Approved by Academic Subcommittee of Bachelor's Programme 'HSE and University of London Double Degree Programme in Economics' on 25 June 2018, Protocol № 5

Mathematics of Finance and Valuation

Lecturer: Mikhail Zhitlukhin.

Abstract

This is an elective course for students specializing in Mathematics and Economics which offers an introduction to an exciting and relatively new area of mathematical application. It is concerned with the valuation (pricing) of 'financial derivatives'. These are contracts which are bought or sold in exchange for the promise of some kind of payment in the future, usually contingent upon the share-price then prevailing (of a specified share, or share index). The course reviews the financial environment and some of the financial derivatives traded on the market. It then introduces the mathematical tools which enable the modelling of the fluctuations in share prices. Inevitably these are modelled by equations containing a random term. It is this term which introduces risk; it is shown how to counterbalance the risks by putting together portfolios of shares and derivatives so that risks temporarily cancel each other out and this strategy is repeated over time. As this procedure resembles hedging a bet – that is, betting both ways - one talks of dynamic hedging. The yield of a temporarily riskless portfolio is equated to the rate of return offered by a safe deposit bank account (that is a riskless bank rate) which is assumed to exist; this equation assumes that the market which values shares and derivatives actually is in equilibrium and hence eliminates the opportunities of 'arbitrage' (such as making sure profit from, say, buying cheap and selling dear). The no-arbitrage approach implies in the continuous time model that the price of a derivative is the solution of a differential equation. One may either attempt to solve the differential equation by standard means such as numerical techniques or via Laplace transforms, though this is not always easy or feasible. However, there is an alternative route which may provide the answer: a calculation of the expected payment to be obtained from the contract by using what is known as the synthetic probability (or the risk-neutral probability. One proves that, regardless of what an investor believes the expected growth rate of the share price to be, the dynamic hedging acts so as to replace the believed growth rate by the riskless growth rate. Though this may seem obvious in retrospect it does require some careful reasoning to justify. The course considers two approaches to risk- neutral calculation, using discrete time and using continuous time. Continuous time requires the establishment of a second-order volatility correction term when using standard first-order approximation from calculus. This leads to what is known as Ito's Rule. Finite time arguments need some apparatus from Linear Algebra like the Separating Hyperplane Theorem. We enter the subject from the discrete time model for an easier discussion of the main issues.

Prerequisites: Abstract Mathematics

Learning Objectives and Outcomes

The course is designed to introduce the main mathematical ideas involved in the modelling of asset price evolution and the valuation of contingent claims (such as call and put options) in a discrete and in a continuous framework.

The student should be able to apply professional knowledge and skills acquired while studying the course in practical areas, including academic research, work in financial institutions, industry, state governance.

Methods of Instruction

The methods used in the course include lectures (2 hours a week), classes (2 hours a week), home assignments, office hours, and self-study.

Grading System and Knowledge Assessment

The grade is made up of

– the written examination (60%),

– the written test (20%)

– the average grade for home assignments (20%)

Sample materials for knowledge assessment are available in ICEF Information system at https://icef-info.hse.ru.

Required reading:

1. Shreve, S. Stochastic Calculus for Finance I, The Binomial asset pricing model. (Springer)

2. Shreve, S. Stochastic Calculus for Finance II, Continuous-time models. (Springer)

3. Pliska, S.R. Introduction to Mathematical Finance – Discrete Time Models. (Blackwell) – 1 экз.!

Optional reading:

1. Hull, J.C. Options, Futures, and Other Derivatives. (Prentice Hall)

2. Follmer, H., Schied, A. Stochastic Finance. (Walter de Gruyter)

3. Musiela, M., Rutkowski, M. Martingale Methods in Financial Modelling. (Springer)

Course plan

1. Financial environment

1.1 Present value of future income

1.2 Bonds, stocks, derivative securities

1.3 Arbitrage arguments

1.4 Hedging

2. One risky asses and two states

2.1 The simplest market model

2.2 Valuation of forward and futures contracts

2.3 Valuation of options

2.4 Valuation by expectation

3. One period many assets

3.1 Description of the model

3.2 The notion of arbitrage

3.3 Fundamental Theorem, of Asset Pricing

4. Multi-period models 4.1 Information trees and related notions

4.2 Valuation of contracts in the multiperiod models

4.3 Arbitrage in the multiperiod model

4.4 Risk-neutral measures

5. The binomial model

5.1 The T-period binomial model

5.2 Valuation of different contracts

6. Continuous-time modelling

6.1 The discrete random walk and Brownian motion 6.2 Stochastic differential equations

6.3 Itoˆ’s formula

7. Black-Scholes model

7.1 Geometric Brownian motion as a stock price model

7.2 The Black-Scholes equation

7.3 The Black-Scholes formula for option prices

8. Perpetual options

8.1 The notion of a perpetual option

8.2 The Cauchy-Euler equation and the value of a perpetual option

Special Equipment and Software Support

Laptop, projector, Internet connection

MS Word, MS Excel