell 1994a), (O'Kelly and Miller 1994), (Labbe and Louveaux 1997), (Klincewicz 1998), (Campbell et al. 2002), (Alumur and Kara 2008), (Campbell and O'Kelly 2012), (Farahani et al. 2013), (Contreras 2015), and (Zabihi and Gharakhani 2018).

Numerous publications consider the uncapacitated multiple allocation p-hub median problem (UMApHMP), first presented in (Campbell 1992). Its modifications are presented in (Campbell 1994b), (Skorin-Kapov et al. 1996), including the uncapacitated multiple allocation hub location problem with fixed costs (UMAHLP), considered by (Campbell 1994b). Exact and heuristics algorithms to solve these problems are proposed, for instance, in (Campbell 1996), (Klincewicz 1996), (Ernst and Krishnamoorthy 1998a), (Ernst and Krishnamoorthy 1998b), (Ebery et al. 2000), (Mayer and Wagner 2002), (Boland et al. 2004), (Hamacher et al. 2004), (Marin 2005) and (Canovas et al. 2007), and they are applicable to solving both the UMAHLP and the UMApHMP problems. A review of a number of heuristic algorithms for solving the p-median problem is presented in (Mladenovic et al. 2007).

Other hub location problems are formulated a) for networks of particular structures such as a line structure ((Martins et al. 2015)), a tree structure ((Contreras et al. 2010)), a star structure ((Labbe and Yaman 2008), (Yaman 2008), and (Yaman and Elloumi 2012)), structures with a particular number of connections (r-allocation) ((Yaman 2011)), and structures with an incomplete hub network ((Nickel et al. 2001), (Yoon and Current 2008), (Calik et al. 2009), and (Alumur et al. 2009)), b) under a number of assumptions on the transportation cost and cargo flows such as the economies of scale ((O'Kelly and Bryan 1998), (Horner and O'Kelly 2001), and (Camargo et al. 2009)), different discounting policies ((Podnar et al. 2002), (Campbell et al. 2005a), and (Campbell et al. 2005b)), and under the presence of arcs with fixed setting costs ((O'Kelly et al. 2015)), c) assuming a possibility to select the capacity of a hub ((Correia et al. 2010)), d) for multimodal hub location problems with different transportation modes ((Kelly and Lao 1991), (Racunica and Wynter 2005), (Limbourg and Jourquin 2009), (Ishfaq and Sox 2011), (Meng and Wang 2011), and (Alumur et al. 2012a)), e) under price sensitive demands ((Kelly et al. 2015)), f) assuming a sequential addition of competing hubs ((Mahmutogullari and Kara 2016)), g) for dynamic multi-period hub location problems ((Gelareh et al. 2015)), and h) for hub-and-spoke models dealing with disruptions at the stage of designing transportation networks with backup hubs and alternative routes ((An et al. 2015)).

Most of the papers on the hub location problem consider the case in which all the data is assumed to be known exactly. In papers addressing the uncertainty in the data, the existence of particular probability distribution over the uncertain parameters is assumed ((Marianov and Serra 2003), (Sim et al. 2009), (Yang 2009), (Contreras et al. 2011), (Alumur et al. 2012b), (Adibi and Razmi 2015) and (Yang et al. 2016)).

A recognized direction of dealing with the uncertainty in parameters of, particularly, networks, including transportation ones, in optimizing both network design and work consists of formulating corresponding problems as robust optimization ones. In these problems, the best solutions in the worst-case combination of parameters assuming values from particular sets is searched. Numerous authors, for instance, (Belenky 1981), (Ben-Tal and Nemirovski 1998), (Ben-Tal and Nemirovski 1999), (Yaman et al. 2001), (Bertsimas and Sim 2003), (Bertsimas and Sim 2004), (Ben-Tal et al. 2004), (Atamturk 2006), (Ordonez and Zhao 2007), (Yaman et al. 2007), (Ben-Tal and Nemirovski 2008), (Mudchanatongsuk et al. 2008), (Shahabi and Unnikrishnan 2014), (Merakli and Yaman 2016, 2017), (Yang and Yang 2017), (Zetina et al. 2017) and (Talbi and Todosijevic 2017) exercise this approach for problems in which sets of uncertain parameters are those described by systems of linear equations and inequalities.

Results that are close to those presented in this paper are discussed in (Merakli and Yaman 2016), (Serper and Alumur 2016), and (Alibeyg et al. 2016). (Merakli and Yaman 2016, 2017) propose a hub location model with a demand uncertainty described by systems of linear constraints. Similar to (Belenky 1981), they formulate a minimax optimization problem on two polyhedra and apply the dual transformation to linearize it and find the best solution in the worst-case of the demand combinations. To solve the linearized problem on CAB, AP, and the Turkish network data, the CPLEX software package, along with particular variants of the Benders decomposition algorithms, is used. An approach presented in (Zetina et al. 2017) and (Talbi and Todosijevic 2017) differs from those proposed in (Merakli and Yaman 2016, 2017). That is, in (Zetina et al. 2017) and in (Talbi and Todosijevic 2017), the change of some problem's parameters (demand, transportation cost) is allowed , and the objective is to find the best solution under the worst set of these parameters. The cardinality of this set can be interpreted as a budget of uncertainty, but this approach is limited and cannot be used for modeling complex relationships among uncertain parameters. (Serper and Alumur 2016) consider the capacitated hub location model with different vehicle types and variable hub capacities. The model lets: a) choose transportation modes (air, ground) and the vehicle type (airplane, trailer, truck) for both hub-to-hub and hub-to-node transportation, and b) choose the capacity level at a hub for each transportation mode. To solve the formulated problem for the Turkish transportation network (with

81 nodes) on CAB, both CPLEX and a neighborhood search (heuristic) algorithm are used. The authors report the average gap between the CPLEX optimal solution and the heuristic algorithm solution to be about 1%. The same approach to modeling variable hub capacities is used in (Alumur et al. 2018), where the authors propose a framework for modeling congestions at hubs in hub location problems with a service time limit. (Alibeyg et al. 2016) introduce a class of hub network design problems with profit-oriented goal functions, which reflect the tradeoff between the profits obtained from moving the commodities and the costs of building transportation networks. The authors propose several profit-oriented models and compare their computational results related to designing networks with those obtained with the use of traditional cost-oriented models based on the CAB data and the use of CPLEX. In (Alibeyg at al. 2018), the authors propose an exact algorithmic framework for solving profit-oriented hub location problems. In this framework, a Lagrangian relaxation is used to obtain efficient bounds at the nodes in a branch-and-bound method taking into account the structure of the goal function. The resulting exact algorithms are tested using a CAB dataset with up to 100 nodes, and they appear to solve more instances of the problems in a limited period of time than CPLEX can solve.

Also, there are publications that do not address the hub location problem itself while studying models related to those used in the hub location problem, which may eventually, be helpful in studying this problem. For instance, (Wang 2016) presents a theoretical study of the optimal hubs network topology, and (Redondi et al. 2011), (Czerny et al. 2014), (Bracaglia et al. 2014), and (Teraji and Morimoto 2014) consider a competition among the hubs. (Small and Ng 2014) study optimization problems of choosing a capacity and the type of access roads to transport hubs, whereas (Nagurney et al. 2015) and (Li and Nagurney 2015) apply a game theory approach to finding equilibrium prices in supply chain networks under competition conditions. That is, (Nagurney et al. 2015) consider supply chain networks with competing manufacturers and freight service providers, whereas (Li and Nagurney 2015) consider supply chain networks with competing suppliers of product components to be assembled by the purchasing firms, which may eventually manufacture some of these components on their own.

A review of publications on public-private partnership in transportation.

(Rouhani et al. 2016) propose a particular framework for analyzing public-private partnership investment projects in transportation from the public welfare viewpoint. In these projects, a share of the revenue that is generated by the project is returned to the citizens who own the public infrastructure involved in the public-private partnership project. (Geddes and Nentchev 2013) assert that such a strategy may increase a public support for a system-wide pricing of the existing roads. The proposed public welfare framework estimates the benefits and the costs of using the investment approach for an urban transportation network with respect to all the major project stakeholders (residents, users, government, and the private sector). (Carpintero and Siemiatycki 2016) study how various political factors affect the formation and the effectiveness of public-private partnership projects with respect to Spain light rail transit projects, and conclude that they affect them significantly.

(Aerts et al. 2014) propose to use of a multi-actor analysis to identify factors being critical for success in implementing public-private partnerships in developing the infrastructure of a port. Based on results presented in several surveys, the authors assert that a) the concreteness and preciseness of the concession agreement, b) the ability to appropriately allocate and share risk, c) the technical feasibility of the project, d) the commitment made by the partners, e) the attractiveness of the financial package, f) a clear definition of responsibilities, g) the presence of a strong private consortium, and h) a realistic cost/benefit assessment are such factors. (Panayides et al. 2015) also consider ports in a study of the influence of institutional factors on the effectiveness of the public-private partnership. An empirical analysis provided by the authors in their paper allows them to suggest that a) "the regulatory quality, b) the market openness, c) the ease of starting a business, and d) the enforcement contracts" are important institutional determinants of the effectiveness of port public-private partnership projects.

(Wang and Zhang 2016) study a road pricing problem in networks belonging to public-private partnerships in the form of a game. Two types of the players are considered by the authors in their game model: a) a set of individual travellers each of whom tries to find her/his own path with the minimal travel cost, and b) a set of transportation firms that cooperate among themselves in an attempt to minimize the total operational cost for every firm. The model allows the authors to find road charging schemes for the players that yield the optimum flows for players of both kind. Also, several other publications dedicated to studying the road pricing problem are listed in that paper. Particularly, among the listed ones, there are a) (Yang and Zhang 2002), where the authors study the tolling design conducted to secure a certain level of the social equity, b) (Sumalee and Xu 2011), where the authors consider optimal pricing schemes under an uncertain demand for services on a transportation network, c) (Zhang and Yang

2004), where the authors research a cordon-based congestion pricing (determining the payment for the right to travel inside a particular city zone), d) (Liu et al. 2014), where the authors analyze a model from (Zhang and Yang 2004) and modify it to take into consideration both the travelling time and the parking time inside the zone, e) (Zhang et al. 2008), where the authors suggest to determine particular prices as components of equilibria in a game model similar to (Wang and Zhang 2016)—where a stochastic nature of the player payoff functions is taken into consideration—and (Meng et al. 2012)—where cordon-based congestion pricing problems are considered, and stochastic equilibria for heterogeneous users are analyzed.

(Zhang and Durango-Cohen 2012) present a game-theoretic model of a concession agreement for examining how a government's tax policy affects the interest of private investors to invest in transportation infrastructure.

Organizational problems associated with forming public-private partnerships for Indian dry (inland) ports are reported in (Haralambides and Gujar 2011) based on the interviews with various stakeholders that the authors have conducted. According to the authors, the excess capacity of the ports, limit pricing policies, and a weak legal framework for setting and running a public-private partnership are among the major obstacles in this field. (Cabrera et al. 2015) consider similar problems for ports in Spain, where the authors list what they believe are primary concerns for public-private partnership schemes in this area of freight transportation services. An improper risk allocation in tendering processes, a failure to meet expectations of the demand for services, and concerns associated with turning the transportation enterprise into a monopoly are listed and discussed there.

(Dementiev and Loboyko 2014) propose a game-theoretic approach to analyzing the chances of establishing public-private partnerships in Russia's suburban railway sector of passenger transport. (Dementiev 2016) further develops this approach by considering the idea to delegate some regulatory functions in public transportation to a public-private partnership. The implementation of this idea is expected to help balance social and commercial interests in line with a predetermined objective. So this idea presents a certain theoretical interest from the viewpoint of welfare comparisons for alternative organizational structures in the public transport sector. Certain optimal corporate structures for such a partnership are determined depending on local costs for public funds and society preferences. The proposed approach was applied for analyzing the effectiveness of a railways suburban transport reform in Russia.

(Carmona 2010) considers general problems of developing a transportation infrastructure in a country in the context of economic regulations in public-private partnership settings. The author proposes to take into account three particular measures of the efficiency. That is, a) the dynamic allocation efficiency (determining whether the whole life-cycle social benefits exceed the costs of the infrastructure provisions), b) the utilization efficiency of the transportation infrastructure (determining whether charging the price that promotes the best possible use of available infrastructure capacity positively affects the infrastructure functioning), and c) the productive efficiency (determining how the services provided by the road infrastructure minimize the transportation cost).

(Galilea and Medda 2010) analyze to what extent and how economic and political statuses of a particular country (mainly the presence of corruption and democracy) may contribute to success of a public-private partnership in developing transportation infrastructures.

One should notice that most of the publications related to public-private partnership problems, in particular, in the field of transportation, are those of general considerations. These publications do not address quantitative approaches to studying issues underlying these problems. Certainly, there are publications in which mathematical models associated with forming public-private partnerships and analyzing their effectiveness are proposed for general interactions of the public and private sector. Also, there are those related to such an interaction in areas other than transportation. However, neither take into consideration any specifics of transportation services, and for this reason, they are not considered in the presented brief review.

Particularly, (Belenky 2014) considers such models in the form of tree-person games in which a state, and investor, and a developer of a project (or a set of projects) interact—in an attempt to find a mutually acceptable conditions for the partnership. In those models the players proceed from a) the minimum volume of investment required for each project from the viewpoint of the state, b) the volume of investment that the state can afford to contribute, c) preferences and requirements of the developer for the compensation of its services, and d) the volume of investment that the investor can afford to contribute. Some other financial factors are also taken into consideration. For this type of the games, under linear constraints describing a set of strategy for each player, necessary and sufficient conditions for the equilibria are established. These conditions allow one to find equilibria in the problem under consideration there by solving linear programming problems forming a dual pair. However, the results presented in that publication are not directly applicable to the problems under consideration in this paper. This is the case due to the presence of Boolean variables in the model underlying mathematical formulations of the problems to be considered in Section III of the present paper.

Thus, the presented review shows that there are classes of problems with formulations being close to the problems mentioned in Section I, which are under consideration in this paper. These close problems have not been modeled and studied in a manner allowing one to use results from the reviewed publications. This is the case even when these results may help work out decisions by the parties negotiating a potential private-public partnership on developing a regional freight transportation infrastructure

III. Mathematical formulations of the problems under consideration

Let

M be the number of locations (nodes) on the regional transportation network under consideration each of which is either a place of cargo origin or a cargo destination point (or both),

N^{new} be the number of points (nodes) on the network suitable for locating new transport hubs,

N^{exist} be the number of points (nodes) on the network with already functioning transport hubs,

s_j be the expected yearly demand for (the volume of) cargo transportation services at node j, $j \in \overline{1, M}$ of the transportation network (from node j to transport hubs in the new (modernized) transportation network and from the hubs to that node),

s_j^{min} be the expected yearly minimum demand for cargo transportation services at node j, $j \in \overline{1, M}$ of the transportation network (from node j to transport hubs in the new (modernized) transportation network and from the hubs to that node),

s_j^{max} be the expected yearly maximum demand for cargo transportation services at node j, $j \in \overline{1, M}$ of the transportation network (from node j to transport hubs in the new (modernized) transportation network and from the hubs to that node),

S^{min} be the expected yearly minimum total demand for cargo transportation services in the region in the planning period,

S^{max} be the expected yearly maximum total demand for cargo transportation services in the region in the planning period,

ϵ_i be the number of variants of the capacity that a new transport hub to be built at node i may have, $i \in \overline{1, N^{new}}$,

μ be the number of the chosen variant of the transport hub capacity at node i, $i \in \overline{1, N^{new}}$, $\mu \in \overline{1, \epsilon_i}$,

$d_{i\mu}^{new}$ be the yearly capacity of a new transport hub at node i under variant μ of the hub capacity, $i \in \overline{1, N^{new}}, \mu \in \overline{1, \epsilon_i}$,

$d_{i\mu}^{max\ new}$ be the maximum yearly capacity of a new transport hub at node i under variant μ of the hub capacity, $i \in \overline{1, N^{new}}, \mu \in \overline{1, \epsilon_i}$,

$d_{i\mu}^{min\ new}$ be the minimum yearly capacity of a new transport hub at node i under variant μ of the hub capacity, $i \in \overline{1, N^{new}}, \mu \in \overline{1, \epsilon_i}$,

$d_{i'}^{exist}$ be the capacity of the existing transport hub at node i', $i' \in \overline{1, N^{exist}}$,

L_i be the number of types of all the new access roads to a new transport hub at node i, $i \in \overline{1, N^{new}}$ that are planned to function on the transportation network as a result of its development during the planning period,

$l_{i'}$ be the number of types of all the access roads to the existing transport hub at node i', $i' \in \overline{1, N^{exist}}$ that are planned to remain on the transportation network as a result of its development during the planning period,

$s_{ji}^{k\ new}$ be the yearly volume of cargo that is planned to be moved between node j, $j \in \overline{1, M}$ of the transportation network and a new transport hub at node i, $i \in \overline{1, N^{new}}$ via a new access road of type k, $k \in \overline{1, L_i}$,

$s_{ji'}^{k'\ exist}$ be the yearly volume of cargo that is planned to be moved between node j, $j \in \overline{1, M}$ of the transportation network and the existing transport hub at node i', $i' \in \overline{1, N^{exist}}$ via an existing access road of type k', $k' \in \overline{1, l_{i'}}$,

$t_{ji}^{k\ new}$ be the (average) cost of transporting a unit volume of cargo between node j and a new transport hub at node i via a new access road of type k to the hub, which cargo owners and cargo carriers are expected to pay to operators of the regional transportation infrastructure (which will act under an agreement with regional transportation authorities or on their behalf) for the access to the new transport hub at node i, $j \in \overline{1, M}$, $i \in \overline{1, N^{new}}$, $k \in \overline{1, L_i}$,

$t_{ji'}^{k'\ exist}$ be the (average) cost of transporting a unit volume of cargo between node j and the existing transport hub at node i' via the existing access road of type k' to the hub, which cargo owners and cargo carriers are expected to pay to operators of the regional transportation infrastructure (which will act under an agreement with regional transportation authorities or on their behalf) for the access to the existing transport hub at node $i',\ j,\ j \in \overline{1,M},\ i' \in \overline{1,N^{exist}},\ k' \in \overline{1,l_{i'}}$,

$Q_{i\mu}^{k\ new}$ be the yearly capacity of a new access road of type k to a new transport hub at node i with the hub capacity $d_{i\mu}^{new}$ on the transportation network, $k \in \overline{1,L_i}, \mu \in \overline{1,\epsilon_i},\ i \in \overline{1,N^{new}}$,

$Q_{i\mu}^{k\ new\ max}$ be the maximum yearly capacity of a new access road of type k to a new transport hub at node i with the hub capacity $d_{i\mu}^{max\ new}$ on the transportation network, $k \in \overline{1,L_i}, \mu \in \overline{1,\epsilon_i},\ i \in \overline{1,N^{new}}$,

$Q_{i\mu}^{k\ new\ min}$ be the minimum yearly capacity of a new access road of type k to a new transport hub at node i with the hub capacity $d_{i\mu}^{min\ new}$ on the transportation network, $k \in \overline{1,L_i}, \mu \in \overline{1,\epsilon_i},\ i \in \overline{1,N^{new}}$,

$Q_{i'}^{k'\ exist}$ be the yearly capacity of the existing access road of type k' to the existing transport hub at node $i',\ k' \in \overline{1,l_{i'}},\ i' \in \overline{1,N^{exist}}$,

$f_{i\mu}$ be the cost of building a new cargo transport hub at node i of variant μ of the hub capacity, $\mu \in \overline{1,\epsilon_i},\ i \in \overline{1,N^{new}}$ on the transportation network,

$g_{i\mu}^{k}$ be the cost of building a new access road of type k to a new transport hub at node i of variant μ of the hub capacity, $\mu \in \overline{1,\epsilon_i},\ i \in \overline{1,N^{new}},\ k \in \overline{1,L_i}$.

$c_{i\mu}^{new}$ be the yearly maintenance cost of a new cargo transport hub at node i of variant μ of the hub capacity, $\mu \in \overline{1,\epsilon_i},\ i \in \overline{1,N^{new}}$ on the transportation network,

$q_{i\mu}^{k\ new}$ be the yearly maintenance cost of a new access road of type k to a new transport hub at node i of variant μ of the hub capacity, $\mu \in \overline{1,\epsilon_i},\ i \in \overline{1,N^{new}},\ k \in \overline{1,L_i}$.

$c_{i'}^{exist}$ be the yearly maintenance cost of the existing cargo transport hub at node i', $i' \in \overline{1,N^{exist}}$ on the transportation network, and

$q_{i'}^{k'\ exist}$ be the yearly maintenance cost of the existing access road of type k' to the existing transport hub at node $i',\ k' \in \overline{1,l_{i'}},\ i' \in \overline{1,N^{exist}}$.

Further, let

$y_{i\mu}$ be a binary (Boolean) variable that equals 1 if a new transport hub of variant μ of the hub capacity will be built at node i and equals 0, otherwise, $\mu \in \overline{1,\epsilon_i},\ i \in \overline{1,N^{new}}$,

$z_{i\mu}^{k}$ be a binary (Boolean) variable that equals 1 if a new access road of type k to a new transport hub at node i of variant μ of the hub capacity will be chosen to be built and equals 0, otherwise, $k \in \overline{1,L_i}, \mu \in \overline{1,\epsilon_i},\ i \in \overline{1,N^{new}}$, and

ψ be the number of years in the planning period of time for which the regional administration is interested in estimating the economic effectiveness of modernizing the existing freight transportation infrastructure.

Basic Assumptions

1. The cost of building a new transport hub of variant μ of the hub capacity at node i is a piecewise linear function of the hub capacity so that for each segment $d_{i\mu}^{min\ new} \leq d_{i\mu}^{new} \leq d_{i\mu}^{max\ new}$, this cost is a linear function

$$f_{i\mu} = a_{i\mu} + \gamma_{i\mu} d_{i\mu}^{new},\ \mu \in \overline{1, \epsilon_i},\ i \in \overline{1, N^{\ new}}.$$

where $a_{i\mu},\ \gamma_{i\mu}$ are positive, real numbers $\mu \in \overline{1, \epsilon_i},\ \ i \in \overline{1, N^{\ new}}$, and the inequalities $d_{i\mu}^{max\ new} \leq d_{i(\mu+1)}^{min\ new}$ hold, $\mu \in \overline{1, \epsilon_i - 1},\ \epsilon_i \geq 2,\ \ i \in \overline{1, N^{new}}$.

2. The cost of building a new access road of type k to a new transport hub of variant μ of the hub capacity at node i is a piecewise linear function of the capacity of this road (which depends on the hub capacity) so that for each segment $Q_{i\mu}^{k\ min\ new} \leq Q_{i\mu}^{k\ new} \leq Q_{i\mu}^{k\ max\ new}$, this cost is a linear function

$$g_{i\mu}^k = b_{i\mu}^k + \beta_{i\mu}^k Q_{i\mu}^{k\ new},\ \mu \in \overline{1, \epsilon_i},\ i \in \overline{1, N^{new}},\ k \in \overline{1, L_i}.$$

where $b_{i\mu},\ \beta_{i\mu}$ are positive real numbers $\mu \in \overline{1, \epsilon_i},\ \ i \in \overline{1, N^{new}}$, and the inequalities $Q_{i\mu}^{k\ max\ new} \leq Q_{k\ i(\mu+1)}^{min\ new}$ hold, $\mu \in \overline{1, \epsilon_i - 1},\ \epsilon_i \geq 2,\ \ i \in \overline{1, N^{new}}$.

One should bear in mind that access roads to a new transport hub are those connecting this hub to the closest element of the existing regional transportation network (a highway segment, a railroad segment, etc.) rather than a new road to be built to connect any node j to this hub. In calculating the cost $t_{ji}^{k\ new}$, the total length of the road between node j and transport hub i, the length of the access road of type k to hub i, and other parameters affecting the cost are taken into account.

Also, the assumption on a piece-wise structure of both costs reflects two features of this type of approximation. First, it allows one to approximate any particular continuous function of one variable that may appear in transportation practice with any needed degree of accuracy (by increasing the number of segments on each of which the function is approximated by a linear one). Second, it helps remain within linear or mixed programming with linear constraints in formulating both ordinary and robust optimization problems formalizing practical problems under consideration in this paper, which makes a difference in calculating solutions to these problems.

3. The expected minimum and maximum yearly demands for (the volumes of) cargo transportation services at node j of the regional transportation network are strictly positive, real numbers $\forall j \in \overline{1, M}$ so that the inequalities

$$0 < s_j^{min} \leq s_j \leq s_j^{max}, \forall j \in \overline{1, M}$$

hold.

4. The inequalities $\sum_{i'=1}^{N^{exist}} d_{i'}^{exist} < S^{min} < S^{max}$, and $Q_{i\mu}^{k\ new\ max} >> 1$, $\mu \in \overline{1, \epsilon_i}$, $k \in \overline{1, L_i}$, $i \in \overline{1, N^{new}}$ hold. No new access roads to already existing transport hubs will be built, and no modernization construction work will be done there in the planning period.

5. The number of types of new access roads that can be built to a new transport hub at node i, $i \in \overline{1, N^{new}}$ to choose from does not depend on the hub capacity. At the same time, the capacities of the new access roads to a new transport hub chosen to be built may depend on the hub capacity.

These tariffs are those expected to be paid by the cargo owners to the transportation carriers based on the situation in the market of transportation services.

6. Cargo flows may originate inside every new transport hub and inside every existing transport hub, and they may go to any node of the transportation network.

7. The amount of the cash flow formed by the taxes to be charged for providing access to the transportation infrastructure of the region is calculated as a particular percentage (ν) of the corresponding transportation tariffs. These tariffs are those expected to be paid by the cargo owners to the transportation carriers based on the situation in the market of transportation services. This percentage is considered to be the same for the whole planning period of time ψ (in years), where $\psi \geq 1$.

8. In negotiations with potential private sector partners, the regional administration chooses an arbitrary length of the planning period ψ for which it estimates the expenses associated with developing the regional freight transportation infrastructure. It proceeds from the yearly capacities of the new transport hubs and new access roads to them to be built during that period. However, the planning period starts once all the new elements of this infrastructure or any particular elements of it (selected by the regional administration) have been built and start functioning.

9. The functioning of the regional freight transportation infrastructure generates revenue in the form of taxes. These taxes start coming in once all the facilities (new transport hubs and new access roads to them) that are expected to be built in the planning period have been built.

In Tables 1-6, reflecting the results of testing the proposed tool on the model data (see Appendix 2), both this revenue and the profit/loss that the potential partnership may receive/sustain are calculated for different periods of time. These time segments begin from the moment at which all the above facilities start their cargo operations (i.e., for different ψ, $\psi \geq 1$).

Remark 2.

To calculate financial parameters, mentioned in basic assumption 9, a) capacities of both the existing facilities and of those to be built (new transport hubs and (new) access roads to them) should be scaled accordingly, and b) the maintenance cost for all the facilities (the existing ones and those to be built) for the whole planning period should be deducted from the revenue. Generally, the maintenance costs (for at least the new facilities to be built) can be considered as a vector belonging to a set of its feasible values (for, instance, to a polyhedron).

Let the regional administration consider the modernization of the existing regional freight transportation infrastructure proceeding from the expected volumes of each cargo flow in the region in this planning period. Let the administration determine that the existing transportation network cannot meet the expected demand for moving cargoes in the region in principle.

Subcase 1. As mentioned in Section I, based upon this determination, the administration then intents

a) to find out what new transport hubs should be built, where these hubs should be located, what types of access roads to each of them, how many, and of what capacities should be built,

b) to analyze the expediency of keeping the existing distribution scheme of at least some of the cargo flows between the nodes on the regional transportation network and the existing freight transport hubs (which is done by estimating the results of possibly redistributing the existing cargo flows by switching portions of these flows to new transport hubs that are planned to be built), and

c) to analyze the expediency of possibly directing parts of the expected new cargo flows to some of (or to all) the existing transport hubs.

These estimates and this analysis should be done to determine an economic strategy of developing the regional freight transportation infrastructure. This strategy depends on the ability of the regional adminstration to obtain federal funds to support this project. It also depends on the administration's ability to convince private investors to contribute to this project on acceptable (to them and to the administration) conditions in the framework of, for instance, a public-private partnership.

In the case under consideration, let the regional administration know the values that the parameters $t_{ji'}^{k'\ exist}$, $t_{ji}^{k\ new}$, $f_{i\mu}$, $g_{i\mu}^{k}$, $c_{i\mu}^{new}$, $q_{i\mu}^{k\ new}$, $c_{i'}^{exist}$, and $q_{i'}^{k'\ exist}$ may assume in the planning period. Then it can estimate the expected total expenses associated with developing the regional freight transportation infrastructure and redistributing the existing cargo flows between the existing transport hubs and those to be built. This can be done by minimizing the function describing these expenses on the set of feasible solutions to the system of linear constraints binding the variables $s_{ji}^{k\ new}$, $s_{ji'}^{k'\ exist}$, $y_{i\mu}$, and $z_{i\mu}^{k}$.

For $\psi \geq 1$, this function takes the form

$$\psi\Big(\sum_{i'=1}^{N^{exist}} c_{i'}^{exist} + \sum_{i'=1}^{N^{exist}} \sum_{k'=1}^{l_{i'}} q_{i'}^{k'\ exist}\Big)+$$

$$\sum_{i=1}^{N^{new}} \sum_{\mu=1}^{\epsilon_i} (f_{i\mu} + \psi c_{i\mu}^{new}) y_{i\mu} + \sum_{i=1}^{N^{new}} \sum_{k=1}^{L_i} \sum_{\mu=1}^{\epsilon_i} (g_{i\mu}^{k} + \psi q_{i\mu}^{k\ new}) z_{i\mu}^{k} -$$

$$\psi\nu\left(\sum_{i'=1}^{N^{exist}} \sum_{j=1}^{M} \sum_{k'=1}^{l_{i'}} t_{ji'}^{k'\ exist} s_{ji'}^{k'\ exist} + \sum_{i=1}^{N^{new}} \sum_{j=1}^{M} \sum_{k=1}^{L_i} t_{ji}^{k\ new} s_{ji}^{k\ new}\right).$$

For the sake of simplifying the reasoning on mathematically modelling the problem under consideration, it is assumed that in every year of the planning period, each of the parameters $t_{ji'}^{k'\ exist}$, $t_{ji}^{k\ new}$, $f_{i\mu}$, $g_{i\mu}^{k}$, $c_{i\mu}^{new}$, $q_{i\mu}^{k\ new}$, $c_{i'}^{exist}$, and $q_{i'}^{k'\ exist}$ assumes the same value. This assumption, however, is not restrictive, and one can consider any values of these parameters for each particular year and add corresponding terms into all the three sums in the above formula (see the last paragraph in Section III).

Subase 2. A deal with potential investors on a public-private partnership associated with developing a regional freight transportation infrastructure is the major goal of the regional administration. However, the version of this infrastructure may substantially depend on what the private sector investors may be interested in considering as the investment subject. That is, depending on what providing transportation services may bring to the service providers, the investors may become interested in both developing the infrastructure and providing these services. Thus, the potential investors may also be interested in signing, for instance, a concession agreement with the regional administration on operating the transportation network that is to be built thanks to their investment. Then the situation changes compared with that in which the development of the regional freight transportation infrastructure is considered as the only subject of the private-public partnership with the investors. That is, the taxes expected to be paid by the providers of transportation services to the regional administration will no longer be considered by the administration as its financial contribution to this partnership. As a signee to the concession agreement, the investors will receive a license to provide transportation services by hiring transportation and other companies to work with both cargo owners and cargo recipients. In this capacity, the private investors will be responsible for paying taxes to the regional administration. In exchange, they will be entitled to receive a profit from providing transportation services (provided this profit may exist in principle).

Certainly, the administration is interested in having a decision-making tool that would allow it to estimate the investor expenses in both situations. It is obvious that the second situation (in which paying taxes becomes the responsibility of the private investors contributing to the development of the regional freight transportation infrastructure) is covered by the previous reasoning. That is, the goal function of the optimization problems to be solved in this situation will differ from the ones to be solved in the first situation only by the sign before its second term (which will be plus instead of minus). Though, for the sake of definiteness, only *Subcase 1* is considered in this paper, calculations on model data are conducted for *Subcase 1* and *Subcase 2* for both *Case A* and *Case B* in *Situation 1* and for *Subcase 1* and *Subcase 2* in *Situation 2* (see Section I). Two regional freight transportation infrastructures corresponding to the calculation results are compared in Section V.

Finding the minimum of the goal function, considered in *Case 1*, requires solving a mathematical programming problem with mixed variables. The problem to be solved then takes the form

$$\sum_{i=1}^{N^{new}} \sum_{\mu=1}^{\epsilon_i} (f_{i\mu} + \psi c_{i\mu}^{new}) y_{i\mu} + \sum_{i=1}^{N^{new}} \sum_{k=1}^{L_i} \sum_{\mu=1}^{\epsilon_i} (g_{i\mu}^{k} + \psi q_{i\mu}^{k\ new}) z_{i\mu}^{k}$$

$$-\nu\psi \left(\sum_{i'=1}^{N^{exist}} \sum_{j=1}^{M} \sum_{k'=1}^{l_{i'}} t_{ji'}^{k'\ exist} s_{ji'}^{k'\ exist} + \sum_{i=1}^{N^{new}} \sum_{j=1}^{M} \sum_{k=1}^{L_i} t_{ji}^{k\ new} s_{ji}^{k\ new} \right) \to \min, \tag{1}$$

$$\sum_{\mu=1}^{\epsilon_i} y_{i\mu} \le 1, \ i \in \overline{1, N^{new}}, \tag{2}$$

$$\sum_{\mu=1}^{\epsilon_i} z_{i\mu}^k \leq 1,\ i \in \overline{1, N^{new}},\ k \in \overline{1, L_i}, \tag{3}$$

$$z_{i\gamma}^{\pi} + z_{i\omega}^{\lambda} \leq 1,\ i \in \overline{1, N^{new}},\ \forall \pi, \lambda \in \overline{1, L_i},\ \forall \gamma, \omega \in \overline{1, \epsilon_i}, \gamma \neq \omega, \tag{4}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{L_i} s_{ji}^{k\ new} \leq \big(S^{max} - \sum_{i'=1}^{N^{exist}} d_{i'}^{exist}\big) \sum_{\mu=1}^{\epsilon_i} y_{i\mu}, i \in \overline{1, N^{new}}, \tag{5}$$

$$y_{i\mu} \leq \sum_{k=1}^{L_i} z_{i\mu}^k \leq \big(\sum_{k=1}^{L_i} Q_{i\mu}^{k\ new\ max}\big) y_{i\mu},\ i \in \overline{1, N^{new}},\ \mu \in \overline{1, \epsilon_i}, \tag{6}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{L_i} s_{ji}^{k\ new} \leq \sum_{\mu=1}^{\epsilon_i} y_{i\mu} d_{i\mu}^{max\ new},\ i \in \overline{1, N^{new}}, \tag{7}$$

$$\sum_{j=1}^{M} s_{ji}^{k\ new} \leq \sum_{\mu=1}^{\epsilon_i} z_{i\mu}^k Q_{i\mu}^{k\ new\ max},\ i \in \overline{1, N^{new}},\quad k \in \overline{1, L_i}, \tag{8}$$

$$\sum_{j=1}^{M} s_{ji'}^{k'\ exist} \leq Q_{i'}^{k'\ exist},\ k' \in \overline{1, l_{i'}},\ i' \in \overline{1, N^{exist}}, \tag{9}$$

$$\sum_{i=1}^{N^{new}} \sum_{k=1}^{L_i} s_{ji}^{k\ new} + \sum_{i'=1}^{N^{exist}} \sum_{k'=1}^{l_{i'}} s_{ji'}^{k'\ exist} = s_j,\ j \in \overline{1, M}, \tag{10}$$

$$s_j^{min} \leq s_j \leq s_j^{max},\ j \in \overline{1, M}, \tag{11}$$

$$S^{min} \leq \sum_{j=1}^{M} s_j \leq S^{max}, \tag{12}$$

$$\sum_{j=1}^{M} \sum_{k'=1}^{l_{i'}} s_{ji'}^{k'\ exist} \leq d_{i'}^{exist},\quad i' \in \overline{1, N^{exist}}, \tag{13}$$

$$\begin{aligned} s_{ji}^{k\ new} \geq 0,\ i \in \overline{1, N^{new}},\ j \in \overline{1, M},\ k \in \overline{1, L_i}, \\ s_{ji'}^{k'\ exist} \geq 0,\ i' \in \overline{1, N^{exist}},\ j \in \overline{1, M},\ k' \in \overline{1, l_{i'}}, \\ y_{i\mu} \in \{0, 1\}, \mu \in \overline{1, \epsilon_i},\ i \in \overline{1, N^{new}}, \\ z_{i\mu}^k \in \{0, 1\}, \mu \in \overline{1, \epsilon_i},\ i \in \overline{1, N^{new}},\ k \in \overline{1, L_i}. \end{aligned} \tag{14}$$

The constraints in Problem (1)-(14) have the following meaning:

(2) – no more than one new transport hub (of any variant of the hub capacity) can be located at each place from the set $\overline{1, N^{new}}$,

(3) – no more than one new access road of each of L_i types to a new transport hub at place i, $i \in \overline{1, N^{new}}$ (whose capacity corresponds to the hub capacity) can be built,

(4) – new access roads of all L_i types to a new transport hub at place i, $i \in \overline{1, N^{mew}}$ can be built for only one chosen variant of the capacity of this hub,

(5) - a cargo flow via a new transport hub at place $i \in \overline{1, N^{new}}$ cannot exceed the maximum of the cargo volume expected to be moved via all the new transport hubs to be located in places from the set $\overline{1, N^{new}}$,

(6) - for any capacity $\mu \in \overline{1, \epsilon_i}$ of a new hub that is to be built at place i, $i \in \overline{1, N^{new}}$, an access road to the hub of at least one type (corresponding to the capacity of this hub) is to be built,

(7) – a cargo flow via every new transport hub that is to be built at place i, $i \in \overline{1, N^{new}}$ cannot exceed the capacity of this hub,

(8) – a cargo flow via a new transport hub at place i, $i \in \overline{1, N^{new}}$ that comes to this hub via any new access road to this hub cannot exceed the capacity of this road,

(9) – a cargo flow via every existing access road $k' \in \overline{1, l_{i'}}$ to existing transport hub i', $i' \in \overline{1, N^{exist}}$ cannot exceed the capacity of this road,

(10) – an expected cargo flow via location j, $j \in \overline{1, M}$ equals the sum of the flows that go there via the existing transport hubs and via new ones to be built,

(11) – each cargo flow volume s_j, $j \in \overline{1, M}$ is considered to be a known number in *Situation 1, Case A*, and it is considered to vary in *Situation 1, Case 2* (see Section II). In *Situation 2*, for each cargo flow via location j, $j \in \overline{1, M}$, the volume of this flow is to remain within certain known limits,

(12) – the total volume of the cargo flow via all the M locations is to remain within certain known limits,

(13) – a cargo flow via every existing transport hub i', $i' \in \overline{1, N^{exist}}$ does not exceed the capacity of this hub,

(14) all the variables in the model are non-negative.

Remark 3.

It is assumed that system of constraints (2)-(14) is compatible. This is easy to verify by solving an auxiliary mixed programming problem that can be formulated in line with the methodology described in (Belenky, 1981). The compatibility, particularly, means that the existing freight transportation network is capable of handling the expected volumes of cargo to be moved between all the M nodes of this network and the regional transport hubs (both the existing ones and those to be built). If this assumption does not hold, the verification by means of solving the above-mentioned auxiliary mixed programming problem determines which existing roads should be modernized to increase their capacities. Also, it determines what new roads should be built to meet the expected demand for moving cargoes via the network. In either case, parameters of all the nodes and roads of all the kinds in the network, except for those associated with new transport hubs and access roads to them (see also Basic Assumption 2), under which the problem of finding the needed volume of investment is formulated can be considered to be known.

One should bear in mind that the goal function in Problem (1)-(14) only partly reflects the expenses that the regional administration (and the potential public-private partnership) should expect to bear. (These expenses are calculated with respect to the end of each year, after all the new transport hubs and (new) access roads to them start functioning.) This is the case since in calculating expenses described by the goal function of the above type, only the expenses associated with developing new transport hubs and (new) access roads to them (along with the revenue to be generated by the project via regional taxes) are taken into account. (It is this revenue that the regional administration would like the potential private partners to consider as its financial contribution to the public-private partnership.)

To calculate the total expenses associated with running the project, one should add the number

$$\psi(\sum_{i'=1}^{N^{exist}} c_{i'}^{exist} + \sum_{i'=1}^{N^{exist}} \sum_{k'=1}^{l_{i'}} q_{i'}^{k'\ exist})$$

to the minimum value of the goal function calculated as a result of solving the Problem (1)-(14). This number reflects the expenses associated with the maintenance of the existing transport hubs and access roads to them. (Since these expenses are represented by a real positive number, there is no need to include them into the goal function of Problem (1)-(14).)

Here, it is assumed that the values of the parameters $c_{i'}^{exist}$ and $q_{i'}^{k'\ exist}$, $i' \in \overline{1, N^{exist}}$, $k' \in \overline{1, l_{i'}}$ do not change during the whole planning period of ψ years. Otherwise, if they remain unchanged only during each year of the planning period, the sum

$$\sum_{\kappa=1}^{\psi}\Big(\sum_{i'=1}^{N^{exist}} c_{i'}^{exist\ \kappa} + \sum_{i'=1}^{N^{exist}}\sum_{k'=1}^{l_{i'}} q_{i'}^{k'\ exist\ \kappa}\Big),$$

where $c_{i'}^{exist\ \kappa}$ and $q_{i'}^{k'\ exist\ \kappa}$ are the values of the corresponding parameters during year κ, $\kappa \in \overline{1,\psi}$, should be added to the above-mentioned expenses.

Also, one should bear in mind that if the inequality $d_{i\mu}^{max\ new} \leq S^{max} - \sum_{i'=1}^{N^{exist}} d_{i'}^{exist}$ holds for all $i \in \overline{1, N^{new}}$, system (5) in the system of constraints of Problem (1)-(14) becomes redundant.

Remark 4. One should bear in mind that to find whether providing transportation services under a potential concession agreement is profitable to the private investors, they should first solve a separate optimization problem. This problem may have the same system of constraints (2)-(14) and the goal function describing the profit expected to be received from providing transportation services. This goal function will have the form

$$\Big(\sum_{i'=1}^{N^{exist}}\sum_{j=1}^{M}\sum_{k'=1}^{l_{i'}} t_{ji'}^{k'\ exist} s_{ji'}^{k'\ exist} + \sum_{i=1}^{N^{new}}\sum_{j=1}^{M}\sum_{k=1}^{L_i} t_{ji}^{k\ new} s_{ji}^{k\ new}\Big) -$$
$$- f_{ex}(s_{ji'}^{k'\ exist}, s_{ji}^{k\ new}),$$

where the function $f_{ex}(s_{ji'}^{k'\ exist}, s_{ji}^{k\ new})$ describes expenses associated with providing transportation services in volumes determined by the vectors $s_{ji'}^{k'\ exist}, s_{ji}^{k\ new}$. This goal function describes the maximum revenue that the investors may expect to receive yearly in the framework of the concession agreement, provided they will be the only operator to provide transportation services to all the cargo owners and cargo recipients in the region. This maximum revenue is calculated by maximizing this function under the system of constraints (2)-(14). This maximum should be multiplied by the number of years, say, ψ under which the concession agreement is signed and compared with the value of the goal function in Problem (1)-(14) in which the second term in the goal function is present with a plus sign. If even in this ideal situation (in which the investors are the only provider of the transportation services for ψ years), the total expenses exceed the expected profit, the investors will be unlikely to be interested in providing any transportation services in the framework of the concession agreement. So, if this is the case, they will be considering only the investment in developing the regional freight transportation infrastructure.

If, however, this expected profit will exceed the expected total expenses, the investors should try to estimate their profit and expenses in a real situation. That is, they should estimate both values under competition conditions, i.e. when the transportation services that they may provide will have only a limited demand, and they may not know exactly what the limitation numbers are. However, considering these problems goes beyond the scope of the present paper.

Problem (1)-(14) is a mathematical programming one with mixed variables, which corresponds to *Situation 1, Case B*, described in Section I. For *Situation 1, Case A* (described in Section I), inequalities (11) are to be replaced by the equalities $s_j = s_j^{max}$, $j \in \overline{1, M}$. Additionally, inequalities (12) are to be excluded from the system of constraints of Problem (1)-(14) (since the equality $\sum_{j=1}^{M} s_j^{max} = S^{max}$ must hold). In both cases, this problem can be solved with the use of standard software packages that are currently widely available (see, for instance, (Mittelmann 2017)).

Since even the average values of the parameters in Problem (1)-(14) may not be known with certainty, the administration may decide to estimate the "worst-case scenario." To this end, the estimates of the areas of the parameter values that these parameter values may belong to (in any particular planning period) should be taken into consideration. This is also the case when a) the parameters $t_{ji}^{k\ exist}$, $t_{ji}^{k\ new}$, $f_{i\mu}$, and $g_{i\mu}^k$ are the piece-wise linear functions described earlier, and b) the parameters determining the maintenance costs for the new transport hubs and those for access roads to these hubs may vary.

In such situations, a robust optimization problem should be formulated and solved.

Let

$$x = (s_{ij}^{k\ new},\ s_{i'j}^{k'\ exist}) \in R_+^{M \sum_{i=1}^{N^{new}} L_i + M \sum_{i'=1}^{N^{exist}} l_{i'}},$$

$$t = (t_{ij}^{k\ new},\ t_{i'j}^{k'\ exist}) \in R_+^{M \sum_{i=1}^{N^{new}} L_i + M \sum_{i'=1}^{N^{exist}} l_{i'}},$$

$y = (y_{i\mu}) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i}$, $f = (f_{i\mu}) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i}$, $c = (c_{i\mu}^{new}) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i}$ $g = (g_{i\mu}^k) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i L_i}$, $q = (q_{i\mu}^{k\ new}) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i L_i}$, $z = (z_{i\mu}^k) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i L_i}$, be vector variables, and let the inclusions

$$\begin{aligned}
t \in \Lambda = \{t \geq 0 : \ tI \leq l\}, \ f \in \Theta = \{f \geq 0 : \ fF \leq r\}, \\
g \in \Gamma = \{g \geq 0 : \ gG \leq e\}, c \in \Delta = \{c \geq 0, \ cW \leq \lambda\}, \\
q \in \Upsilon = \{q \geq 0, \ q\Psi \leq \eta\} \\
(x, y, z) \in \Phi = \{(x, y, z) \geq 0 : P(x, y, z) \geq \delta\}, \\
x \in MX = \{x \geq 0 : \ Ax \geq b\}, \\
y \in \Omega Y = \{y \geq 0 : \ By \geq \pi, y \in T_{\sum_{i=1}^{N^{new}} \epsilon_i}\}, \\
z \in HZ = \{z \geq 0 : Kz \geq h, \ z \in T_{\sum_{i=1}^{N^{new}} L_i \epsilon_i}\},
\end{aligned}$$

where $I, F, G, W, \Psi, P, A, B, K$ are matrices and $l, r, e, \lambda, \eta, \delta, b, \pi, h$ are vectors of corresponding dimensions, $T_{\sum_{i=1}^{N^{new}} \epsilon_i}$ is a unit cube in $R_+^{\sum_{i=1}^{N^{new}} \epsilon_i}$, and $T_{\sum_{i=1}^{N^{new}} L_i \epsilon_i}$ is a unit cube in $R_+^{\sum_{i=1}^{N^{new}} \epsilon_i L_i}$, hold.

Here, it is assumed that a) the sets $MX, \ \Lambda, \ \Theta, \ \Gamma, \ \Delta$, and Υ are (nonempty) polyhedra in Euclidean spaces of corresponding dimensions, i.e., the systems of linear inequalities describing these sets are compatible (see *Remark* 3), b) each of the sets ΩY and HZ is a subset of a convex polyhedron in a finite-dimensional space of a corresponding dimension ($R^{\sum_{i=1}^{N^{new}} \epsilon_i}$ and $R^{\sum_{i=1}^{N^{new}} L_i \epsilon_i}$, respectively) and consists of only the vectors from the polyhedron being its vertices each coordinate of which is either 0 or 1 (i.e., belong to the unit cubes $T_{\sum_{i=1}^{N^{new}} \epsilon_i}$ and $T_{\sum_{i=1}^{N^{new}} L_i \epsilon_i}$, respectively), and c) $P(x, y, z) \geq \delta$ determines a subset of a polyhedron in which each vector (x, y, z) with a particular pair of the integer components (y, z) is located.

Under the assumption made and with the use of this notation, one can formulate a problem that corresponds to *Situation 2* (see Section I) and generalizes Problem (1)-(14). A solution to the generalized problem allows the regional administration to estimate the above-mentioned "worst-case scenario." For the planning period of $\psi \geq 1$ years and under the Basic Assumptions 1-9, this problem can be written in the vector-matrix form, for instance, as follows:

$$\begin{aligned}
\max_{(t,f,c,g,q) \in \Lambda \times \Theta \times \Delta \times \Gamma \times \Upsilon} \big(-\nu\psi \, \langle t, x \rangle + \langle (f + \psi c), y \rangle + \langle (g + \psi q), z \rangle \big) \\
\to \min_{(x,y,z) \in (MX \times \Omega Y \times HZ) \cap \Phi}
\end{aligned} \quad . \tag{15}$$

Let $u = (t, (f + \psi c), (g + \psi q)),\ v = (x, y, z)$, let $D = \begin{pmatrix} -\nu\psi E_1 & 0_1 & 0_2 \\ 0_3 & E_2 & 0_4 \\ 0_5 & 0_6 & E_3 \end{pmatrix}$ be a quadratic matrix with the number of rows equalling the sum of the numbers of all the vector components belonging to the vectors t, f, and g, where $E_1\ E_2,\ E_3$ are unit matrices of the sizes $\left(M \sum_{i=1}^{N^{new}} L_i + M \sum_{i'=1}^{N^{exist}} l_{i'}\right)$, $\sum_{i=1}^{N^{new}} \epsilon_i$, and $\sum_{i=1}^{N^{new}} \epsilon_i L_i$, respectively, $0_\kappa,\ \kappa \in \overline{1,6}$ are zero matrices of the corresponding sizes, and let $\Pi = (MX \times \Omega Y \times HZ) \cap \Phi$.

Then Problem (15) can be rewritten as

$$\max_{u \in \Lambda \times \Theta \times \Delta \times \Gamma \times \Upsilon} \langle u, Dv \rangle \to \min_{v \in \Pi}. \tag{16}$$

Remark 5.

One should emphasize the difference between the statement underlying the formulation of Problem (15) and that of another problem statement that may, eventually, be considered by the regional administration. This other problem statement may appear in an attempt to deal with the expected volumes of cargo to be moved via the transportation network (being part of the regional freight transportation infrastructure) to be built. That is, in Problem (15), the administration chooses both locations for new transport hubs and types of new access roads to these hubs to be built, along with cargo flows to go via the hub locations for each particular cargo flow. This makes the flow volumes a part of the variables that the regional administration controls. However, a) the costs of cargo transportation, b) the costs of building new transport hubs, c) costs of building new access roads to them, and d) the maintenance costs for both the hubs and access roads to them are considered as market variables, not controlled by the administration.

In the above-mentioned other statement of the problem of determining optimal locations for new transport hubs and types of access roads to them, the transport costs are considered to be under the regional administration control. The costs of building new transport hubs, costs of building new access roads to them, and the maintenance costs for both the hubs and access roads to them are still considered to be market variables. However, the volumes of cargoes to be moved in particular directions (flow volumes) are considered to be market variables as well. In this case, a different minimax problem is to be formulated.

If $\hat{u} = (x, (f + \psi c), (g + \psi q)),\ \hat{v} = (t, y, z)$, and $\hat{\Pi} = (\Lambda \times \Omega Y \times HZ) \cap \Phi$, this minimax problem can then be written as

$$\max_{\hat{u} \in MX \times \Theta \times \Delta \times \Gamma \times \Upsilon} \langle \hat{u}, D\hat{v} \rangle \to \min_{\hat{v} \in \hat{\Pi}}. \tag{16'}$$

However, since the vector x is present in the description of both the set MX and the set $\hat{\Pi}$, this minimax problem turns out to be the one with connected variables. That is, while the maximization of the function $\langle \hat{u}, D\hat{v}\rangle$ is done over the vector variables that include the vector x, the minimization of the maximum function is done over the vector variables $\hat{v} = (t, y, z)$. However, the vector x is present in the description of the set $\hat{\Pi}$ (via the description of the set Φ), binding the variables y, z and x, which makes Problem (16') a problem with connected variables. Even when all the variables are continuous (which is not the case in Problem (16')), problems with connected variables are more complicated than Problem (15) (Belenky 1997). In any case, Problem (16') is not a subject of considerations in the present paper.

Remark 6.

The goal function in Problem (15) can also be rewritten as follows:

$$\max_{(t,f,c,g,q)\in\Lambda\times\Theta\times\Delta\times\Gamma\times\Upsilon}\big(-\nu\psi\,\langle t,x\rangle+\langle f,y\rangle+\psi\,\langle c,y\rangle+\langle g,z\rangle+\psi\,\langle q,z\rangle\big) \rightarrow \min_{(x,y,z)\in(MX\times\Omega Y\times HZ)\cap\Phi}. \tag{17}$$

Let now $(f,c)=\tilde{f},\ (g,q)=\tilde{g}$, and let

$$\tilde{f}\in\tilde{\Theta}=\{\tilde{f}=(f,c)\geq 0:\ fF\leq r,\ cW\leq\lambda\},$$
$$g\in\tilde{\Gamma}=\{\tilde{g}=(g,q)\geq 0:\ gG\leq e,\ q\Psi\leq\eta\}.$$

Here, $\tilde{\Theta}$ and $\tilde{\Gamma}$ are polyhedra, and Problem (15) can be rewritten in the form

$$\max_{(t,\tilde{f},\tilde{g})\in\Lambda\times\tilde{\Theta}\times\tilde{\Gamma}}\left(-\nu\psi\,\langle t,x\rangle+\left\langle\tilde{f}\tilde{E}_2,y\right\rangle+\left\langle\tilde{g}\tilde{E}_3,z\right\rangle\right) \rightarrow \min_{(x,y,z)\in(MX\times\Omega Y\times HZ)\cap\Phi} \tag{17'}$$

where $\tilde{E}_2=\begin{pmatrix}E_2\\ \psi E_2\end{pmatrix}$, $\tilde{E}_3=\begin{pmatrix}E_3\\ \psi E_3\end{pmatrix}$, and $E_2,\ E_3$ are unit matrices (see (15) and (16)). Thus, Problem (16) can be rewritten as

$$\max_{\tilde{u}\in\Lambda\times\tilde{\Theta}\times\tilde{\Gamma}}\left\langle\tilde{u},\tilde{D}v\right\rangle\rightarrow\min_{v\in\Pi}, \tag{18}$$

where $\tilde{u}=(t,\tilde{f},\tilde{g}),\ v=(x,y,z)$, and $\tilde{D}=\begin{pmatrix}-\nu\psi E_1 & 0 & 0\\ 0_7 & E_2 & 0_8\\ 0_9 & \psi E_2 & 0_{10}\\ 0_{11} & 0_{12} & E_3\\ 0_{13} & 0_{14} & \psi E_3\end{pmatrix}$, and $0_\kappa,\ \kappa\in\overline{7,14}$ are zero vectors of corresponding sizes.

A solution to minimax Problem (17') provides the estimate of only a part of the expenses of the potential public-private partnership associated with developing a regional freight transportation infrastructure with newly built transport hubs and access roads to them. As mentioned in considering Problem (1)-(14), this estimate does not take into consideration the expenses associated with the maintenance of the already existing transport hubs and access roads to them (during the planning period of ψ years). To take these expenses into consideration in estimating the economic effectiveness of the regional freight transportation infrastructure for ψ years, $\psi \geq 1$, one should add either the number

$$\psi\left(\sum_{i'=1}^{N^{exist}} c_{i'}^{exist} + \sum_{i'=1}^{N^{exist}} \sum_{k'=1}^{l_{i'}} q_{i'}^{k'\ exist}\right)$$

or the number

$$\sum_{\kappa=1}^{\psi}\left(\sum_{i'=1}^{N^{exist}} c_{i'}^{exist\ \kappa} + \sum_{i'=1}^{N^{exist}} \sum_{k'=1}^{l_{i'}} q_{i'}^{k'\ exist\ \kappa}\right),$$

to the minimax value to be obtained as a result of solving Problem (17'). Here, $c_{i'}^{exist\ \kappa}$ and $q_{i'}^{k'\ exist\ \kappa}$ are the values of the corresponding parameters during year κ, $\kappa \in \overline{1,\psi}$. Finally, one can, of course, consider these two costs to be as uncertain as are the costs $c_{i\mu}^{new}$ and $q_{i\mu}^{k\ new}$ and include corresponding vector variables into the formulation of the minimax problem. This can be done in just the same way this takes place for the variable vectors c and q.

IV. The Basic Assertion

For the sake of definiteness, the Basic Assertion is formulated with respect to Problem (18) assuming that the parameters $c_{i'}^{exist}$ and $q_{i'}^{k'\ exist}$, $i' \in \overline{1,l_{i'}}$, $i' \in \overline{1,N^{exist}}$ are not variables over the planning period of ψ years.

Basic Assertion.

The equality

$$\min_{v\in\Pi} \max_{\tilde{u}\in\Lambda\times\tilde{\Theta}\times\tilde{\Gamma}} \left\langle \tilde{u}, \tilde{D}v \right\rangle = \min_{v\in\Pi,\ Jw\geq\tilde{D}v} \langle \omega, w \rangle$$

holds, where J is a matrix and ω is a vector of corresponding dimensions.

Proof is presented in Appendix 1.

Corollary 1.

Problem (18) is reducible to a mixed programming problem with linear constraints.

Corollary 2.

Let $Y = \{y \geq 0 : \; By \geq \pi\}$, and $Z = \{z \geq 0 : Kz \geq h\}$. Then the number

$$\min_{v \in (MX \times Y \times Z) \cap \Phi, \; Jw \geq \tilde{D}v} \langle \omega, w \rangle$$

is the lower bound for the number

$$\min_{v \in \Pi, \; Jw \geq \tilde{D}v} \langle \omega, w \rangle,$$

and this lower bound can be found by solving a linear programming problem

$$\langle \omega, w \rangle \to \min_{v \in (MX \times Y \times Z) \cap \Phi, \; Jw \geq \tilde{D}v}.$$

According to (Belenky 1981), a solution to this linear programming problem, along with that to the dual (to this linear programming) one, determine a saddle point in an antagonistic game on the polyhedra $\Lambda \times \tilde{\Theta} \times \tilde{\Gamma}$ and $MX \times Y \times Z$ with the payoff function $\left\langle \tilde{u}, \tilde{D}v \right\rangle$.

V. Testing the proposed tool on model data

The proposed tool has been tested on several sets of model data collected by the authors using open source (see https://github.com/ggfedin/Model_dataset). The aim of the testing was to demonstrate how the proposed decision-support tool can be used in negotiations between a regional administration and potential investors. That is, for any set of model data, which the negotiating parties may change many times, calculation results obtained with the use of this tool can be presented in the form of easily read tables and observable illustrative pictures. The tables are presented in Appendix 2, whereas the pictures are presented in Appendix 3.

A model region with 32 cargo origin/destination points was "designed" based on the information taken from these open sources. It was assumed that there are two already functioning transport hubs ($i' \in \{1, 2\}$) and eight locations for potentially locating new transport hubs to be built ($i \in \overline{3, 10}$). Two types of access roads (railways and highways) to both the new and existing transport hubs were considered ($k, k' \in \{1, 2\}$), and it was assumed that each type of access road can have two capacities to choose from. Three different tax rates included in the transportation tariffs (that constitute

ν percents of these tariffs, $\nu \in \{1, 7, 13\}$), and three particular planning periods (with the number of ψ years, $\psi \in \{1, 3, 5\}$) were considered. Picture 1 from Appendix 3 shows the geographic locations of the cargo origin/destination points in the “designed” region, possible locations for new transport hubs in this region, and the location of the existing transport hubs there.

Picture 1 from Appendix 3 shows the geographic locations of the cargo origin/destination points in the ’designed” region, possible locations for new transport hubs in this region, and the location of the existing transport hubs there.

Calculations for four variants of the mixed programming problem (“*Situation 1, Case A, Subcase 1*”, “*Situation 1, Case A, Subcase 2*”, “*Situation 2, Case B, Subcase 1*” and “*Situation 1, Case B, Subcase 2*”) and for two variants of a robust (minimax) optimization problem (“*Situation 2, Subcase 1*” and “*Situation 2, Subcase 2*”) were conducted.

For “*Situation 1, Case A, Subcase 1*” and for “*Situation 1, Case A, Subcase 2*,” described in Sections I and III, the following data was assigned: a) the capacity of a new transport hub for each of the two variants of the hub capacity at each of the eight potential locations of new transport hubs ($i \in \overline{3, 10}$), b) the capacities of the existing two transport hubs ($i' \in \{1, 2\}$), c) the capacity of a new access road to each new transport hub for each of the two variants of the hub capacity, d) the capacities of each access road to each of the two existing transport hubs, e) the expected yearly demand for cargo services at each of the above 32 cargo origin/destination points, f) the expected total yearly demand for cargo services in the region, g) the cost of building a new transport hub for each of the two variants of the hub capacity, h) the cost of building a highway to a new transport hub for both variants of the highway capacity, i) the cost of building a railway to a new transport hub for both variants of the hub capacity, and j) the maintenance cost for highways for both variants of the highway capacity and for railways for both variants of the railway capacity. The costs for transporting a unit volume of cargo between each of the 32 cargo origin/destination points and between each of these points and the existing and new transport hubs were calculated.

The calculations were conducted proceeding from a) the length of the route between the above points and the hubs (computed via Google Maps), b) the average cost of transporting a unit volume of cargo per kilometer, and c) a discount that depends on the length of the route. As mentioned in Section III, *Subcase 2* differs from *Subcase 1* only by the sign before the second term in the goal function. For *Subcase 2* the goal function takes the form

$$\sum_{i=1}^{N^{new}} \sum_{\mu=1}^{\epsilon_i} (f_{i\mu} + \psi c_{i\mu}^{new}) y_{i\mu} + \sum_{i=1}^{N^{new}} \sum_{k=1}^{L_i} \sum_{\mu=1}^{\epsilon_i} (g_{i\mu}^k + \psi q_{i\mu}^{k\ new}) z_{i\mu}^k$$

$$+\nu\psi\Bigg(\sum_{i'=1}^{N^{exist}} \sum_{j=1}^{M} \sum_{k'=1}^{l_{i'}} t_{ji'}^{k'\ exist} s_{ji'}^{k'\ exist} + \sum_{i=1}^{N^{new}} \sum_{j=1}^{M} \sum_{k=1}^{L_i} t_{ji}^{k\ new} s_{ji}^{k\ new} \Bigg) \rightarrow \min . \qquad (1')$$

Problems (1)-(14) and (1')-(14) were solved with the assigned and calculated parameters as described above, and the calculation results for "*Situation 1, Case A, Subcase 1.*" and "*Situation 1, Case A, Subcase 2*" are presented in Tables 1 and 2 from Appendix 2, respectively. Two examples of the optimal hub locations and their capacities in Problem (1)-(14) for *Case A, Subcase 1* and in Problem (1')-(14) for *Case A, Subcase 2* for the values of the parameters ($\nu = 7, \psi = 5$), which correspond to line 8 in Table 1 and in Table 2, respectively, are shown on Picture 2 and Picture 3 from Appendix 3, respectively.

For "*Situation 2, Case B, Subcase 1*" and for "*Situation 1, Case B, Subcase 2*," described in Sections I and III, it was assumed that the expected demands on transporting cargo at each of the network nodes, as well as the expected total demand for cargo services in the region, vary. It was assumed that for the demand on transporting cargo at each of the above 32 locations, the maximum demand could not exceed 30% of the demand specified in e) in the above (for *Case A*). Further, it was assumed that the minimum demand could not be lower than 20% of this demand, whereas the total demand for cargo services in the region could not exceed 15% of the demand specified in f) in the above (for *Case A*). Finally, it was assumed that the minimum of the total demand could not be lower than 5% of the demand specified in f).

All the other assumptions, i.e., a)-d) and g)-i), were the same as in *Case A*. Problems (1)-(14) and (1')-(14) were solved with thus chosen, assigned, and calculated data, and the calculation results are presented in Tables 3 and 4 from Appendix 2, respectively. Two examples of the optimal hub locations and their capacities in Problems (1)-(14) for *Case B, Subcase 1* and in Problem (1')-(14) for *Case B, Subcase 2* for the values of the parameters ($\nu = 7, \psi = 5$), which correspond to line 8 in Table 3 and in Table 4, respectively, are shown on Picture 4 and Picture 5 from Appendix 3, respectively.

For the robust (minimax) problem in *Situation 2*, described in Section I, with *Subcases 1 and 2*, described in Section III, particular values of the following parameters in this model were used based on the expert opinions:

— The maximum and the minimum average cost of transporting a unit volume of cargo between node j and a new (or an existing) transport hub at point i (i') via access road of type k (k') for each $i \in \overline{3,10}$, ($i' \in \{1,2\}$), for each $k \in \{1,2\}$, and for both variants of the hub capacity,

— the maximum and the minimum average costs of building a new transport hub at each place i, $i \in \overline{3,10}$,

— the maximum and the minimum average costs of building a new access road of each of the two types (highways and railways) to a new transport hub i, $i \in \overline{3,10}$,

— the maximum and the minimum average maintenance costs for every new transport hub i, $i \in \overline{3,10}$,

— the maximum and the minimum average maintenance costs for highways and railways to every new transport hub, and

— the maximum and the minimum average maintenance costs for highways and railways to every existing transport hub.

Also, the following assumptions were made:

a) The unknown cost of transporting a unit volume of cargo between node j and a new (or existing) transport hub at point i (i') via access road of type k (k') could not exceed 30% and could not be lower than 30% of its current value (i.e., corresponding to the existing scheme of moving the cargo), $k, k' \in \{1,2\}$, $i \in \overline{3,10}$, $i' \in \{1,2\}$,

b) the unknown cost of building a new transport hub at point i, $i \in \overline{3,10}$ of both variants of the hub capacity could not exceed 20% and could not be lower than 9% of its current value (i.e., corresponding to the existing market value),

c) the unknown cost of building a highway to a new transport hub at point i, $i \in \overline{3,10}$ of both variants of the hub capacity could not exceed 15% and could not be lower than 15% of its current value (i.e., currently existing market value),

d) the unknown cost of building a railway to a new transport hub i, $i \in \overline{3,10}$ of both variants of the hub capacity could not exceed 17.5% and could not be lower that 17.5% of its current value (i.e., currently existing market value),

e) the unknown maintenance cost for a new cargo transport hub at point i, $i \in \overline{3,10}$ of both variants of the hub capacity could not exceed 25% and could not be lower than 10% of its current value (i.e., currently existing market value),,

f) the unknown maintenance cost for highways and railways to a new cargo transport hub at point i, $i \in \overline{3,10}$ could not exceed 50% and could not be lower than 50% of its current value (i.e., currently existing market value),, and

g) the maintenance costs for the existing cargo transport hubs and the maintenance costs for the roads to the existing cargo transport hubs were known exactly.

For *Situation 2, Subcase 1* the robust (minimax) problem is formulated as Problem (18), and for *Situation 2, Subcase 2* the problem takes the form

$$\max_{\tilde{u}\in\Lambda\times\tilde{\Theta}\times\tilde{\Gamma}} \left\langle \tilde{u}, \tilde{D}'v \right\rangle \to \min_{v\in\Pi}, \tag{18'}$$

where

$$\tilde{D}' = \begin{pmatrix} \nu\psi E_1 & 0 & 0 \\ 0_7 & E_2 & 0_8 \\ 0_9 & \psi E_2 & 0_{10} \\ 0_{11} & 0_{12} & E_3 \\ 0_{13} & 0_{14} & \psi E_3 \end{pmatrix}.$$

Problems (18) and (18') were solved with the input data, which was described above. The calculation results for *Situation 2, Subcase 1* are presented in Table 5, and the calculation results for *Situation 2, Subcase 2* are presented in Table 6 from Appendix 2, respectively.

Two examples of optimal hub locations and their capacities, and optimal assignments of cargo origin/destination points to the hubs in Problem (18) for *Situation 2, Subcase 1* and in Problem (18') for *Situation 2, Subcase 2* for the values of the parameters ($\nu = 7, \psi = 5$), which correspond to line 8 in Table 5 and in Table 6, respectively, are shown on Pictures 6 and Picture 7 from Appendix 3, respectively.

Both in *Situation 1* (in Problem (1)-(14) for *Case A* and in Problem (1')-(14) for *Case B*) and *Situation 2* (in Problem (18) for *Subcase 1*and in Problem (18') for *Subcase 2*), the systems of constraints and the goal functions were formed in accordance with their description presented in Section III. These problems were solved with the use of the solver *Intlinprog*, being part of the MatLab interactive environment installed in a personal laptop. The laptop was equipped with a 2.5-GHz Intel Core i5 CPU and 16-GB RAM, based on the Windows platform. The optimal solutions were obtained in less that 0.5 second for Problem (1)-(14) and for Problem (1')-(14) in all the nine combinations of the values for $\nu = 7$ and $\psi = 5$ in each of the two subcases described by each of these two problems. The optimal solutions were obtained in less than three seconds for Problem (18) and Problem (18') in all the nine combinations for $\nu = 7$ and $\psi = 5$ in one subcase described by each of these two problems.

For *Situation 2, Subcase 1*, the calculation results show that under a particular set of the input data, in considering both Problem (1)-(14) and Problem (18), the expected yearly revenue does not cover the required expenses within one year after all the new facilities start functioning. However, in more than a year, the functioning of the regional freight transportation infrastructure generates substantial revenue. Based on these estimates, which are quite expectable for large-scale projects, the regional administration may make at least two strategic decisions.

1. The regional administration may offer a part of this (substantial) amount as its financial contribution to the public-private partnership in negotiations with potential partners from the private sector.

2. The regional administration may decide not to form any partnership with the private sector on the project for providing the functioning of the regional freight transportation infrastructure. This may happen if a) the project is expected to generate profit in a relatively short period of time after all facilities of the new regional freight transportation infrastructure start functioning, and b) the regional administration can get a loan from, say, a bank under acceptable conditions. (Certainly, what period of time should be viewed as a short one is to be determined.)

According to Tables 1, 3, and 5, the second strategic decision may be the case

a) for the estimates obtained by solving Problem (1)-(14) for *Situation 1, Case A and Case B* for three years and for five years (under 7% and under 13% of the tax value both in Case A and in Case B),

b) for the estimates obtained by solving Problem (18) for three years and for five years (under 13% of the tax value).

Depending on the potential loan conditions, there could be certain combinations of both strategies. Further, a determination of how much to borrow and how much to ask the private investors to contribute may require the use of mathematical methods. Finally, other strategies that are based upon the above estimates of the expected financial results of the project functioning could be formed. At the same time, the regional administration should bear in mind that the calculation results are those for a particular set of the data, and changing the data can lead to calculating a strategy that may seem more promising. Also, the calculation results may suggest that the values of some parameters reflecting the uncertainty conditions should be reconsidered. For instance, the boundaries within which the expected volumes of cargo flows may vary could be such parameters.

Thus, the estimates that can be calculated with the use of the proposed decision-support tool may provide a certain flexibility to the regional administration in choosing its financial strategy for developing a regional freight transportation infrastructure.

Section III describes how private investors participating in negotiations with the regional administration on potential investments in developing a new regional fright transportation infrastructure can benefit from using the proposed decision-making tool. That is, first, they are to calculate the maximum yearly revenue that they may expect to receive by operating the new transportation network in the framework of the new regional freight transportation infrastructure. (They can do this by solving an optimization problem discussed in Section III, provided they know the function describing their expenses associated with providing transportation services in the framework of the new regional freight transportation infrastructure.) Second, they are to calculate the total expenses associated with both developing this new infrastructure and with paying taxes to the regional administration and to compare these expenses with the revenue. (To calculate the expenses, they should solve Problem (1')-(14) and Problem (18').)

With respect to the model data, as one can see from Tables 2, 4, and 6 and from Pictures 6 and 7, if the revenue exceeds the total expenses, the investors may agree to finance the infrastructure corresponding to Picture 7. This infrastructure differs from the one depicted on Picture 6 while the regional administration may prefer them to finance the infrastructure depicted on Picture 6.

It is clear that in a) *Situation 1, Case A, Subcase 2*, b) *Situation 1, Case B, Subcase 2*, and c) *Situation 2, Subcase 2*, the regional administration may affect the total expenses of its potential partners from the private sector by changing the tax value. The calculation results presented in Appendix 2 under a particular set of the input data show that the share of taxes in total expenses is relatively small for one year projects. The lowest share for these projects is about 1% (in Problem (1')-(14), *Case B* under 1% of the tax value), and the highest share is about 12% (in Problem (1')-(14), *Case A* under 13% of the tax value). At the same time, for five year projects the lowest share is about 2.6% (in Problem (18') under 1% of the tax value), and the highest share is about 30.8% (in Problem (1')-(14), *Case A* under 13% of the tax value).

Based on these calculations, which were conducted for a particular set of model data, solving Problem (1')-(14) and Problem (18'), long term projects are more sensitive to changes in the tax value than short projects.

VI. Discussion

1. From the authors' viewpoint, the present paper makes a contribution to solving large-scale problems that appear in transportation economics. Particularly, it suggests how a problem associated with making strategic management decisions on investing in the development of a regional freight transportation infrastructure can be formalized as a solvable mathematical problem. That is, it shows that a substantially nonlinear problem with mixed variables can be solved with the use of mixed programming techniques implemented in the framework of standard software packages, for instance, MILP. Thus, solving this strategic management problem does not, generally, require developing any heuristics or special software for practically reasonable sizes of the problem. This finding distinguishes the authors' approach to economic problems in transportation from those proposed by some other authors, including (Merakli and Yaman 2016).

2. Though the number of points on the regional cargo transportation network may be quite high, the number of points suitable for locating new transport hubs there is usually relatively small (for instance, does not usually exceed 10). Also, a) the number of the hub capacity options to choose from does not usually exceed 4, b) the number of types of new roads that are planned to be built to a new transport hub does not exceed 3, and c) the road capacity of each type is usually determined by the hub capacity. Thus, the total number of Boolean variables in practical problems formulated as Problem (1)-(14) or Problem (18) is relatively small. This allows one to solve these practical problems with the use of standard software packages such as MILP or CPLEX quite quickly, even when these packages are implemented in laptops. (For optimization software packages see, for instance, (Bixby 2002) and (Mittelmann 2017).)

3. Even if the number of Boolean variables in any practical problem under consideration in this paper were high, the lower estimates of the expenses and the profit/loss, could be calculated with the use of linear programming techniques. These techniques are described, particularly, in (Bertstimas and Tsitsiklis 1997) and in (Yudin and Golshtein 1965), and their use is possible due to the results from (Belenky 1981). That is, as

shown in (Belenky 1981), calculating the minimax of a bilinear function with continuous variables, for instance, in a continuous analog of Problem (18), is reducible to solving linear programming problems forming a dual pair. Calculating these lower estimates requires solving such a continuous analog.

4. Finally, as is known, in operations management in general and in transportation systems in particular, experimental findings obtained in one system can usually be used to improve daily operations in another one, at least for a short period of time. In strategic management, however, the situation is different, particularly in transportation systems. That is, strategic decisions in transportation systems are not universal for obvious reasons. They are unique for every particular system, and they cannot usually be replicated in other systems. Regularities established by researchers based upon any chosen set of data do not matter much to decision makers involved in developing strategic management decisions. This is the case since such regularities may change dramatically when a different set of data is used, whereas these decisions are made for a long period of time. In contrast, these decision makers feel "armed" when they have an easy-to-operate tool helping them quickly calculate solutions to the problems they face with any sets of the data.

The "value" of strategic recommendations that are based on regularities drawn from experiments with a particular set (or even with several particular sets) of data is usually doubtful. This is the case unless these regularities allow researchers a) to indicate a class of situations in each of which (within this class) these regularities always hold, and b) to establish verifiable criteria to determine whether a particular situation belongs to this class. Otherwise, not only do such regularities not contribute to any theory, they may be misleading and even damaging to those who apply them in practice. This is especially so with respect to financial decisions that are to be made by regional administrations or by the country governments, since, usually, the taxpayers' money is at stake. When this is the case, any unsubstantiated decisions that are based on experimental data may cause financial troubles at least to the region for which these decisions are made. However, the authors are not aware of such classes of situations in strategic management either in general or in transportation systems. At the same time, finding such classes of situations is not within the goals of the present paper.

This, of course, does not mean that all the experimental calculation results related to strategic management decisions in particular systems, including those from transportation, are useless. Nor does this mean that decision makers responsible for strategic management decisions will always ignore them. These calculation results may eventually reveal extremely helpful strategic business information. However, for that very reason, one should bear in mind that real data will very unlikely be made available to interested researchers by regional administrations.

For obvious business reasons (especially if negotiations with potential investors from the private sector are under way or are planned), as well as for security ones, regional administrations prefer to have a tool that allows them to make calculations with real data themselves, with any data they may decide to put in. At the same time, as mentioned earlier, experimental calculation results conducted on any set of data (real or not) may change dramatically when a different set of data is used.

VII. Results

1. A mathematical model to formalize problems associated with finding quantitative estimates of investments needed from the private sector for developing (modernizing) a regional freight transportation infrastructure is proposed.

2. Depending on the information available to decision makers, three optimization problems are formulated on the basis of the proposed mathematical model.

Two of these three problems allow one to find the estimates assuming that the information on the values of the parameters of the model is known exactly either for all the parameters or for a part of them. In both cases, the corresponding optimization problems are formulated as mixed programming ones.

A robust optimization problem is formulated on the basis of the same mathematical model under uncertainty on the values of all the parameters of the model. It is proven that this robust optimization problem is reducible to a mixed programming one under natural assumptions on the boundaries within which the values of the parameters can vary.

3. In the above-mentioned (three) mixed programming problems, all the integer variables are Boolean, and the number of these variables is relatively small. This allows one to use standard software packages like MILP and CPLEX, implemented even on laptops, to find the quantitative estimates of the investment volumes needed from the private sector for developing (modernizing) a regional freight transportation infrastructure.

Thus, any of these three problems, which decision makers from regional administrations responsible for making strategic management decisions may choose to solve, can be solved on laptops.

4. The proposed model was used to formulate three above-mentioned mixed programming problems based upon the model data taken from open sources. Solutions to these problems are presented in Appendix 2. Possible strategic management decisions that these solutions may suggest are described in Section V.

5. Two brief surveys of publications relevant to the subject of the present paper are offered. One of the surveys is on hub location problems, whereas the second one is on modelling public-private partnership in transportation projects. Both surveys, particularly, help substantiate the need for developing the decision-making tool proposed in the present paper.

VIII. Concluding remarks

1. One of the goals of this paper is to describe a decision-making tool that may help a regional administration in its negotiations with both the federal government and private investors on developing (modernizing) a regional freight transportation infrastructure. By estimating the needed volume of investment in this project, the tool may substantiate the need for a private-public partnership if the federal government and the regional administration cannot finance the project in full.

2. The proposed approach to modelling the problem under consideration in this paper by taking into account the uncertainty in the values of all its parameters consists of considering this problem as a robust (minimax) one on a Descartes product of two sets of vector variables. One of these sets is a polyhedron, and the other is a subset of another polyhedron formed by the vectors each of whose components equals either 0 or 1. The minimax of a bilinear function of these two vector variables is sought, and it is proven that finding this minimax is reducible to solving a mixed programming problem.

3. Any decision-support system for analyzing the existing regional freight transportation infrastructure and/or for developing an optimal one that has a chance to work effectively should meet certain criteria. Particularly, it should allow the administration of a region

a) to find and to estimate variants of this infrastructure (that the administration may consider to be of interest to the region) in an acceptable time, despite the fact that large-scale problems are to be solved to this end,

b) to depict the locations of all the infrastructure elements on a geographic map graphically, in an easy-to-understand form,

c) to input new and to change already existing information relating to the transportation infrastructure from easy-to-operate interfaces,

d) to obtain solutions (infrastructure variants) based on the available information only, including the data that can be known only approximately in principle, as well as on statistical estimates that can be calculated based on this information, and

e) to be flexible in incorporating both new information and new regularities formalizing relations between variables and parameters in the mathematical models that are in use as they become known in the course of developing and analyzing strategic decisions related to a regional freight transportation infrastructure.

The described features of the approach presented in this paper bear evidence that a decision-support system for the considered purposes can easily be assembled based on the decision-making tool proposed in this paper. Such a decision-support system should include software for a) solving linear and mixed programming problems, and b) graphically depicting solutions to mathematical problems. As mentioned earlier, standard software packages for solving mixed programming problems are widely available even on laptops. A geographic information system software applicable to transportation problems is described, for instance, in (Abulizi et.el 2016)).

4. One can easily be certain that the proposed approach can be used in estimating the needed investment in developing regional passenger transportation infrastructures, as well as in developing the regional freight and passenger transportation infrastructures concurrently.

5. Basic Assumptions 1 and 2 consider piecewise linear approximations of the costs of building both new hubs and new access roads to them, which are usually described by convex functions with positive values. A description of known techniques for approximating, particularly, such convex functions by piecewise linear functions can be found in many scientific publications, including (Gavrilovich 1975).

6. Basic Assumption 4, which is about not building new access roads to the existing transport hubs and not doing any modernization construction work there in the planning period, is not restrictive. That is, by introducing new variables, one can include a modernization of the existing transport hubs and access roads to them as part of the activities associated with developing a new regional freight transportation infrastructure. These new variables should be present in the system of constraints of Problem (1)-(14).

7. Basic Assumption 9, determining that the revenue in the form of regional taxes (that the administration expects to receive as a result of the

functioning of the regional freight transportation infrastructure) comes in after all the facilities (new transport hubs and access roads to them) that are planned to be built start functioning, is not restrictive. In fact, the regional administration may solve Problem (1)-(14) or Problem (18) several times taking into account the schedule of developing new facilities within any particular period of time. To this end, this period should be divided into a corresponding number of parts. During each of these parts, the regional administration expects a particular set of facilities to be built and to start functioning, while considering already built new facilities as the existing ones.

Authors' contribution

Alexander S. Belenky: Problem Statement and Mathematical Formulation, Developing a Mathematical Model of the Problem, Proof of the Assertion, Manuscript Writing and Editing

Gennady G. Fedin: Developing a Mathematical Model of the Problem, Literature Review, Collecting Model Data, Conducting Numerical Calculations, Manuscript Writing

Alain L. Kornhauser: Content Planning, Developing a Mathematical Model of the Problem, Manuscript Writing

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Appendix 1

Proof of the Basic Assertion.

1. Let $v = v^* = (x^*, y^*, z^*) \in \Pi = (MX \times \Omega Y \times HZ) \cap \Phi$. Since $\Lambda \times \tilde{\Theta} \times \tilde{\Gamma}$ is a (non-empty) convex polyhedron, the linear function $\left\langle \tilde{u}, \tilde{D}v^* \right\rangle$ is bounded from above on this polyhedron, so it attains its maximum on the set $\Lambda \times \tilde{\Theta} \times \tilde{\Gamma}$. By the duality theorem of linear programming (see, for instance, Yudin and Golshtein (1965)), this means that the set of feasible solutions to the problem that is dual to the problem of maximizing this linear function on the set $\Lambda \times \tilde{\Theta} \times \tilde{\Gamma}$ is non-empty.

Let the problem of maximizing the linear function $\left\langle \tilde{u}, \tilde{D}v^* \right\rangle$ on the set $\Lambda \times \tilde{\Theta} \times \tilde{\Gamma}$ be written as

$$\begin{gathered} \left\langle \tilde{u}, \tilde{D}v^* \right\rangle \to \max_{\tilde{u} \in \Lambda \times \tilde{\Theta} \times \tilde{\Gamma}} \\ \tilde{u}J \leq \omega, \ \tilde{u} \geq 0, \end{gathered} \tag{19}$$

where $J = \begin{pmatrix} I & 0 & 0 & 0 & 0 \\ 0 & F & 0 & 0 & 0 \\ 0 & 0 & W & 0 & 0 \\ 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & \Psi \end{pmatrix}$, $\omega = (l, r, \lambda, e, \eta)$, and $\tilde{u} = (t, f, c, g, q)$.

Then the set of feasible solutions to the problem that is dual to Problem (19) is determined by the system of linear inequalities $Jw \geq \tilde{D}v^*$, $w \geq 0$, where w is the vector of (dual) variables in the problem being dual to Problem (19). This set is non-empty, and the maximum of the goal function in Problem (19) is attainted.

2. Let now $y = y^*$, $z = z^*$, and $x \in MX$ so that $(x, y^*, z^*) \in (MX \times \Omega Y \times HZ) \cap \Phi$. Further, let $v(y^*, z^*) = (x, y^*, z^*) \in \big(MX \times \{y^*\} \times \{z^*\}\big) \cap \Phi$, where $\big(MX \times \{y^*\} \times \{z^*\}\big) \cap \Phi$ is a convex polyhedron, which is a subset of the set $(MX \times \Omega Y \times HZ) \cap \Phi$. Based on the results from Belenky (1981), one can easily be certain that for every pair of the vectors $(y^*, z^*) : (x, y^*, z^*) \in (MX \times \Omega Y \times HZ) \cap \Phi$, the inequality

$$\begin{aligned} &\min_{v(y^*,z^*) \in (MX \times \{y^*\} \times \{z^*\} \cap \Phi} \ \max_{\tilde{u} \in \Lambda \times \tilde{\Theta} \times \tilde{\Gamma}} \left(\left\langle \tilde{u}, \tilde{D}v(y^*, z^*) \right\rangle \right. \\ &= \min_{v(y^*,z^*) \in (MX \times \{y^*\} \times \{z^*\} \cap \Phi} \ \min_{Jw \geq \tilde{D}v(y^*,z^*)} \langle \omega, w \rangle \end{aligned}$$

holds.

3. Since the set $\big(MX \times \{y^*\} \times \{z^*\}\big) \cap \Phi$ is a subset of a polyhedron for any pair of the vectors (y^*, z^*), and the number of the sets of these pairs for which the inclusion $(x, y^*, z^*) \in (MX \times \{y^*\} \times \{z^*\}) \cap \Phi$ holds is finite, the equalities

$$\min_{v\in\Pi} \max_{\tilde{u}\in\Lambda\times\tilde{\Theta}\times\tilde{\Gamma}} \left\langle \tilde{u}, \tilde{D}v \right\rangle$$
$$= \min_{(y^*,z^*)\in\Omega Y\times HZ} \; \min_{v(y^*,z^*)\in(MX\times\{y^*\}\times\{z^*\})\cap\Phi} \; \min_{Jw\geq\tilde{D}v(y^*,z^*)} \langle \omega, w\rangle$$
$$= \min_{(x,y,z)\in(MX\times\Omega Y\times HZ)\cap\Phi,\ Jw\geq\tilde{D}(x,y,z)} \langle \omega, w\rangle = \min_{v\in\Pi,\ Jw\geq\tilde{D}v} \langle \omega, w\rangle$$

hold. The Basic Assertion is proved.

Appendix 2

Table 1. Problem (1)-(14),
Situation 1. Case A. Subcase 1.

ψ (years)	ν (%)	New hubs to be constructed	New highways to build	New railways to build	Revenue (USD, mln)	Expenses (USD, mln)	Investments needed (neg. numbers)/Profit (pos. numbers) (USD, mln)
1	1	8(2),9(2),10(2)	8, 9	8, 9, 10	268	4 750	-4 482
1	7	8(2),9(2),10(2)	8, 9	8, 9, 10	1 875	4 750	-2 875
1	13	8(2),9(2),10(2)	8, 9	8, 9, 10	3 482	4 750	-1 268
3	1	8(2),9(2),10(2)	8, 9	8, 9, 10	803	5 450	-4 647
3	7	8(2),9(2),10(2)	8, 9	8, 9, 10	5 624	5 450	174
3	13	3(2),8(2),9(2)	8, 9	3, 8, 9	10 597	5 550	5 047
5	1	8(2),9(2),10(2)	8, 9	8, 9, 10	1 339	6 150	-4 811
5	7	3(2),8(2),9(2)	8, 9	3, 8, 9	9 511	6 250	3 261
5	13	3(2),8(2),9(2)	8, 9	3, 8, 9	17 662	6 250	11 412

Table 2. Problem (1')-(14),
Situation 1. Case A. Subcase 2.

ψ (years)	ν (%)	New hubs to be constructed	New highways to build	New railways to build	Taxes (USD, mln)	Construction expenses (USD, mln)	Total expenses (USD, mln)
1	1	7(2),8(2),10(2)	7, 10	7, 8, 10	68	4 750	4 818
1	7	5(2),7(2),10(2)	7, 10	5, 7, 10	357	4 850	5 207
1	13	5(2),7(2),10(2)	7, 10	5, 7, 10	664	4 850	5 514
3	1	7(2),8(2),10(2)	7, 10	7, 8, 10	205	5 450	5 655
3	7	5(2),7(2),10(2)	7, 10	5, 7, 10	1 072	5 550	6 622
3	13	5(2),7(2),8(2)	5, 7, 8	5, 7, 8	1 837	5 680	7 517
5	1	7(2),8(2),10(2)	7, 10	7, 8, 10	342	6 150	6 492
5	7	5(2),7(2),10(2)	7, 10	5, 7, 10	1 787	6 250	8 037
5	13	3(2),5(2),8(2)	3, 5, 8	3, 5, 8	2 899	6 500	9 399

TABLE 3. Problem (1)-(14),
Situation 1. Case B. Subcase 1.

ψ (years)	ν (%)	New hubs to be constructed	New highways to build	New railways to build	Revenue (USD, mln)	Expenses (USD, mln)	Investments needed (neg. numbers)/Profit (pos. numbers) (USD, mln)
1	1	8(2),9(2),10(1)	8, 9	8, 9, 10	264	4 450	-4 186
1	7	8(2),9(2),10(1)	8,9	8, 9, 10	1 851	4 450	-2 599
1	13	8(2),9(2),10(2)	8, 9,10	8, 9, 10	3 855	4 860	-1 005
3	1	8(2),9(2),10(1)	8,9	8, 9, 10	793	5 150	-4 357
3	7	8(2),9(2),10(2)	8, 9,10	8, 9, 10	6 228	5 580	648
3	13	3(2),8(2),9(2)	3, 8,9	3, 8, 9	11 681	5 680	6 001
5	1	8(2),9(2),10(1)	8,9	8, 9, 10	1 322	5 850	-4 528
5	7	3(2),8(2),9(2)	3, 8,9	3, 8, 9	10 483	6 400	4 083
5	13	3(2),8(2),9(2)	3, 8, 9	3, 8, 9	19 468	6 400	13 068

Table 4. Problem (1')-(14),
Situation 1. Case B. Subcase 2.

ψ (years)	ν (%)	New hubs to be constructed	New highways to build	New railways to build	Taxes (USD, mln)	Construction expenses (USD, mln)	Total expenses (USD, mln)
1	1	7(2),8(1),10(2)	7, 10	7, 8, 10	47	4 450	4 497
1	7	5(2),7(2),8(1)	5, 7	5, 7, 8	217	4 550	4 767
1	13	5(2),7(2),8(1)	5, 7	5, 7, 8	403	4 550	4 953
3	1	7(2),8(1),10(2)	7, 10	7, 8, 10	140	5 150	5 290
3	7	5(2),7(2),8(1)	5, 7	5, 7, 8	651	5 250	5 901
3	13	3(2),5(2),8(1)	3, 5	3, 5, 8	1 091	5 350	6 441
5	1	7(2),8(1),10(2)	7, 10	7, 8, 10	233	5 850	6 083
5	7	3(2),5(2),8(1)	3, 5	3, 5, 8	979	6 050	7 029
5	13	3(2),5(2),8(1)	3, 5	3, 5, 8	1 818	6 050	7 868

Table 5. Problem (18),
Situation 2. Subcase 1.

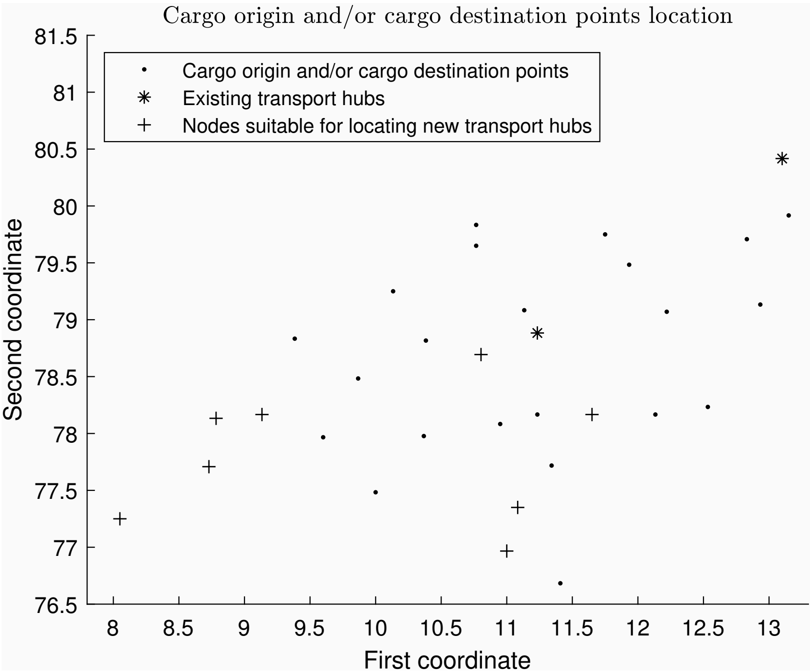
ψ (years)	ν (%)	New hubs to be constructed	New highways to build	New railways to build	Revenue (USD, mln)	Expenses (USD, mln)	Investments needed (neg. numbers)/Profit (pos. numbers) (USD, mln)
1	1	8(2),9(2),10(1)	8, 9	8, 9, 10	185	5 380	-5 195
1	7	8(2),9(2),10(1)	8, 9	8, 9, 10	1 296	5 380	-4 084
1	13	8(2),9(2),10(1)	8, 9	8, 9, 10	2 407	5 380	-2 973
3	1	8(2),9(2),10(1)	8, 9	8, 9, 10	555	6 280	-5 725
3	7	8(2),9(2),10(1)	8, 9	8, 9, 10	3 888	6 280	-2 392
3	13	8(2),9(2),10(2)	8, 9, 10	8, 9, 10	8 096	6 840	1 256
5	1	8(2),9(2),10(1)	8, 9	8, 9, 10	926	7 180	-6 254
5	7	8(2),9(2),10(2)	8, 9	8, 9, 10	7 266	7 770	-504
5	13	3(2),8(2),9(2)	3, 8, 9	3, 8, 9	13 628	7 870	5 758

TABLE 6. Problem (18'),
Situation 2. Subcase 2.

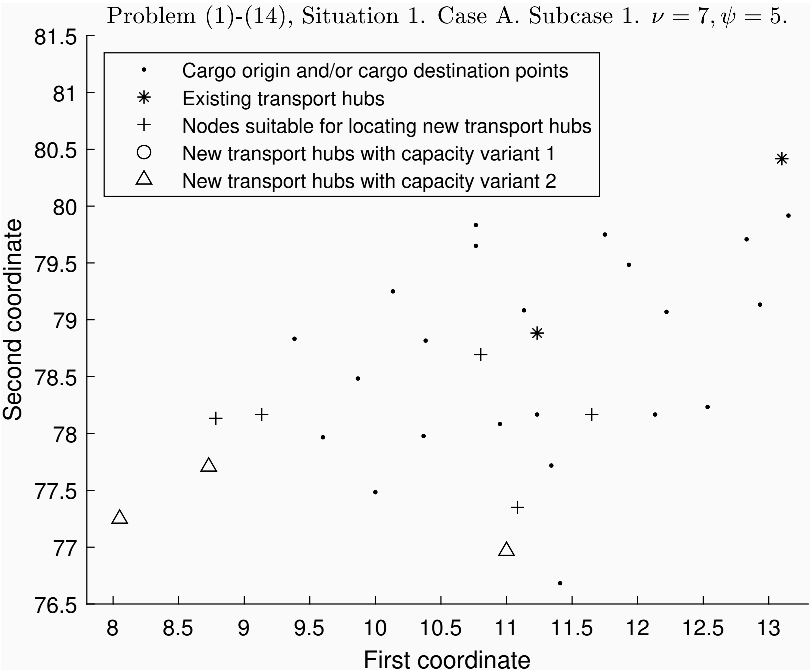
ψ (years)	ν (%)	New hubs to be constructed	New highways to build	New railways to build	Taxes (USD, mln)	Construction expenses (USD, mln)	Total expenses (USD, mln)
1	1	7(2),8(1),10(2)	7, 10	7, 8, 10	61	5 380	5 441
1	7	5(2),7(2),8(1)	5, 7	5, 7, 8	282	5 480	5 762
1	13	5(2),7(2),8(1)	5, 7	5, 7, 8	524	5 480	6 004
3	1	7(2),8(1),10(2)	7, 10	7, 8, 10	182	6 280	6 462
3	7	5(2),7(2),8(1)	5, 7	5, 7, 8	846	6 380	7 226
3	13	3(2),5(2),8(1)	3, 5	3, 5, 8	1 418	6 480	7 898
5	1	5(2),7(2),8(1)	5, 7	5, 7, 8	201	7 280	7 481
5	7	3(2),5(2),8(1)	3, 5	3, 5, 8	1 273	7 380	8 653
5	13	3(2),5(2),8(1)	3, 5	3, 5, 8	2 363	7 380	9 743

Appendix 3

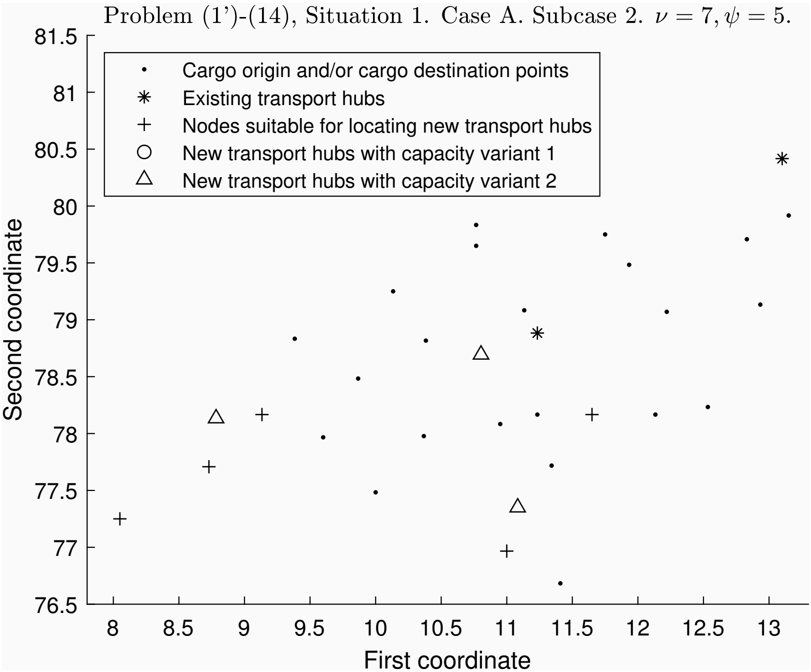
Picture 1



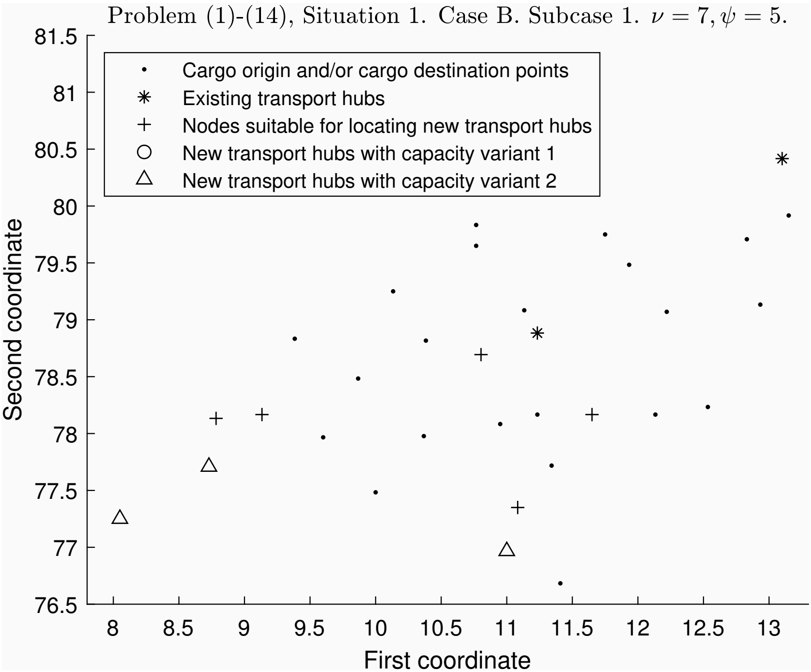
Picture 2. Example of solution for Problem (1)-(14), Case A, *Subcase 1.*



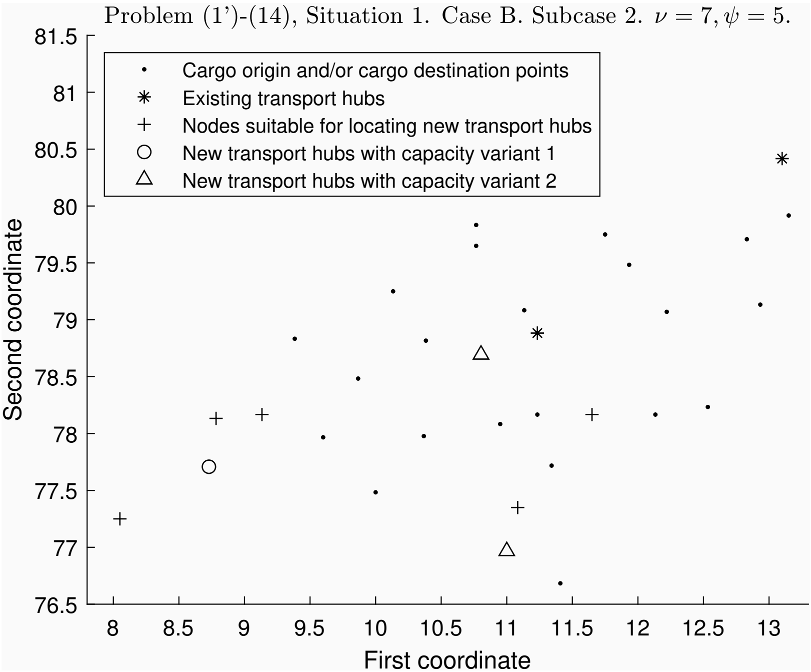
Picture 3. Example of solution for
Problem (1)-(14), Case A, *Subcase 2.*



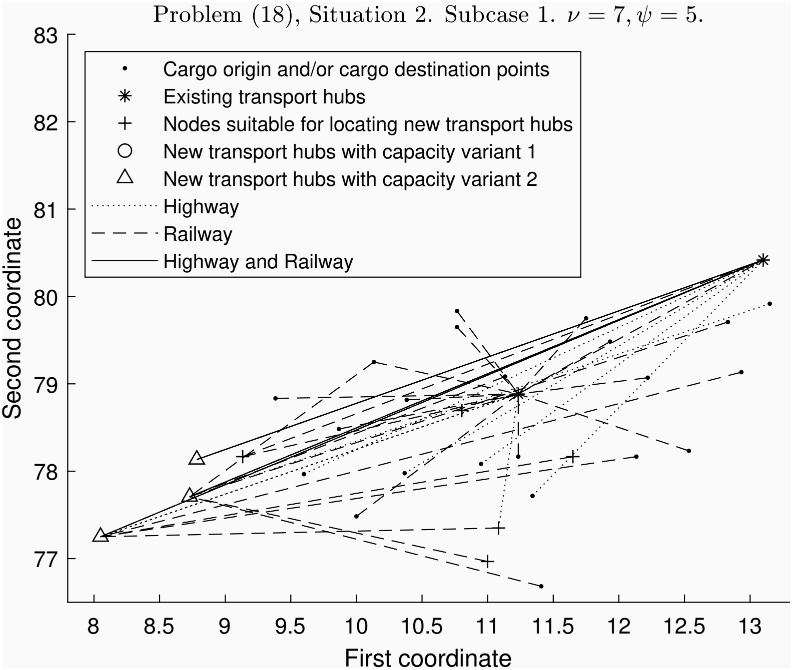
PICTURE 4. Example of solution for Problem (1)-(14), Case B, *Subcase 1.*



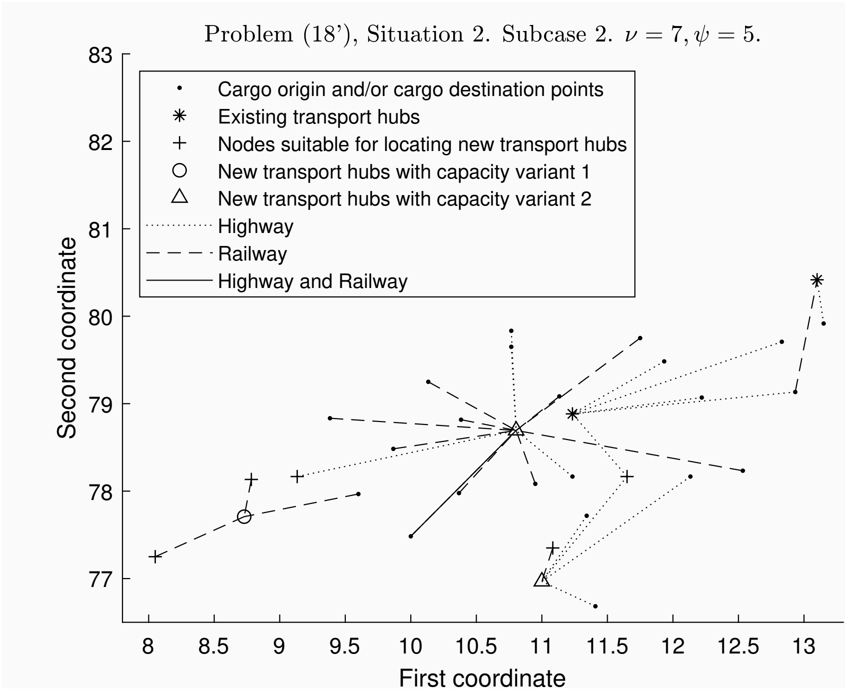
PICTURE 5. Example of solution for Problem (1)-(14), Case B, *Subcase 2*.



Picture 6. Example of solution
for Problem (18), *Subcase 1.*



Picture 7. Example of solution
for Problem (18), *Subcase 2*.



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Предложена система поддержки принятия решений для оценки объема инвестиций, требуемых для создания/модернизации региональной инфраструктуры грузового транспорта с целью рационализации использования территории региона. Эта оценка производится путем решения задач выбора а) мест для строительства и мощностей новых транспортных узлов, б) мощностей действующих транспортных узлов, которые подлежат модернизации, и в) типов дорог ко всем транспортным узлам и их оптимальных мощностей, наряду с оптимальным распределением транспортных потоков между всеми видами транспорта, которые должны функционировать в регионе. Эти задачи математически формулируются в виде двух задач математического программирования со смешанными переменными и минимаксной задачи, решение которой сводится к решению задачи математического программирования со смешанными переменными, в которой все целочисленные переменные являются Булевыми. Предложенная система поддержки принятия решений включает в себя а) математическую модель с линейной структурой ограничений, на базе которой формулируются все три вышеупомянутые задачи математического программирования со смешанными переменными, б) стандартные пакеты прикладных программ для решения задач математического программирования со смешанными переменными и линейными ограничениями и в) стандартные пакеты прикладных программ для обработки данных и графического изображения решений задач. Примеры решения указанных выше трех задач математического программирования со смешанными переменными с исходными данными, взятыми из открытых источников, демонстрируют эффективность предложенной системы.

Ключевые слова: билинейные функции векторных аргументов, инвестиции в инфраструктуру грузовых перевозок региона, минимаксная задача с линейными ограничениями, смешанное математическое программирование, государственно-частное партнерство, транспортные узлы и подъездные пути к ним

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(*на английском языке*)

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