Course syllabus «Stochastic models»

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Programme Academic Council
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Number of credits	4
Contact hours	48
Self-study hours	104
Course	1,2
Educational format	Without use of online course

I. Goals and Results of Mastering the Discipline; Prerequisites

Mathematical models based on probability theory prove to be extremely useful in describing and analyzing complex systems that exhibit random components. The goal of this course is to introduce several classes of stochastic processes, analyze their behavior over a finite or infinite time horizon, and help students enhance their problem solving skills. The course combines classic topics such as martingales, Markov chains, renewal processes, and queuing systems with approaches based on Stein's method and on concentration inequalities.

The course focuses mostly on discrete-time models and explores a number of applications in operations research, finance, and engineering. This is an elective course, offered to MASNA students, and examples used in class may differ depending on students' interests.

As a result of taking the course, students should:

Know:

- the theoretical foundation of stochastic processes
- modern extensions to stochastic modeling
- the basic principles behind working with all types of data for using stochastic components in models

Be able to:

- explore the advantages and disadvantages of stochasticity in the models and demonstrate how it contributes to the analysis.
- work with major linear modeling programs, especially R, so that they can use them and interpret their output.
- develop and/or foster critical reviewing skills of published empirical research using applied statistical methods.
- to criticize constructively and determine existing issues with applied linear models in published work

Have:

- an understanding of the basic principles of stochastic models and lay the foundation for future learning in the area.

- the skill to meaningfully develop an appropriate model for the research question
- the skill to work with statistical software, required to analyze the data

Basic knowledge of introductory statistics are required for this course.

The basics of this discipline should be used in all other program related courses

The course is strongly related and complementary to other compulsory courses provided in the first year (e.g. Applied Linear Models II, Contemporary Data Analysis) and sets a crucial pre-requisite for later courses and research projects as well as for the master thesis. The course gives students an important foundation to develop and conduct their own research as well as to evaluate research of others.

II. Content of the Course

Please note: due to their complexity, some of the sessions will run over multiple class periods.

SESSION ONE: Understanding randomness

The first session will focus on understanding randomness and the statistical relationships between random events. It will also review expectation and integration, almost sure convergence and the dominated convergence theorem, convergence in probability and in distribution, the e law of large number and the ergodic theorem.

SESSION TWO: Stein's method and central limit theorems

The session introduces the notions of coupling, Poisson approximation and Le Cam's theorem, the Stein-Chen method, and Stein's method for the geometric and the normal distribution.

SESSION THREE: Conditional expectation and martingales

The focus of this session will be on conditional expectation, martingales, the martingale stopping theorem, the Hoeffding-Azuma inequality, the martingale convergence theorem, and the uniform integrability.

SESSION FOUR: Probability inequalities

This sessions builds the understanding of Jensen's inequality, probability bounds via the importance sampling identity, Chernoff bounds, second moment and conditional expectation inequalities.

SESSION FIVE: Discrete-time Markov chains

This session covers the Chapman-Kolmogorov equations and classification of states, the strong Markov property, stationary and limiting distributions, transition among classes, the gambler's ruin problem, and mean times in transient states; branching processes and time reversibility.

SESSION SIX: Renewal theory

This session will introduce the limit theorems, renewal reward processes, and Blackwell's therem.

SESSION SEVEN: Queueing theory (multiple class meetings)

This session will focus on the Poisson process, and a range of queueing systems: M/M/1, M/G/1, G/M/1, and G/G/1.

III. Grading

Course grade will be completed as follows:

Course Element	**% Towards Final Grade**
Final Exam	**50%**
Final In-Class or Take-home exam (at the discretion of the instructor)	*50%*
Participation and responsibility grade	**50%**
Homework Assignments (5 x Varied points)	*20%*
In-Class Labs (9-10 x Varied points)	*20%*
Quizzes (Best 9 of 10, Varied points)	*10%*
Total	100%

If the final grade is non-integer, it is rounded according to algebraic rules. If has a half (.5) at the end, we are rounding upward. Rounding of cumulative grades and other rounding issues are performed according to the HSE rules.

IV. Grading Tools

This class contains several assignments that test student knowledge and understanding throughout the course.

<u>Quizzes</u>
You cannot meaningfully participate in the seminar if you have missed my lecture and did not do any reading. Therefore, to encourage you to prepare for seminars, every seminar will have a quiz on the lecture material and all assigned readings for the week. This includes the very first seminar, which will focus on Lecture 1 material. You are allowed to miss any one quiz (skip a seminar, not prepare, etc.) – in other words, I will count the best 9 out of 10 quizzes that we will have. If you submit all ten, I will count best nine. All quizzes will be done online and submitted to me via SurveyMonkey (links will be given in class).

Important: I record IP addresses and only accept quizzes submitted from with the HSE IP address. Quizzes submitted from other locations are NOT counted towards your grade. In other words, to participate in a quiz, you have to be present in class.

<u>In-class Labs</u>
There will be a lab assignment in almost every seminar, depending on our progress. Since we will be learning SAS, and learning quickly, you will need to devote a substantial time to it. Seminar labs should help you with this task. At the end of the lab, you will submit your completed assignment for the day (or as much as you were able to complete) to me via LMS.

Homework assignments

There will be several homework assignments that will provide additional hands-on practice for the concepts we've learned in class and practiced during the seminar. Homeworks will be assigned as needed throughout the semester. All homework submissions must be done by the stated deadline via the LMS system.

V. Resources

5.1 Main Literature

1. Medhi, J.. Stochastic Models in Queueing Theory, Elsevier Science & Technology, 2002. ProQuest Ebook Central, https://ebookcentral.proquest.com/lib/hselibrary-ebooks/detail.action?docID=311392.
2. Meerschaert, Mark M., and Alla Sikorskii. Stochastic Models for Fractional Calculus, De Gruyter, Inc., 2011. ProQuest Ebook Central, https://ebookcentral.proquest.com/lib/hselibrary-ebooks/detail.action?docID=835465.
3. Ziemba, W. T, and R. G Vickson. Stochastic Optimization Models in Finance, World Scientific Publishing Co Pte Ltd, 2006. ProQuest Ebook Central, https://ebookcentral.proquest.com/lib/hselibrary-ebooks/detail.action?docID=1679671.
4. Insua, David, et al. Bayesian Analysis of Stochastic Process Models, John Wiley & Sons, Incorporated, 2012. ProQuest Ebook Central, https://ebookcentral.proquest.com/lib/hselibrary-ebooks/detail.action?docID=877779.
5. Jan-Frederik, Mai. Simulating Copulas : Stochastic Models, Sampling Algorhythms, and Applications, Imperial College Press, 2012. ProQuest Ebook Central, https://ebookcentral.proquest.com/lib/hselibrary-ebooks/detail.action?docID=3050920.

5.2 Additional Literature

1. Rachev, Svetlozar T., et al. Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization : The Ideal Risk, Uncertainty, and Performance Measures, John Wiley & Sons, Incorporated, 2008. ProQuest Ebook Central, https://ebookcentral.proquest.com/lib/hselibrary-ebooks/detail.action?docID=331611.
2. Tan, Wai-Yuan. Stochastic Models with Applications to Genetics, Cancers, AIDS and Other Biomedical Systems, World Scientific Publishing Co Pte Ltd, 2002. ProQuest Ebook Central, https://ebookcentral.proquest.com/lib/hselibrary-ebooks/detail.action?docID=1679538.
3. Li, Quan-Lin. *Constructive computation in stochastic models with applications: the RG-factorizations*. Springer Science & Business Media, 2011. URL https://link.springer.com/book/10.1007/978-3-642-11492-2. Springer Link.

5.3 Software

№ п/п	Name	Access conditions
1.	MicrosoftWindows 7 Professional RUS MicrosoftWindows 10 MicrosoftWindows 8.1 Professional RUS	*From the university's internal network (contract)*
2.	Microsoft Office Professional Plus 2010	*From the university's internal network (contract)*

3.	R, R studio	*Open access. URL: https://www.r-project.org/*

5.3 Material and technical support

Classrooms for lectures on the discipline provide for the use and demonstration of thematic illustrations corresponding to the program of the discipline, consisting of:

- PC with Internet access (operating system, office software, antivirus software);
- multimedia projector with remote control.