

A sample of a placement test
on math refresher course for the Master degree students, ICEF

1. (25 points) A firm's production function is $Q = K + L + 2\sqrt{KL}$, where $K > 0$ and $L > 0$ are capital and labor, respectively. The firm is perfectly competitive and seeks to maximize its output, but the firm is run by accountants who have imposed a fixed budget on the production of C dollars per hour, which means satisfying constraint $wL + rK = C$, where w and r are hourly wage rate and rental rate of capital, respectively.

State the constrained optimization problem associated with that production and solve it by the Lagrange multiplier method (it is sufficient to find the optimal values of capital and labor alone). No second-order conditions are required to check.

2. (25 points) Using Implicit Function Theorem or otherwise find $\frac{dy}{dx}$, given equation $xe^{2y} - ye^{2x} = 0$.

3. (25 points) Find all the values of λ for which the quadratic form is positive definite

$$q(x_1, x_2, x_3) = 5x_1^2 + x_2^2 + \lambda x_3^2 + 4x_1x_2 - 2x_1x_3 - 2x_2x_3.$$

4. (25 points) In the Solow's one-sector growth model k is the capital per capita, s , ($0 < s < 1$) is the savings rate and α , ($0 < \alpha < 1$) is a parameter. Consider the equation $\dot{k} = sk^\alpha - k$. Answer the following questions:

- a) Does there exist a steady-state solution? Explain.
- b) Let $s = 1/2$ and $\alpha = 1/3$. Solve the initial-value problem given $k(0) = 8$.