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We analyze how reputation concerns of a partially informed decision-maker affect her ability to extract information from reputation-concerned advisors. Too high decision-maker's reputation concerns destroy her incentives to seek advice. However, when such concerns are low, she is tempted to solicit advice regardless of her private information, which can undermine advisors' truth-telling incentives. The optimal strength of the decision-maker's reputation concerns maximizes advice-seeking while preserving advisors' truth-telling. Prior uncertainty about the state of nature calls for a more reputation-concerned decision-maker. Higher expected competence of the decision-maker or advisors may worsen information aggregation, unless the reputation concerns are properly adjusted.

Keywords: reputation concerns, information aggregation, advice

JEL classification: D82, D83

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1 Introduction

According to the case study by Huy et al. (2016)[1], one of the causes of the fall of Nokia was the failure of top managers to aggregate information from middle managers in the face of the iPhone challenge. Although middle managers received signals suggesting that a radical change in the strategy was needed, they did not communicate those signals to top managers, despite being routinely asked for information. Apparently, one of the reasons for such behavior was the failure of the latter to credibly convey the seriousness of the threat to the former. As a result, middle managers succumbed to the top managers' displayed optimism about Nokia's current strategy and hid warning signals.

We analyze how incentive problems of a decision-maker can undermine the incentives of advisors to provide the former with truthful information. In our story, the decision-maker's incentive problems arise due to her either excessive or *insufficient* reputation concerns, which can provoke insufficient or *excessive advice-seeking* respectively. Our focus is on the latter problem. Although we do not claim that our model fully explains the demise of Nokia, some evidence suggests this problem was relevant in the company. While we start with the Nokia case as a motivating example, our setup is rather general and fits a variety of real-life settings. For instance, the decision-maker can be a CEO, a politician, a head of a university department, and the advisors can be her colleagues, subordinates, designated advisors, or any kind of experts in the domain of the decision-maker's responsibilities.

[1] See also Vuori and Huy (2016).

The problem of insufficient advice-seeking is well-known in the literature. Several works document that people can be reluctant to ask for advice or help from other people, even when such advice/help can improve the quality of their decisions (e.g., Lee (2002), Brooks et al. (2015)). One frequently cited reason for such behavior in the management and psychology literature is the fear to appear incompetent, inferior, or dependent (e.g., DePaulo and Fisher (1980), Lee (1997), Lee (2002), Brooks et al. (2015)). Levy (2004) provides a model in which a decision-maker excessively ignores/neglects the opportunity to ask for advice in order to be perceived competent.

Overall, the existing studies suggest that too high reputation concerns of a decision-maker may be detrimental to her ability to collect information from potential advisors. We argue that *low* reputation concerns generate the opposite problem – excessive advice-seeking, which is also detrimental to information aggregation. Consequently, some intermediate level of reputation concerns is generally optimal. The key feature of our story, which distinguishes it from the previous literature, is that the decision-maker's advice-seeking behavior affects advisors' truthtelling. Without reputation concerns the decision-maker will *always* ask for advice. As we explain below, this adversely affects the advisors' incentives to provide truthful information. The positive role for reputation concerns then is to ensure that the decision-maker asks for advice more often when it is needed more, that is, when her available information leaves high uncertainty about the state of the world. This behavior improves the advisors' information provision incentives and, therefore, results in better aggregation of information.

In our model, a decision-maker needs to take a decision/action from a binary set. The optimal action depends on the unknown state of nature, which is also binary. Prior to taking an action, the decision-maker receives an informative binary signal about the state. In addition, she can solicit advice from other agents ("advisors"), each of whom has also received an informative binary signal. The crucial feature of the model is that both the decision-maker and the advisors have reputation concerns — they want to appear competent, i.e., able to receive precise signals. The decision-maker can be one of two types: good and bad, the difference being that the good type receives more informative signals. Similarly, each of the advisors can also be one of two types: high and low. Neither the decision-maker nor any of the advisors knows her or his own type. All advisors are ex-ante identical. The decision-maker cares both about taking the right action and appearing competent (i.e., being of a good type), whereas the advisors only have reputation concerns (for simplicity).

In this setup, similarly to Ottaviani and Sørensen (2001), an advisor reports truthfully in (the most informative) equilibrium if and only if his belief about the state *before* accounting for his own signal (i.e., based only on the prior and decision-maker's decision to ask for advice[2]) is sufficiently close to 1/2, so that different signals result in different states appearing more likely for the advisor. Otherwise, no informative advice takes place ("babbling" or "herding" by the advisors).

Now, if the decision-maker cares *only* about the quality of decisions, she will *always* want to ask for advice. This means that, in equilibrium,

[2] We assume that all advisors speak simultaneously. Sequential advice would not alter our results qualitatively, as we argue in Section 5.

no information can be inferred by the advisors from the decision-maker's behavior. This ensures truthful reporting when the prior belief about the state is close to 1/2. Suppose, instead, the prior is sufficiently far from 1/2 (as in the case of Nokia's self-confidence of having the best strategy in the mobiles industry). Then the advisors will herd on the prior, and no informative advice will be provided. This is what we call the problem of "excessive advice-seeking": the decision-maker's "unrestrained" advice-seeking behavior destroys provision of advice.

Now suppose the decision-maker could *commit* to ask for advice only when she receives a signal that contradicts the prior. When unrestrained advice-seeking leads to herding by the advisors, such commitment could induce the advisors to report truthfully, provided that the combination of the prior and the decision-maker's signal results in a belief sufficiently close to 1/2. As a consequence, the decision-maker would manage to receive decision-relevant information precisely when it is most needed (when her signal confirms the prior, extra information is of much lower value).

We show that the decision-maker's reputation concerns can help to implement such commitment as a separating equilibrium. The key intuition can be explained through a kind of "single-crossing" argument. A decision-maker who received the signal confirming the prior has a strong reputational motive to show this. In contrast, a decision-maker with the signal contradicting the prior has a weaker reputational incentive (or even a disincentive) to be perceived as having received the signal confirming the prior. Coupled with a higher need for advisors' information of the latter signal-type, these incentives generate separation of the two signal-types on the asking/not asking decision, provided that the weight of reputation in the decision-maker's utility function is sufficiently high.

We also show that, for a range of weights on reputation, there exists an equilibrium with even more information aggregation. In this equilibrium, the decision-maker always asks for advice when her signal contradicts the prior and *mixes* between asking and not asking when her signal confirms the prior, and the advisors report truthfully when asked. We call this equilibrium partially separating. Then, the optimal weight on reputation is the one that maximizes the frequency of asking for advice in the partially separating equilibrium, without damaging the advisors' truthtelling incentives. A further rise in the reputation concerns destroys this equilibrium and results in excessive advice-avoidance.

Next, we study the interaction between the prior uncertainty about the state of the world and the decision-maker's reputation concerns. We show that greater uncertainty leads to a higher optimal weight on reputation. The intuition is that higher prior uncertainty increases the decision-maker's incentives to ask for advice even when her signal confirms the prior. A higher weight on reputation is then needed to restrain this temptation. However, when the prior uncertainty becomes so high that truthtelling by the advisors arises even if the decision-maker *always* asks for advice, restraining advice-asking is not needed anymore, and any weight on reputation from 0 up to a certain value becomes optimal.

There may be various ways of adjusting reputation concerns in an organization. One way is to pick managers with certain characteristics (for instance, younger managers are likely to have stronger reputation concerns).

Another way is to calibrate practices of rewarding and punishing managers: increasing explicit rewards for high performance or raising the likelihood of dismissal for underperformance is equivalent to lowering the weight of reputation. Then, our findings imply that, as uncertainty about the right strategy for an organization kicks in, one should relieve the anxiety of the manager on the correct decision by making explicit rewards and/or the probability of dismissal less sensitive to performance.

Going back to Nokia, Huy et al. (2016) argue that Nokia's top managers were not technological experts (in contrast to Apple's Steve Jobs) and routinely relied on information provided by middle managers (that is, they "always asked for advice", in our terminology). In addition, the top managers were constantly under strong pressure from investors to deliver short-term results. Our model suggests that greater top managers' concerns for being perceived as technological experts and lower external pressure would generate advice-seeking behavior conducive to truthful information provision by middle managers.

We also study the impact of the prior competence of the decision-maker and the advisors on information aggregation. Higher prior competence of either party allows to aggregate more information, provided that the organization can adjust the decision-maker's reputation concerns accordingly. If the advisors are more confident about their own information, they reveal it truthfully also when the decision-maker asks for advice more frequently (with a signal confirming the prior). If the decision-maker receives signals of higher quality, she can avoid asking for advice less frequently (with a signal confirming the prior) and still transmit to the advisors sufficient uncertainty about the state for them to be willing to report their signal truthfully. This result provides an additional rationale for the decision-maker's reputation concerns: A decision-maker who is perceived smarter by her subordinates will be more able to steer the organization along a truthful revelation path.

Yet, if the organization does not adjust the decision-maker's incentives properly, these opportunities may not be exploited, and higher prior competence of the decision-maker or of the advisors can undermine information aggregation and worsen the quality of decisions. Our model is able to capture a variety of channels, often observed in real-life settings, through which this effect can materialize. For low reputation concerns, higher quality of the advisors may provoke excessive advice-seeking. Instead, when reputation concerns are high, it can cause excessive advice-avoidance. The latter effect arises because higher-quality advice is more likely to be followed by the decision-maker independently of her private information, with the result that a correct decision will not be ascribed to her ability. Analogously, receiving higher-quality signals can induce the decision-maker to refrain from asking for advice, if the weight of reputation in her preferences is not reduced.

Finally, we note that our main results would arguably hold in an alternative setup in which advisors' reputation concerns are replaced with concerns about right decisions but acquisition and/or transmission of information is costly. Such a setup generates the same problem of excessive advice-seeking by the decision-maker with a signal confirming the prior, for if the advisors believe that they face such a decision-maker, they will lose incentives to acquire/transmit information. We elaborate more on this in the Conclusion section.

Related literature

There are a number of papers arguing that reputation concerns can be detrimental for efficiency, because they distort behavior of agents (e.g., Scharfstein and Stein (1990), Trueman (1994), Prendergast and Stole (1996), Effinger and Polborn (2001), Morris (2001), Levy (2004), Prat (2005), Ottaviani and Sørensen (2001, 2006a, 2006b), Ely and Välimäki (2003)).[3] In these papers, like in our work, reputation concerns are "career concerns for expertise" which arise due to the future gains from being perceived smart (except for Morris (2001) and Ely and Valimaki (2003), in which the agent have concerns for being perceived as having certain preferences). Of these papers, Levy (2004) and Ottaviani and Sørensen (2001, 2006a, 2006b) most closely relate to our work. Ottaviani and Sørensen consider aggregation of information from agents possessing private signals about the state of nature. Due to their reputation concerns, agents have incentives to misreport their signals, which may result in herd behavior in reporting. Levy (2004) presents a model in which a decision-maker, who knows her ability, cares both about the outcome of her action and the public perception of her ability. Levy shows that the decision-maker excessively contradicts prior public information or may abstain from asking for valuable advice in order to raise her perceived competence.

[3]A few papers provide a positive view of reputation concerns. Suurmond et al. (2004) present a model in which reputation concerns of an agent incentivize him to acquire more information. Klein and Mylovanov (2017) show that reputation concerns may provide incentives for truthful reporting in a model of long-term dynamic interaction between the agent and the principal. Also, in Morris (2001), reputation concerns of an advisor may actually make the reporting behavior of a misaligned advisor less biased.

In our model, we have a reputation-concerned decision-maker who decides whether to ask for advice or not, like in Levy (2004), and reputation-concerned advisors who are tempted to herd on the public belief in their reporting behavior, like in the papers by Ottaviani and Sørensen. The crucial distinction of our paper is the strategic interaction between reputation-concerned agents.[4] In our model, the strategy of the decision-maker (to ask for advice or not depending on her signal) impacts on the advisors' behavior. Absent such influence, the problem of excessive advice-seeking would not exist, and the results would be similar to the ones in Levy (2004), i.e., the decision-maker's reputation concerns could only harm.

Our paper is also related to works on communication with two-sided private information, especially those in which the decision-maker can (attempt to) reveal her private information before the expert talks. In de Bettignies and Zabojnik (2015) there is a manager and a worker. The manager decides whether to reveal or to conceal her signal about the optimal action for an organization. This signal is hard information but the manager does not always receive it, thus she can pretend she does not have it even when she actually does. The worker can then exert effort to search for additional information and improve the accuracy of the action. Revealing the manager's signal allows pointing the worker the right direction for his search but

[4]Levy (2004) has an extension in which she considers a strategic advisor, who has both instrumental and reputational payoff. However, in contrast to our model, the decision-maker does not exercise any influence on the advisor's truthtelling incentives. Instead, it is the advisor who tries to affect the decision-maker's actions by distorting the information he transmits. Thus, strategic interactions in Levy (2004) are orthogonal to those in our paper.

may dampen the worker's effort. The authors show that, in equilibrium, the manager conceals information more often than it would be optimal for her if she could commit to a signal-revelation strategy.

Chen (2009) considers a Crawford and Sobel (1982) type of framework, but assumes that the decision-maker also has private information about the state. She provides conditions under which the decision-maker fails to reveal her signal to the expert in equilibrium and discusses when such revelation (full or partial) is possible. In a subsequent paper, Chen and Gordon (2014) argue that full revelation of the decision-maker's information is possible only if her signal is sufficiently informative. However, these papers do not discuss whether the decision-maker would benefit or lose from the ex-ante perspective by hiding her information.

Chen (2012) considers the effects of public information in a Crawford-Sobel framework. The paper shows that, depending on the magnitude of the bias and the precision of the public signal, the receiver may be either better or worse off when the sender is asked to report after the public signal arrives. Since in Chen (2012) the public signal always arrives prior to the decision-maker choosing her action, her setting is equivalent to a setup in which the receiver has private information and can choose *ex-ante* whether to *commit* to reveal or to conceal it before the sender's communication.

In all mentioned studies on communication with two-sided private information, the incentive problems of the sender(s) are driven by either costly effort provision or divergence of preferences with the receiver over optimal actions. In contrast, in our paper, the advisors' incentive problem stems from their reputation concerns. More importantly, in these papers the decision-maker has the only goal of extracting information from the sender. In our model, instead, the decision-maker's incentives are shaped by the trade-off between the desire to receive information and the desire to appear competent.

The rest of the paper is organized as follows. In Section 2 we set up the model. Section 3 carries out the equilibrium analysis. In Section 4 we examine the effects of the prior uncertainty about the state as well as the impact of advisors' and the decision-maker's expected competence. Section 5 shows the robustness of our results to alternative modeling assumptions. Section 6 concludes the paper. The Appendix contains the proofs for Section 3. The Supplemental Appendix mostly contains the proofs for Section 4, complementary material to Section 5 and a numerical example.

2 The model

2.1 Players and information

There is a state of the world $\omega \in \{0,1\}$. A decision-maker has to take a decision $d \in \{0,1\}$. The instrumental utility for the decision-maker from the decision is 1 if the decision matches the state of the world and 0 otherwise. The decision-maker receives a private signal $\sigma \in \{0,1\}$ about the state. There are N advisors, each of whom has also received a private signal $s_i \in \{0,1\}$, $i \in \{1,...,N\}$. Conditional on the state, all signals are independent.

The decision-maker can be of two types, $\theta \in \{G, B\}$, which influence the precision of her signal. Specifically, for any ω,

$$g := \Pr(\sigma = \omega | \theta = G) > b := \Pr(\sigma = \omega | \theta = B) \geq 1/2,$$

That is, the *G*ood type of the decision-maker receives a more informative signal than the *B*ad type.

Analogously, each advisor $i = 1, ..., N$ can be of type $t_i \in \{H, L\}$, with the *H*igh type receiving a more informative signal than the *L*ow type. Namely, for any ω:

$$h := \Pr(s_i = \omega | t_i = H) > l := \Pr(s_i = \omega | t_i = L) \geq 1/2.$$

The types of all agents are independent of each other and of the state of the world. No agent knows his/her own type and types of others. There are common priors about the state of the world, the type of the decision-maker, and the type of each advisor, namely:

$$p := \Pr(\omega = 0),\ q := \Pr(\theta = G),\ r := \Pr(t_i = H),\ \forall i = 1, ..., N;\ p, q, r \in (0, 1)$$

Without loss of generality, we assume that $p \geq 1/2$.

We will call the decision-maker "signal-type 0" when she has received signal $\sigma = 0$ and "signal-type 1" otherwise (not to confuse the private information of the decision-maker with her unknown type θ.)

2.2 Sequence of the events and payoffs

The sequence of events is as follows:

1. Nature draws the state ω and the competences of all players.

2. All players receive their private signals.

3. The decision-maker decides whether to ask for advice or not. This is a binary choice $m \in \{m^0, m^1\}$, where m^0 and m^1 denote "not asking" and "asking" respectively. It is impossible to ask a subgroup of advisors: Either all advisors are invited to provide advice or none. If the decision-maker does not ask, the game proceeds to stage 5. If she asks, the game proceeds to the next stage.

4. If asked, the advisors provide their advice publicly to the decision-maker. Specifically, all advisors *simultaneously* and *publicly* send binary *cheap-talk* messages $a_i \in \{0, 1\}$, $i \in \{1, ..., N\}$.

5. The decision-maker takes a decision $d \in \{0, 1\}$.

6. The state is revealed and players receive their payoffs.

The decision-maker cares about matching her action with the state (instrumental objective). However, she would also like to appear informed (reputation concerns). We model the decision-maker's reputational payoff as the posterior belief about her ability in the eyes of an "external observer", who observes the whole course of the game (m, d, and $a = (a_i)_{i=1}^N$ if $m = m^1$) and the realized state (ω): $\Pr(G|m, a, d, \omega)$ (a to be omitted if $m = m^0$). The observer could be a decision-maker's boss (say, the board of directors). Alternatively, the decision-maker could care about his reputation in the eyes of the advisors (who may be her colleagues or subordinates).[5]

[5]In Section 4 we show that *a priori* more competent decision-makers, under the right reputational incentives, are able to receive truthful advice more often. This provides a ground for why the decision-maker may care about her reputation in the eyes of the advisors.

The decision-maker's aggregate payoff is a convex combination of the instrumental and reputational objectives with weight $\rho \in [0,1]$ attached to reputation:

$$u_D(m,a,d,\omega) = (1-\rho)I(d,\omega) + \rho \Pr(G|m,a,d,\omega), \text{ where}$$

$$I(d,\omega) = \begin{cases} 1 \text{ if } d = \omega; \\ 0, \text{ if } d = 1-\omega. \end{cases}$$

For simplicity, we assume that the advisors only have reputation concerns: Each advisor cares only about his reputation in the eyes of the decision-maker. An advisor's payoff is thus

$$u_i(m,a,d,\omega) = \Pr(H|a_i,\omega), \ \forall i = 1,...,N,$$

provided that the decision-maker asked for advice.[6]

To avoid uninteresting cases, we make the following assumptions.

A1 If all advisors receive the same signal $\overline{s} \in \{0,1\}$, then $\Pr(\omega = \overline{s}|\sigma, s = (\overline{s}...\overline{s})) > 1/2$, regardless of σ.

A2 Upon inferring that the decision-maker has received signal 0, each advisor believes that state 0 is more likely *regardless of the own signal*, i.e., $\Pr(\omega = 0|\sigma = 0, s_i) > 1/2$, regardless of s_i; upon inferring that the decision-maker has received signal 1, an advisor who received signal 1 believes that state 1 is more likely, i.e., $\Pr(\omega = 1|\sigma = 1, s_i = 1) > 1/2$.

[6]If the decision-maker did not ask for advice, an advisor's payoff is simply the prior belief r, but this does not play any role in the model.

A1 means that both signal-types can change their mind after truthful advice, that is, advice is potentially useful for the decision-maker regardless of her own signal. For our analysis it is important that at least signal-type 1 can change her mind after advice (otherwise advice is totally useless). Assuming that advice is potentially useful also for signal-type 0 greatly simplifies the exposition. In Section 5 we discuss what happens when A1 is violated.

A2 eliminates the trivial cases in which the advisors' opinions about which state is more likely are independent of what they infer about the decision-maker's signal. The first part of A2 is true if

$$\Pr(\omega = 0|s_i = 1, \sigma = 0) \equiv \frac{\Pr(s_i = 1|\omega = 0)\Pr(\omega = 0|\sigma = 0)}{num. + \Pr(s_i = 1|\omega = 1)\Pr(\omega = 1|\sigma = 0)} > 1/2.$$

Since $\Pr(s_i = \omega) = rh + (1 - r)l$ for any ω, this condition boils down to

$$\Pr(\omega = 0|\sigma = 0) > rh + (1 - r)l,$$

that is, the average precision of an advisor's signal is smaller than the combined strength of the initial prior *and* a signal 0 to the decision-maker. Conversely, the second part of A2 is true if

$$\Pr(\omega = 0|\sigma = 1) < rh + (1 - r)l.$$

The model uses a number of assumptions. First, no player knows his/her own type. Second, if the advisors are not asked, they cannot report anything. Third, asking cannot be accompanied by any additional statements from the decision-maker. Fourth, advice is simultaneous rather than sequential. Fifths, both asking for advice and providing advice are public. Sixth, the decision-maker is allowed to ask either all advisors or none only. Finally, the advisors only care about their reputation in front of the decision-maker. In Section 5 we argue that relaxing these assumptions does not have a qualitative impact on our results.

3 Equilibrium analysis

All the results of this section except for Lemma 2 are proved in the Appendix.

3.1 The decision stage

Proceeding by backward induction, we start the equilibrium analysis from the final decision stage. In terms of expected instrumental utility, it is always optimal for the decision-maker to take the action that corresponds to the state she considers more likely at that moment. In terms of expected reputation, intuitively, the decision-maker always prefers to be perceived as the signal-type corresponding to the state she considers more likely rather than the opposite signal-type. Thus, both considerations give rise to the following equilibrium.

Lemma 1 *Consider an arbitrary history of events ψ prior to the decision stage (that is, ψ is either m^0 or (m^1, a)). Then, for any beliefs about the signal-types after history ψ, the following behavior is a Bayesian equilibrium of the game that starts after ψ: the decision-maker always takes the decision that corresponds to the state that she considers more likely; when she considers two states equally likely, she takes the decision that corresponds to her signal.*

Apart from the considered equilibrium, there may exist other equilibria at the decision stage. However, this is arguably the most natural equilibrium. In addition, picking a different equilibrium at the decision stage would not change our qualitative results.

3.2 The advising stage

We borrow the analysis of the advisors' behavior from Ottaviani and Sørensen (2001). Each advisor cares only about his reputation. Thus, he always prefers to be perceived as having received the signal corresponding to the state she considers more likely rather than the opposite signal. Therefore, when an advisor considers different states more likely for different signals, there is a natural equilibrium, in which he always reports his signal truthfully. In contrast, when an advisor considers the same state more likely regardless of his signal, there cannot be any informative communication, due to a strong temptation to "herd" on the more likely state.

So, we simply reformulate Lemma 1 from Ottaviani and Sørensen (2001) using our notation. Let $\overline{\omega}$ be the more likely state from the perspective of an advisor conditional on being asked but ignoring the own signal. An advisor with signal $s_i \neq \overline{\omega}$ still believes that the state corresponding to his signal is (weakly) more likely if $\Pr(\omega \neq \overline{\omega}|s_i \neq \overline{\omega}, m^1) \geq 1/2$. Similarly to the derivations following the statement of A2, one can show that this inequality is equivalent to[7]

$$\Pr(\overline{\omega}|m^1) \leq rh + (1-r)l. \tag{TR}$$

The reformulated lemma is:

Lemma 2 *When (TR) holds, advisors report their true signals in the most informative equilibrium of the advising stage; when (TR) is not satisfied, there exists no equilibrium with informative reporting.*

Thus, when the two signal-types of an advisor consider different states (weakly) more likely, we will say that the advisors *report truthfully.*[8] When the two signal-types consider the same state strictly more likely, we will say that the advisors *herd*[9] (on the corresponding message).

3.3 First best and second best

Let us note first that, in any equilibrium of the game, the *ex-ante* expected reputation of any player is equal to the prior belief about her/him, i.e., does not depend on a particular equilibrium. Thus, since the agents' payoffs are

[7] Condition (TR) is equivalent to condition $q \leq \rho^I$ from Ottaviani and Sørensen (2001), where q denotes the prior belief before advisors speak, and ρ^I is the average precision of an advisor's signal. To be precise, in Ottaviani and Sørensen (2001) the condition is $1-\rho^I \leq q \leq \rho^I$, because they do not restrict q to be greater than 1/2.

[8] When the two signal-types consider different states more likely, there is also a partially informative communication equilibrium, in which one of the signal-types randomizes between reporting his signal and lying (Ottaviani and Sørensen, 2001). Our qualitative results would remain intact if we assumed that the advisors play in this way.

[9] Equivalently, we could say that they "babble" instead of herding. Either way, what matters is that their communication is totally uninformative in equilibrium.

linear in reputation, the *ex-ante* welfare comparisons boil down to comparing the likelihoods of taking a correct decision.

By A1, advice is potentially valuable for both signal-types. Hence, the first-best solution is that both signal-types receive truthful advice and then take the decision that corresponds to the state that emerges as more likely.

Yet, if both signal-types always ask for advice, the advisors may not have the incentive to provide truthful advice. So, we ask the question: What is the maximum aggregation of information subject to the incentive compatibility constraint of the advisors?

To be precise, suppose signal-type 1 always asks for advice. What is the maximum probability μ of asking by signal-type 0 compatible with (TR)? Let us denote this value of μ by $\overline{\mu}$.

Note first that, when $\mu = 1$, $\Pr(\omega = 0|m^1) = p$ (asking is not informative about the state) and $\omega = 0$ is the more likely state $\overline{\omega}$.

Suppose $p \leq rh + (1-r)l$. Then, when $\mu = 1$, $\Pr(\overline{\omega}|m^1) \leq rh + (1-r)l$, that is, (TR) is satisfied. Hence $\overline{\mu} = 1$, which is the first-best solution.

Suppose now $p > rh + (1-r)l$. Then, when $\mu = 1$, $\Pr(\overline{\omega}|m^1) > rh + (1-r)l$: (TR) is violated and the first best cannot be achieved. For $\mu = 0$, as shown in Section 2.2, A2 implies that $\Pr(\omega = 0|m^1) = \Pr(\omega = 0|\sigma = 1) < rh + (1-r)l$. So, if we gradually raise μ starting from $\mu = 0$, $\Pr(\omega = 0|m^1)$ will increase until it reaches $rh + (1-r)l$ for some $\mu < 1$, which is precisely what we call $\overline{\mu}$. Indeed, $\omega = 0$ is the more likely state at this point, and a further increase in μ would violate (TR). We say that, in such a case ($p > rh + (1-r)l$), at $\mu = \overline{\mu}$ the *second best* is realized.

It is easy to see that signal-type 1 must indeed ask with probability 1 in the second best. If she did not, then efficiency could be improved in either of the two following ways without violating (TR). Suppose both signal-types are asking with probability below 1, and (TR) holds. Then, obviously, the probabilities of asking by both signal-types could be increased in such a way that $\Pr(\overline{\omega}|m^1)$ would not change, meaning that (TR) would remain satisfied. Suppose now signal-type 0 is asking with probability 1, while signal-type 1 is not, and (TR) holds. Then, the more likely state conditional on asking, $\overline{\omega}$, is 0, and increasing the probability of asking by signal-type 1 would reduce $\Pr(\overline{\omega}|m^1)$; thus, (TR) would remain satisfied while efficiency would improve.

3.4 The choice between asking and not asking and overall equilibrium behavior

When solving the first stage of the game, we make the following assumption regarding the off-the-path beliefs of the observer.

- A3 After observing a sequence of events that has probability 0 in equilibrium, the observer puts probability 1 on the signal-type that corresponds to the observed decision.

A3 is compatible with our solution of the decision stage: whenever we pin down a pooling equilibrium at the decision stage, it is sustained by A3; whenever we pin down a separating equilibrium at the decision stage, A3 is compatible with Bayes rule. Note that according to A3, the observer believes that an unexpected decision is taken by the corresponding signal-type, *even if the observed asking or not asking action was supposed to be chosen only by* *the other signal-type*.[10] A3 may seem rather restrictive, but we make it for simplicity. Weaker assumptions on off-the-path beliefs would not alter our qualitative results, but the exposition would get more complicated.[11]

Before presenting our main propositions, we formulate two auxiliary lemmas. The first one concerns the behavior of expected reputation for signal-type 0.

[10] Therefore, A3 implies that the observer *strongly believes* (Battigalli and Siniscalchi, 2002) in the behavior prescribed by Lemma 1.

[11] For example we could only assume that after observing an out-of-equilibrium sequence of events ending with decision i, the observer puts probability 1 on signal-type i if the other signal-type considers state $j \neq i$ weakly more likely given the pre-decision history.

Lemma 3 *The expected reputation of signal-type* 0 *conditional on a given* $m \in \{m^0, m^1\}$ *(i.e. conditional on not asking or asking) is:*

i) Strictly increasing in $\nu := \Pr(m|\sigma = 0)/\Pr(m|\sigma = 1)$ *for* $m = m^1$ *and* $\nu \leq 1$, *and also for any* m *and any* ν *if* $p > qg + (1 - q)b$.

ii) Strictly higher for $m = m^0$ *than for* $m = m^1$ *when* $\Pr(m^1|\sigma = 1) = 1$ *and* $p > 1/2$.

Lemma 3 implies that when signal-type 1 always asks, the expected reputation of signal-type 0 after asking is increasing in the probability that she asks (by part (i)) and is anyway higher after not asking (part (ii)). This conclusion is intuitive but far from trivial. Indeed it does not always hold if signal-type 0 asks more often than signal-type 1: in this case, in terms of expected reputation, she may be better off leaving more uncertainty about her signal, especially when she is not very confident about the state. This is so because the "downside" of revealing a signal opposite to the state of the world is higher that the "upside" of revealing a signal that corresponds to the state of the world.

Now we can formulate the key "single crossing" result that we outlined in the Introduction.

Lemma 4 *Consider a strategy of the decision-maker such that:*

1. *given the asking/not asking behavior prescribed by this strategy, truthful reporting occurs after asking, i.e., (TR) holds;*

2. *signal-type* 1 *always asks, and signal-type* 0 *does not always ask;*

3. signal-type 0 *weakly prefers to ask.*

Then signal-type 1 *strictly prefers to ask.*

3.4.1 Equilibria with information aggregation

First, we partition the space of parameters according to the following driver: Which state does signal-type 1 consider more likely? By Bayes rule, we get:

$$\Pr(\omega = 1|\sigma = 1) = \frac{[qg + (1-q)b]\,(1-p)}{[qg + (1-q)b]\,(1-p) + [q(1-g) + (1-q)(1-b)]\,p}.$$

It is straightforward to show that $\Pr(\omega = 1|\sigma = 1) < 1/2$ if and only if:

$$qg + (1-q)b < p,$$

that is, the average signal precision is weaker than the prior.

Now, suppose asking implies $\sigma = 1$. If $\Pr(\omega = 1|\sigma = 1) < 1/2$ (i.e., $qg + (1-q)b < p$), an advisor with $s_i = 0$ will clearly believe that $\omega = 0$ is more likely. At the same time, by A2, an advisor with $s_i = 1$ will believe that $\omega = 1$ is more likely, which implies that (TR) holds. Then, by Lemma 2 we have truthful reporting by advisors.

If $\Pr(\omega = 1|\sigma = 1) \geq 1/2$ (i.e., $qg+(1-q)b \geq p$), we further partition the space of parameters according to the following driver: Do advisors report truthfully if they learn that the decision-maker has received signal 1? This is true if condition (TR) is satisfied. When $\Pr(\omega = 1|\sigma = 1) \geq 1/2$ and asking implies $\sigma = 1$, (TR) takes the form $\Pr(\omega = 1|\sigma = 1) \leq rh + (1-r)l$.

So, we have three cases:

Case 1. $qg + (1-q)b < p$;

Case 2. $qg + (1 - q)b \geq p$ and $\Pr(\omega = 1|\sigma = 1) \leq rh + (1 - r)l$;

Case 3. $qg + (1 - q)b \geq p$ and $\Pr(\omega = 1|\sigma = 1) > rh + (1 - r)l$.

We are interested in the existence of equilibria with at least some information aggregation, meaning that the decision-maker sometimes asks for advice, and the advisors report truthfully. Three types of equilibria will be of primary importance for us:

- Pooling on asking: both signal-types always ask for advice;
- Separating: signal-type 0 never asks for advice, signal-type 1 always asks;
- Partially separating: signal-type 0 asks with probability μ, signal-type 1 always asks.

We start with the existence conditions for the separating and the partially separating equilibria. The following result provides the main insight of the paper.

Proposition 1 *Consider Cases 1 and 2.*

i) A separating equilibrium in which signal-type 0 *never asks for advice and signal-type* 1 *always asks for advice exists if and only if* $\rho \in [\underline{\rho}, \overline{\rho}]$*, with* $\underline{\rho} \in (0,1)$ *and* $\overline{\rho} \in (\underline{\rho}, 1]$*, where* $\overline{\rho} < 1$ *in Case 1 and* $\overline{\rho} = 1$ *in Case 2;*

ii) A partially separating equilibrium in which signal-type 0 *is indifferent between asking and not asking for advice and signal-type* 1 *always asks exists if and only if* $\rho \in [\underline{\rho}, \widehat{\rho}]$*, where* $\widehat{\rho} \in (\underline{\rho}, 1]$*.*

iii) In both equilibria the advisors report truthfully. In the partially separating equilibrium μ *is strictly increasing in* ρ*, ranging from* 0 *at* $\underline{\rho}$ *to* $\overline{\mu}$ *at* $\widehat{\rho}$*.*

The intuition behind Proposition 1 is as follows. A decision-maker who received the signal confirming the prior (signal-type 0) has a strong reputational incentive to convey this news to the observer. At the same time, her need for extra information is low, because she is already quite confident about the state. In contrast, a decision-maker who received the signal contradicting the prior (signal-type 1) has either a weaker reputational incentive to be perceived as signal-type 0 (when the signal is weaker than the prior – Case 1) or even a reputational incentive to reveal her true signal (when the signal is stronger than the prior – Case 2). At the same time, such decision-maker cares more about information aggregation, because the signal contradicting the prior results in higher uncertainty compared to the signal confirming the prior.

Thus, the rationale for separation (full or partial) of the two signal-types arises. However, the weight of reputation should generally be sufficiently high for such separation to emerge. When ρ is below $\underline{\rho}$, the instrumental incentive to receive additional information dominates and signal-type 0 prefers to deviate to asking for advice.

At $\rho = \underline{\rho}$, the incentive compatibility of signal-type 0 binds. Hence, by Lemma 4, in the separating equilibrium, signal-type 1 strictly prefers to ask for advice at $\underline{\rho}$ as well as for some $\rho > \underline{\rho}$, by continuity. However, when signal-type 1 believes that $\omega = 0$ is more likely (Case 1), she has a reputational incentive to mimic signal-type 0. As we increase ρ, this incentive grows and finally prevails once ρ passes $\overline{\rho}$. Consequently, for $\rho > \overline{\rho}$, full separation cannot be supported anymore.

Consider now the partially separating equilibrium. If signal-type 1 always asks for advice, then, by Lemma 3, part (ii), the expected reputation of signal-type 0 from asking is lower than from not asking for any probability of asking, μ. However, by part (i) of the lemma, it grows with μ, thus making asking more attractive to her. Since asking generates a higher instrumental payoff, then, provided that ρ is neither too low nor too high, there will be μ that makes signal-type 0 indifferent between asking and not asking (given that the advisors report truthfully).

Since an increase in ρ makes not asking more attractive, μ has to go up with ρ in equilibrium, in order to preserve the indifference. Eventually ρ becomes so high that μ hits $\overline{\mu}$ – the maximum μ compatible with truthtelling by the advisors. The corresponding value of ρ is denoted $\widehat{\rho}$. A further increase in ρ, while keeping μ at $\overline{\mu}$, will make signal-type 0 deviate to not asking.

Let us consider Case 3 now.

Proposition 2 *Consider Case 3. There exists a separating equilibrium for every value of* ρ *but it does not trigger truthful reporting. There exists a partially separating equilibrium in which signal-type* 0 *is indifferent between asking and not asking for advice and signal-type* 1 *always asks if and only if* $\rho \in [\underline{\widehat{\rho}}, \widehat{\rho}]$*, where* $\underline{\widehat{\rho}} \in (0,1)$*,* $\widehat{\rho} \in (\underline{\widehat{\rho}}, 1]$*. In the partially separating equilibrium the advisors report truthfully and* μ *is strictly increasing in* ρ*, ranging from some* $\underline{\mu} > 0$ *at* $\underline{\widehat{\rho}}$ *to* $\overline{\mu}$ *at* $\widehat{\rho}$*.*

Similarly to Case 2, in Case 3 each signal-type prefers to be recognized as such rather than the opposite signal-type. However, now full separation does not trigger truthful reporting after asking. Thus, full separation, albeit with no information provision, becomes possible in equilibrium for any value of ρ.

In the partially separating equilibrium, to induce truthtelling by the advisors, signal-type 0 needs to ask at least with probability $\underline{\mu}$ that makes the incentive compatibility condition of the advisors binding ($\Pr(\omega = 1|m^1) = rh + (1-r)l$). The lower bound on reputation concerns, $\underline{\widehat{\rho}}$, is the value of ρ that makes signal-type 0 indifferent between asking and not asking for $\mu = \underline{\mu}$.

When pooling on asking triggers truthful reporting, the first best can be implemented in a pooling equilibrium up to precisely $\widehat{\rho}$. Indeed, if pooling triggers truthful reporting, the partially separating equilibrium at $\widehat{\rho}$ coincides with the pooling equilibrium with weak incentive to ask for signal-type 0.

Proposition 3 *If* $p \leq rh + (1-r)l$ *a pooling equilibrium in which both signal-types always ask for advice and the advisors report truthfully exists if and only if* $\rho \in [0, \widehat{\rho}]$. *If* $p > rh + (1-r)l$ *such an equilibrium does not exist.*

Beside the three described equilibria, there may exist other equilibria with information aggregation. We will prove in Proposition 4 that none of these equilibria exist for $\rho < \underline{\rho}$ in Cases 1 and 2, and for $\rho < \underline{\widehat{\rho}}$ in Case 3. Moreover, each of these equilibria, (i) if it exists for $\rho \leq \widehat{\rho}$, it is ex-ante worse than the pooling-on-asking equilibrium or the partially separating equilibrium for the same ρ, and (ii) if it exists for $\rho > \widehat{\rho}$, it is ex-ante strictly worse than the pooling-on-asking equilibrium or the partially separating equilibrium arising at $\rho = \widehat{\rho}$. For (ii), just note that any profile of strategies in which signal-type 1 asks with probability less than one is strictly worse than the second best (see Section 3.3). To show (i), we compare all the possible equilibria with information aggregation in the Supplemental Appendix, Section I.

3.4.2 General picture and the effect of reputation concerns

Consider first $p \leq rh + (1-r)l$. The pooling-on-asking equilibrium exists, and thus the first best can be implemented in equilibrium, if and only if $\rho \in [0, \widehat{\rho}]$. Any equilibrium existing for $\rho > \widehat{\rho}$ is obviously inferior. Thus, for $p \leq rh + (1-r)l$, we reach the conclusion (familiar from Levy, 2004) that too high reputation concerns hamper information aggregation.

Consider now $p > rh + (1-r)l$. For $\rho > \widehat{\rho}$ the second best cannot be implemented anymore; thus the conclusion is qualitatively the same as in the case when $p \leq rh + (1-r)l$: too high reputation concerns are harmful.

However, for low ρ the picture changes drastically. Specifically, the following holds:

Proposition 4 *Assume $p > rh + (1-r)l$. Then, for $\rho < \underline{\rho}$ in Cases 1 and 2, and for $\rho < \underline{\widehat{\rho}}$ in Case 3, there exists no equilibrium with any information aggregation.*

Thus, *when the prior is sufficiently strong ($p > rh+(1-r)l$), too low reputation concerns are unambiguously bad* as they result in a complete failure of information aggregation. The intuition is simple: when the decision-makers cares little about her reputation, she is tempted to ask for advice regardless of her signal. But then, the advisors have no incentive to report truthfully, as they keep believing in the state suggested by the prior.

Given the negative effect of crossing $\widehat{\rho}$, our overall analysis suggests that the effect of the decision-maker's reputation concerns on information aggregation is generally non-monotonic. Both too high and too low reputation concerns are detrimental for information aggregation. Too low reputation concerns provoke excessive advice-seeking, which undermines the advisors' reporting incentives. Too high reputation concerns result in excessive advice avoidance.

4 Comparative statics

Now we ask: What is the impact of the priors (about state of nature, competence of the advisors, competence of the decision-maker) on the optimal level of reputation concerns and on the ultimate quality of the decisions?

We start from the role of prior uncertainty about the state. If the prior uncertainty is so high that each advisor believes the more likely state coincides with the own signal (p close to $1/2$), the decision-maker can always ask for advice and obtain truthful reporting, hence reputation concerns do not matter (as long as they are not so high that signal-type 0 prefers to reveal herself by not asking). Else, as the prior uncertainty decreases (p goes up), signal-type 0 becomes more confident about the state and less tempted to ask for advice. Therefore, she will refrain from asking (every time) for lower levels of reputation concerns, that is, the equilibrium thresholds $\underline{\rho}$ (or $\underline{\widehat{\rho}}$) and $\widehat{\rho}$ tend to decrease. Also, she must ask less frequently for asking to transmit sufficient uncertainty to the advisors and induce them to report truthfully. This makes not asking even more tempting, to avoid being perceived as signal-type 1. So, the thresholds $\underline{\widehat{\rho}}$ and $\widehat{\rho}$ decrease further.

Proposition 5 *When the prior uncertainty is not too high, $p > rh+(1-r)l$, greater prior uncertainty calls for higher reputation concerns, as $\underline{\rho}$, $\underline{\widehat{\rho}}$ and $\widehat{\rho}$ rise, and $\underline{\rho} = \underline{\widehat{\rho}}$ when we switch from Case 2 to Case 3. When the prior uncertainty is high enough, $p \leq rh + (1-r)l$, the first best can be achieved for all levels of reputation concerns up to a threshold ($\widehat{\rho}$), which also increases in the prior uncertainty.*

Proof. The second statement of Proposition 5 follows directly from Proposition 3, the first statement is formally proved in the Supplemental Appendix. ■

It seems obvious that more competent advisors or decision-maker improve the quality of decisions. This is certainly the case when asking and reporting behavior of the parties is fixed. However, the competence of the advisors and/or the decision-maker do affect both asking and reporting, and, as we argue below, these changes in behavior can be detrimental to information aggregation.

If the organization is able to adjust the relative reputation concerns of the decision-maker, the effect can only be positive: the second-best frequency of asking increases in the prior competence of decision-maker and advisors. For the competence of the advisors, the argument is very simple: More confident advisors believe that the state that corresponds to their signal is more likely for a wider range of prior beliefs about the state (i.e., condition (TR) is relaxed). For the competence of the decision-maker, the mechanism is a bit more subtle. As it increases, a smaller difference between the probabilities of asking by signal-types 1 and 0 is sufficient to move the advisors' belief $\Pr(\omega = 0|m^1)$ close enough to $1/2$ and induce truthful reporting.

Proposition 6 *The second-best probability of asking by signal-type* 0*,* $\overline{\mu}$*, is increasing in the decision-maker's or the advisors' prior competence.*

Proof. See the Supplemental Appendix for a formal proof. ■

Yet, for fixed values of ρ, higher prior quality of the advisors or of the decision-maker can surprisingly harm information aggregation. For low levels of reputation concerns, higher prior competence of the advisors can induce excessive advice-seeking (in other words, $\underline{\rho}$ increases), which can completely destroy the incentive of the advisors to report truthfully (cf. Proposition 4).

For higher values of reputation concern, higher prior competence of the advisors or of the decision-maker can *both* induce excessive advice avoidance.

For the decision-maker, the reason is obvious: Higher signal precision makes signal-type 0 more confident about the state and more tempted to signal her signal-type by not asking, which can destroy (for instance) the pooling-on-asking equilibrium.

For the advisors, the reason is subtle. Higher quality of advice induces the decision-maker to follow it more often independently from her private information. This reduces the opportunity for signal-type 0 to reveal herself through the decision *after* asking, thereby lowering her chances to take credit for a correct decision. As a result, signal-type 0 may prefer to abstain from asking.

The above arguments lead to the following proposition.

Proposition 7 *For given reputation concerns, greater prior competence of advisors or decision-maker can* ***worsen*** *information aggregation and the quality of decisions.*

Proof. See the Supplemental Appendix for a formal proof. ■

5 Robustness

Asking a subset of advisors

Suppose the decision-maker could choose to ask any subset of advisors. Assume this choice is observed by everyone (we address secret advice-seeking below). First, our separating, partially separating, and pooling-on-asking equilibria of the baseline model survive. This is ensured by the off-the-path belief that asking a proper subset of advisors (rather than all advisors) implies that the decision-maker has received $\sigma = 0$, thus resulting in no truthtelling, by A2.

There can be other equilibria, but the crucial thing is that signal-type 0 cannot ask a subset of advisors different from the one approached by signal-type 1 and receive informative advice at the same time: in any such equilibrium, she will be recognized and, hence, provided with no information. Thus, all other equilibria look qualitatively similar to those of the baseline model, with the full set of advisors being substituted by a proper subset. We elaborate more on these equilibria in the Supplemental Appendix, Section III.A.

Secret asking and publicly unobservable advice

If we introduce the option of secret asking, our three baseline model equilibria survive for the same reason as in the previous subsection: We just need to impose the off-the-path belief that any asking behavior except asking publicly all advisors implies that the decision-maker has received $\sigma = 0$. In other words, public advice-seeking emerges endogenously in equilibrium.

In the Supplemental Appendix, Section III.D, we argue that consideration of other potential equilibria under the possibility of secret asking would not change our qualitative results.

A separate issue is observability of the advisors' messages by the external observer. This issue is irrelevant for the behavior of the advisors, as they only care about their reputation in the eyes of the decision-maker (we discuss what happens if they have other concerns in subsection "Advisors' incentives" below). As for the decision-maker, making the advisors' messages unobservable by the external observer would generally affect her incentives. This is because the decision is affected by advice, and, therefore, the observer's inference about the decision-maker's signal is affected by information on both the decision and the advice. However, intuitively, our equilibria would not qualitatively change, as the asking/not asking behavior would clearly be driven by the same trade-off as in the baseline model.

Impossibility of not asking and asking accompanied by statements

In some real cases it may be impossible to shut down advice-giving by simply not asking. Then, instead of "asking" and "not asking", m^1 and m^0 can be interpreted as two non-verifiable statements by the decision-maker about her signal before receiving advice. It is clear that the three equilibria of the baseline model survive without any changes: Due to A2, the advisors herd after hearing m^0 (assuming that in the pooling-on-m^1 equilibrium a deviation to m^0 triggers the belief that $\sigma = 0$); hence m^0 becomes equivalent to just not asking.

The above conclusion also holds if asking can be accompanied with a statement about σ: For any of our baseline model equilibria there will be an equivalent equilibrium in which both signal-types make the same statement after asking, and a deviation is interpreted as $\sigma = 0$.

In the Supplemental Appendix (Sections III.B and III.C), we argue that considering other equilibria does not change our qualitative conclusions.

Sequential public advice

First of all, notice that in our setup, for a given advisors' belief conditional on being asked, sequential public advice always provides the decision-maker with less information. If the advisors herd under simultaneous advice, so will they under sequential advice starting from the first speaker. At the same time, if the advisors tell the truth under simultaneous advice, they will still start herding under sequential advice once the number of messages in one direction exceeds that in the other direction by one or two (depending on the direction of messages).

Thus, if the choice of the advice scheme (sequential versus simultaneous) is part of the game, then the conclusions we reached in the discussion of asking a subset of advisors apply here as well (in particular, all baseline model equilibria survive) If, in contrast, sequentiality of advice is exogenous, our results still stay qualitatively intact: Although sequential advice is less informative, the fundamental trade-off between reputation and receiving information remains, generating the familiar types of equilibria.

Privately known advisors' types

First of all, what is crucial for our story is the distortion of the advisors' incentives when the confidence about the state rises. Although in our model this distortion arises due to reputation concerns, costly information acquisition by advisors would generate a similar effect (we elaborate more on that in subsection "Advisors' incentives" below), even when they know their types.

Second, while unawareness of an advisor about his type may be an extreme assumption, full awareness is equally extreme. Presumably, an advisor could learn his type through experience, i.e., by assessing correctness of his signals in the past. However such learning is limited: Even for good advisors signals are never perfectly precise and, moreover, advisors may not always receive accurate ex-post information on whether their signals matched the state.

Finally, even under the assumption that the advisors know their types, herd behavior does not fully disappear. By Lemma 4 of Ottaviani and Sørensen (2001), low types still herd with positive probability whenever $\Pr(\overline{\omega}|m^1) > l$, where $\overline{\omega}$ is the more likely state conditional on being asked. Therefore, the problem of "excessive asking", though becoming less severe, remains relevant. Hence, having a (moderately) reputation-concerned decision-maker remains beneficial, similarly to the baseline model.

Privately known decision-maker's type

Let us now return to the assumption of privately *un*known advisors' types, and consider what happens if the decision-maker knows her type. Instead of two signal-types there will be four privately known competence-signal-types (call them just "types"), which can be denoted $G0$, $G1$, $B0$, $B1$, as each of the competence-types $\{G, B\}$ can receive either $\sigma = 0$ or $\sigma = 1$.

The first thing to notice is that Proposition 4 qualitatively holds. If $\rho = 0$ or is sufficiently small, all types will be tempted to ask. Consequently, when $p > hr + l(1-r)$, the advisors will herd.

We also argue that comparative statics with respect to p remains qualitatively similar to that in the baseline model. Of course, due to richer private information, the set of equilibria will be richer. However, essentially the same trade-off determines the decision of a given type, and an increase in p reduces the benefit from asking for advice (always for $G0$ and $B0$, and eventually for all types). We provide a more detailed account of this logic (together with a specific description of possible equilibria) in the Supplemental Appendix, Section III.E.

Intuitively, a version of Proposition 6 will also hold: higher competence of the advisors or of competence-types G or B, or higher prior probability of G all allow for higher frequency of asking by signal-types 0 to be consistent with advisors' truthtelling. The same concerns Proposition 7: For fixed ρ, higher advisors' competence can provoke excessive advice-seeking, and a higher ability of competence-type G may destroy pooling-on-asking by raising her temptation to reveal her competence-type through abstaining from advice-seeking.

Advisors' incentives

Our setup can be modified to allow an advisor to care about the quality of decisions in addition to reputation. The optimal weight of the advisors' reputation concerns would then be as small as possible, to maximize their truthtelling incentives. However, in reality, it is hardly possible to eliminate the reputation concerns altogether. Therefore, the herd behavior would still be a problem (albeit for a smaller set of beliefs), and all our qualitative results would survive.

In addition to reputation in the eyes of the decision-maker, an advisor may care about his reputation in front of other people. If advice is public, this is immaterial. In contrast, if the advisors' messages are observed only by the decision-maker, such extra reputation concerns may help truthtelling indirectly, through the incentive to reduce the probability of wrong decisions. However, provided that *some* concerns for reputation in the eyes of the decision-maker remain, the argument in the previous paragraph applies here as well.

A key ingredient of our story is that the advisors are willing to provide information only when they feel uncertain about the state of nature. Apart from reputation concerns, there may be other reasons that generate a similar incentive. For example, assume that advisors have no reputation concerns and care about the quality of decisions, but need to incur a cost of acquiring (or transmitting) a signal. Then their incentives to acquire information will be stronger (and hence the quality of information received by the decision-maker will be higher) the more undecided they think the decision-maker is. Consequently, like in our baseline model, it will be crucial to avoid "excessive asking" by a decision-maker with the signal confirming the prior. At the same time, the temptation to ask for advice should increase in the prior uncertainty and the competence of advisors. Thus, we conjecture that such a framework will generate the same main results as the current one.[12]

[12] One difference of such a setting from the current one is that it is not the uncertainty about the state per se that would matter for the advisors' incentives, but whether they believe that they face a *decision-maker* who is undecided. This would matter when the decision-maker after receiving signal 1 is rather confident that $\omega = 1$. In the current model, pooling on asking triggers truthful reporting in such a case. Yet, in the alternative setup, the advisors will have weak incentives, for they know that the decision-maker is not undecided.

Abandoning A1

Suppose that, differently than what A1 imposes, signal-type 0 believes that $\omega = 0$ is more likely even when all advisors truthfully report 1. In this case, the first-best solution only requires signal-type 1 to always ask for advice. Moreover, the "stubbornness" of signal-type 0 acts as a commitment device for her not to ask for advice. Therefore, in Cases 1 and 2, the separating equilibrium with truthful reporting exists for all weights of reputation from 0 to $\overline{\rho}$. So, the residual role for reputation concerns is only to provide a *strict* (rather than weak) incentive to signal-type 0 to refrain from asking and not "disturb" truthful reporting. Any arbitrarily small value of ρ does the job. In Case 3, truthful reporting requires instead signal-type 0 to ask for advice with some probability. Then, reputation concerns can only harm, and only for $\rho = 0$, in the continuum of equilibria where signal-type 1 always asks, there are some where signal-type 0 asks with a frequency that ensures truthful advice provision.

Everything else being equal, signal-type 0 never changes her mind only for sufficiently high values of p, i.e., sufficiently low uncertainty. Then, the analysis of this extreme case confirms (in a continuous fashion) the findings that we presented in Section 4: Lower uncertainty calls for lower reputation concerns.

6 Conclusion

In this paper we have studied how reputation concerns of a decision-maker affect her ability to extract decision-relevant information from potential advisors. Too high reputation concerns provoke excessive advice-avoidance due to the decision-maker's desire to appear well informed. Too low reputation concerns result in excessive advice-seeking, which destroys advisors' incentives to provide truthful information. In general, some intermediate reputation concerns are optimal, as they create a credible commitment (in equilibrium) to abstain from asking for advice too frequently and, at the same time, do not trigger too much advice-avoidance.

A rise in the prior uncertainty about the state of nature increases the temptation to ask for advice. This may disrupt aggregation of information when the prior uncertainty is not too high, i.e., when the problem of excessive advise-seeking is relevant. In such a case, higher optimal reputation concerns are needed in order to restrain excessive advice-seeking.

We have also shown that an increase in the prior competence of the advisors or the decision-maker has a non-trivial effect. Both improve information aggregation, provided that the reputation concerns of the decision-maker are properly adjusted. However, absent such an adjustment, higher prior competence of either party can worsen information aggregation and the quality of decisions. Better quality advisors may provoke excessive advice-seeking (when the decision-maker's reputation concerns are not strong enough) or excessive advice-avoidance (when the reputation concerns are sufficiently high). Higher prior competence of the decision-maker may induce her to refrain from asking for advice, if the weight of reputation in her preferences is not reduced.

A legitimate question is how an organization can adjust the relative weight of reputation concerns in the decision-maker's utility function. One factor that can affect reputation concerns is the age of the decision-maker: Other things being equal, younger managers should have stronger career concerns. Alternatively, an organization could adjust practices of rewarding and punishing managers: Higher explicit rewards for good performance or higher likelihood of dismissal for underperformance is equivalent to a lower weight of reputation. In particular, our findings imply that, as uncertainty about the right strategy for an organization kicks in, one should relieve the anxiety of the manager on the correct decision by making explicit rewards and/or the probability of dismissal less sensitive to performance.

Appendix

Proofs of the propositions of Section 3.

Proof of Proposition 1. Take a candidate separating equilibrium in which signal-type 1 always asks and signal-type 0 never asks. It is easy to observe that the difference in expected reputation between asking and not asking is negative for signal-type 0 and, in Case 1, signal-type 1,[13] whereas it is zero for signal-type 1 in Case 2.[14] By truthful reporting after asking and A1, the difference in expected instrumental payoff between asking and not

[13]The decision maker prefers to be perceived as the signal-type that corresponds to the state that she considers more likely rather than as the opposite signal-type. For the formalization of this argument, see the proof of Lemma 1.

[14]In Case 2, after not asking signal-type 1 decides 1, so by A3 she is perceived as signal-type 1, just like after asking.

asking is positive for both signal-types.[15] Hence, the difference in expected payoff between asking and not asking is strictly decreasing in ρ for both signal-types. For $\rho = 0$, both signal-types strictly prefer to ask. For $\rho = 1$, signal-type 0 strictly prefers not to ask and signal-type 1 strictly prefers not to ask in Case 1 and is indifferent in Case 2. Thus, each signal-type is indifferent in the candidate separating equilibrium only for one value of ρ. Let $\underline{\rho}$ be the value at which signal-type 0 is indifferent and let $\overline{\rho}$ be the value at which signal-type 1 is indifferent. By Lemma 4, at $\underline{\rho}$ signal-type 1 strictly prefers to ask. Thus $\overline{\rho} > \underline{\rho}$ ($\overline{\rho} = 1$ in Case 2) and at $\overline{\rho}$ signal-type 0 strictly prefers not to ask. Therefore, the separating equilibrium exists if and only if $\rho \in [\underline{\rho}, \overline{\rho}]$.

Consider now the partially separating equilibrium. For $\rho < \underline{\rho}$, no such equilibrium can exist: Since signal-type 0 strictly prefers to ask when $\mu = 0$, by Lemma 3 (part (i)) she strictly prefers to ask also when $\mu > 0$ (when signal-type 1 always asks, ν of Lemma 3 is identical to μ). For $\rho = 1$, by Lemma 3 (part (ii)) (and by continuity for the case $p = 1/2$), signal-type 0 weakly prefers not to ask for any value of μ. For a fixed μ, the expected payoff after asking or not asking is the convex combination of two constant terms (expected reputation and expected instrumental utility) with weights ρ and $(1 - \rho)$. Hence, the observations above about $\rho < \underline{\rho}$ and $\rho = 1$ imply that the difference in expected payoff between asking and not asking is strictly decreasing in ρ for signal-type 0. Therefore, for any μ, there must be a unique value of $\rho \in [\underline{\rho}, 1]$ such that signal-type 0 is indifferent between asking and

[15] This is because, by A1, advisors' information is decision-relevant with a positive probability. See the proof of Lemma 4 for the formal argument.

not asking. For $\mu = 0$ such value of ρ is obviously $\underline{\rho}$. Furthermore, this value must be strictly increasing in μ. This is because, by Lemma 3 (part (i)), expected reputation of signal-type 0 after asking is increasing in μ. Hence, a higher μ requires a higher ρ to keep signal-type 0 indifferent. Let $\widehat{\rho}$ be such value for $\mu = \overline{\mu}$, i.e., the maximum value of μ compatible with truthful reporting by the advisors.

Thus, given that signal-type 1 always asks for advice, the range of ρ for which signal-type 0 is indifferent between asking and not asking for some μ is $[\underline{\rho}, \widehat{\rho}]$. By Lemma 4, whenever signal-type 0 is indifferent, signal-type 1 strictly prefers to ask if $\mu < 1$, and, by continuity, weakly prefers to ask if $\mu = 1$. Thus she will not deviate. Therefore, the partially separating equilibrium exists if and only if $\rho \in [\underline{\rho}, \widehat{\rho}]$. ■

Proof of Proposition 2. It is straightforward to observe that the candidate separating equilibrium is always an equilibrium. Since $\Pr(\omega = 1|\sigma = 1) > rh + (1-r)l$, the advisors herd, hence there is no difference in expected instrumental utility between asking and not asking. In terms of expected reputation, since $\Pr(\omega = 1|\sigma = 1) > 1/2$, both signal-types prefer to be perceived as such rather than as the other one (see the proof of Lemma 1 for a formal argument).

For the partially separating equilibrium, the formal argument is exactly the same as in the proof of Proposition 1, with the only difference that $\mu = 0$ is no longer compatible with truthful reporting by advisors. Notice that: (1) $\Pr(\omega = 1|m^1)$ is decreasing in μ, (2) $\Pr(\omega = 1|m^1)$ equals $1-p \leq rh+(1-r)l$ for $\mu = 1$ and $\Pr(\omega = 1|\sigma = 1) > rh + (1-r)l$ for $\mu = 0$. Hence, there exists a value of μ, denoted by $\underline{\mu}$, such that $\Pr(\omega = 1|m^1) = rh + (1-r)l$.

This is the lowest value of μ compatible with (TR). The value of ρ making signal-type 0 indifferent between asking and not asking for $\mu = \underline{\mu}$ is denoted by $\underline{\widehat{\rho}}$. Since $\overline{\mu}$ is either 1 or determined by $\Pr(\omega = 0|m^1) = rh + (1-r)l$, $\underline{\mu} < \overline{\mu}$, which implies $\underline{\widehat{\rho}} < \widehat{\rho}$. ■

Proof of Proposition 3. If $p \leq rh + (1-r)l$, $\overline{\mu} = 1$. So, by Proposition 1 for Cases 1 and 2 and by Proposition 2 for Case 3, at $\rho = \widehat{\rho}$ there exists a "partially separating" equilibrium with $\mu = 1$, i.e. the pooling-on-asking equilibrium. Since the expected instrumental utility of both signal-types is strictly higher after asking, we have: (i) for $\rho < \widehat{\rho}$ they strictly prefer to ask in the candidate pooling equilibrium, which is, therefore, an equilibrium; (ii) for $\rho > \widehat{\rho}$ (given $\widehat{\rho} < 1$), signal-type 0 strictly prefers not asking to pooling on asking, because at $\widehat{\rho}$ she is indifferent, hence she will deviate. ■

Proof of Proposition 4. Note first that, since $p > rh + (1-r)l$, truthful reporting by the advisors requires that signal-type 1 asks for advice more often than signal-type 0, that is, $\nu \equiv \Pr(m^1|\sigma = 0)/\Pr(m^1|\sigma = 1) < 1$. Suppose that signal-type 1 always asks and signal-type 0 never asks ($\nu = 0$). Then, by Lemma 3 (part (i)), the expected reputation of signal-type 0 from asking is the lowest possible, as $\nu = 0$. At the same time, her expected reputation from not asking is the highest possible in Case 1 (by Lemma 3, part (i)) and constant in Case 2 (after not asking each signal-type is perfectly revealed through decision d). Thus, the expected reputational loss from asking for signal-type 0 is the highest possible under perfect separation. Moreover, for $\rho < \underline{\rho}$, by Proposition 1 there is no separating equilibrium, because signal-type 0 would strictly prefer to ask. The two things combined imply that for $\rho < \underline{\rho}$, in any hypothetical equilibrium with truthful reporting, signal-type 0 strictly prefers to ask. But then, $\Pr(\overline{\omega}|m^1) \geq p > rh + (1-r)l$, implying no truthful reporting by the advisors.

In Case 3, by the same logic, the expected reputational loss from asking is the highest possible under partial separation with $\nu \equiv \mu = \underline{\mu}$, under the constraint that the advisors report truthfully, i.e., that $\nu \geq \underline{\mu}$. Moreover, for $\rho < \underline{\widehat{\rho}}$, by Proposition 2 there is no partially separating equilibrium, because signal-type 0 would strictly prefer to ask. The two things combined bring to the same conclusion as for Cases 1 and 2. ■

Proofs of the lemmas of Section 3.

Throughout, we assume that the decision-maker takes the decision that corresponds to the state that she considers strictly more likely (and follows her own signal if she considers both states equally likely), and that the advisors report their signals truthfully. We start with some preliminaries.

Vectors of advisors' signals

For any profile of advisors' truthfully reported signals s, let $o(s)$ denote the number of 0's in s. The decision after s is 1 if and only if $o(s) < j$ for some $j \leq n$ when $\sigma = 0$ and $o(s) < j'$ for some $j' \geq j$ when $\sigma = 1$. By A1, $j > 0$ and $j' \leq n$. Denote by S the set of all possible s. Let $\overline{S}$ be the set of s such that $j \leq o(s) < j'$ and $\widehat{S}$ its complement. In other words, $\overline{S}$ is the subset of S where, for any $s \in \overline{S}$, both signal-types take the decision corresponding to their own signal. In contrast, for any $s \in \widehat{S}$, both signal-types take the same decision, suggested by s. While $\overline{S}$ is empty when $j' = j$, $\widehat{S}$ is never empty.

For a profile s to belong to $\widehat{S}$, it must contain either enough 0's to let signal-type 1 believe that state 0 is more likely, or sufficiently many 1's (definitely more than $n/2$) to make signal-type 0 believe that state 1 is more likely. However, since $\omega = 0$ is weakly more likely a priori, the minimum number of 1's needed to "change the mind" of signal-type 0 is weakly higher than the minimum number of 0's needed to "change the mind" of signal-type 1. Therefore, the likelihood that s falls into $\widehat{S}$ should be weakly higher when $\omega = 0$.

To formalize this argument, consider first all profiles $s \in \widehat{S}$ such that $o(s) \leq n/2$. It must be that either $\Pr(\omega = 1|\sigma = 0, s) > 1/2$ (s contains so many 1's that signal-type 0 considers $\omega = 1$ more likely) or $\Pr(\omega = 0|\sigma = 1, s) > 1/2$ (despite $o(s) \leq n/2$, s contains enough 0's to let signal-type 1 still believe that $\omega = 0$ is more likely). Then, the profile $s' = \overrightarrow{1} - s$ with $o(s') = n - o(s)$ also belongs to $\widehat{S}$, because: (1) if $\Pr(\omega = 1|\sigma = 0, s) > 1/2$, then $\Pr(\omega = 0|\sigma = 1, s') > 1/2$ as well (s' contains as many 0's as s contains 1's, and $p \geq 1/2$), (2) if $\Pr(\omega = 0|\sigma = 1, s) > 1/2$, then $\Pr(\omega = 0|\sigma = 1, s') > 1/2$ (s' contains more 0's than s does).

Since all advisors are identical and, for every i, $\Pr(s_i = \omega|\omega)$ does not depend on ω, $\Pr(s|\omega = 1) = \Pr(s'|\omega = 0)$ and $\Pr(s|\omega = 0) = \Pr(s'|\omega = 1)$.

If there are any remaining profiles $s'' \in \widehat{S}$, they must have $o(s'') > n/2$, implying $\Pr(s''|\omega = 0) \geq \Pr(s''|\omega = 1)$. Thus, we conclude that

$$\Pr(\widehat{S}|\omega = 0) \geq \Pr(\widehat{S}|\omega = 1). \tag{1}$$

This formula will be used in the proof of Lemma 4.

Decision-maker's reputation at terminal nodes

Fix a terminal history ξ. Let

$$\gamma := \Pr(\sigma = \omega) = qg + (1-q)b > 1/2.$$

Suppose first that, after observing ξ, the observer concludes that the decision-maker has definitely received a specific signal σ: $\Pr(\sigma|\xi) = 1$. Then, when state ω is observed, the reputation depends only on whether $\sigma = \omega$ or $\sigma \neq \omega$, i.e., one of these two values of reputation is realized:

$$\Pr(G|\xi,\omega) = \Pr(G|\sigma = \omega) = \frac{\Pr(\sigma = \omega|G)\Pr(G)}{\Pr(\sigma = \omega)} = \frac{gq}{\gamma} =: x;$$
$$\Pr(G|\xi,\omega) = \Pr(G|\sigma \neq \omega) = \frac{\Pr(\sigma \neq \omega|G)\Pr(G)}{\Pr(\sigma \neq \omega)} = \frac{(1-g)q}{1-\gamma} := y.$$

It is straightforward to show that, since $1/2 \leq b < g$, we have $x > y$.

Suppose now that ξ does not necessarily reveal the signal-type perfectly. Specifically, suppose that either of these two cases is realized: (i) $\xi = (m^1, a, d)$ with $a = s \in \widehat{S}$ and d being the decision corresponding to the state that both signal-types consider strictly more likely, or (ii) $\xi = (m^0, d = 0)$ with $\Pr(m^0|\sigma) \neq 0$ for both σ and signal-type 1 considers state $\omega = 0$ strictly more likely. Then:

$$\Pr(\xi = (m^1, a, d)|\omega, \sigma) = \Pr(m^1|\sigma) \cdot \Pr(s|\omega) \cdot \Pr(d|\sigma, s, m^1)$$
$$= \Pr(m^1|\sigma) \cdot \Pr(s|\omega),$$
$$\Pr(\xi = (m^0, d = 0)|\omega, \sigma) = \Pr(m^0|\sigma) \cdot \Pr(d = 0|\sigma, m^0) = \Pr(m^0|\sigma).$$

In the formulas above we have used the fact that m depends only on σ, $a = s$ and s depends only on ω, and d is deterministic given σ and s (when $m = m^1$) or just σ (when $m = m^0$).

So, the reputation of the decision-maker at ξ when state ω is observed is

$$\Pr(G|\xi,\omega) = \Pr(G|\sigma=\omega)\Pr(\sigma=\omega|\xi) + \Pr(G|\sigma\neq\omega)\Pr(\sigma\neq\omega|\xi) =$$
$$= x\frac{\Pr(\xi|\sigma=\omega)\Pr(\sigma=\omega)}{numerator + \Pr(\xi|\sigma\neq\omega)\Pr(\sigma\neq\omega)} + y\frac{\Pr(\xi|\sigma\neq\omega)\Pr(\sigma\neq\omega)}{num. + \Pr(\xi|\sigma=\omega)\Pr(\sigma=\omega)} =$$
$$= x\frac{\Pr(m|\sigma=\omega)\cdot\gamma}{num. + \Pr(m|\sigma\neq\omega)\cdot(1-\gamma)} + y\frac{\Pr(m|\sigma\neq\omega)\cdot(1-\gamma)}{num. + \Pr(m|\sigma=\omega)\cdot(1-\gamma)}$$
$$= \Pr(G|m,\omega).$$

The formula is the same for cases (i) and (ii) because, after expressing $\Pr(\xi|\omega,\sigma)$ as $\Pr(m|\sigma)\cdot\Pr(s|\omega)$ in case (i), $\Pr(s|\omega)$ cancels out.

Let $\nu = \Pr(m|\sigma=0)/\Pr(m|\sigma=1)$. From the formula above, we get:

$$\Pr(G|m,\omega=1) = \frac{gq+\nu(1-g)q}{\gamma+\nu(1-\gamma)} =: v(\nu);$$
$$\Pr(G|m,\omega=0) = \frac{\nu gq+(1-g)q}{\nu\gamma+1-\gamma} =: w(\nu).$$

It is easy to observe that:

$$x = v(0) > w(0) = y;$$
$$x > v(\nu) > w(\nu) > y \qquad \text{for } \nu\in(0,1);$$
$$x > v(1) = w(1) > y;$$
$$x > w(\nu) > v(\nu) > y \qquad \text{for } \nu > 1.$$

Moreover, for any $\nu > 0$,

$$v(\nu) + w(\nu) > x + y,$$

because

$$v(\nu) = x \cdot \Pr(\sigma = 1|m, \omega = 1) + y \cdot \Pr(\sigma = 0|m, \omega = 1),$$

$$w(\nu) = x \cdot \Pr(\sigma = 0|m, \omega = 0) + y \cdot \Pr(\sigma = 1|m, \omega = 0),$$

$x > y$, and

$$\Pr(\sigma = 1|m, \omega = 1) + \Pr(\sigma = 0|m, \omega = 0)$$
$$> \Pr(\sigma = 0|m, \omega = 1) + \Pr(\sigma = 1|m, \omega = 0).$$

Proofs

Proof of Lemma 1. Consider an arbitrary history of events ψ prior to the decision stage (that is, ψ is either m^0 or (m^1, a)). Fix a signal-type $\overline{\sigma}$, and without loss of generality suppose that she considers state 0 weakly more likely, that is $\Pr(\omega = 0|\overline{\sigma}, \psi) \geq 1/2$. Suppose that if she takes $d = 1$, she is perceived as signal-type 1. This would be the equilibrium belief if signal-type 1 considers state 1 weakly more likely or an off-the-path belief when signal-type 1 considers state 0 strictly more likely.

Then, if signal-type $\overline{\sigma}$ takes $d = 1$, her expected reputation is

$$\begin{aligned}\Pr(\omega &= 0|\overline{\sigma}, \psi) \cdot \Pr(G|\sigma \neq \omega) + [1 - \Pr(\omega = 0|\overline{\sigma}, \psi)] \cdot \Pr(G|\sigma = \omega) = \\ &= \Pr(\omega = 0|\overline{\sigma}, \psi) \cdot y + [1 - \Pr(\omega = 0|\overline{\sigma}, \psi)] \cdot x,\end{aligned}$$

with x and y defined in Preliminaries.

If signal-type $\overline{\sigma}$ takes $d = 0$ and the other signal-type, at ψ, considers state 1 weakly more likely (which implies $\overline{\sigma} = 0$), the expected reputation of signal-type $\overline{\sigma}$ is:

$$\begin{aligned}\Pr(\omega &= 0|\overline{\sigma},\psi)\cdot\Pr(G|\sigma=\omega)+[1-\Pr(\omega=0|\overline{\sigma},\psi)]\cdot\Pr(G|\sigma\neq\omega)= \\ &= \Pr(\omega=0|\overline{\sigma},\psi)\cdot x+[1-\Pr(\omega=0|\overline{\sigma},\psi)]\cdot y.\end{aligned}$$

Since $\Pr(\omega = 0|\overline{\sigma},\psi) \geq 1/2$ and $x > y$, $d = 0$ yields non lower reputation than $d = 1$ to signal-type $\overline{\sigma}$.

If signal-type $\overline{\sigma}$ takes $d = 0$ and the other signal-type, at ψ, considers state 0 strictly more likely, the expected reputation of signal-type $\overline{\sigma}$ is:

$$\begin{gathered}\Pr(\omega=0|\overline{\sigma},\psi)\cdot\Pr(G|\psi,d=0,\omega=0)+ \\ +[1-\Pr(\omega=0|\overline{\sigma},\psi)]\cdot\Pr(G|\psi,d=0,\omega=1)= \\ =\Pr(\omega=0|\overline{\sigma},\psi)\cdot w+[1-\Pr(\omega=0|\overline{\sigma},\psi)]\cdot v.\end{gathered}$$

Here v and w are as defined in Preliminaries, because, given that both signal-types take the same decision after ψ, $\Pr(G|\psi,d,\omega) = \Pr(G|m,\omega)$. Since $\Pr(\omega = 0|\overline{\sigma},\psi) \geq 1/2$, $w \geq y$, and $w + v \geq x + y$, $d = 0$ yields non lower reputation than $d = 1$ to signal-type $\overline{\sigma}$.

Obviously, instrumental utility only reinforces the no-deviation incentives. ■

Proof of Lemma 3. From Bayes rule, we get:

$$\Pr(\omega=0|\sigma=0)=\frac{p\gamma}{p\gamma+(1-p)(1-\gamma)}.$$

For $m = m^0, m^1$ and $\nu = \Pr(m|\sigma = 0)/\Pr(m|\sigma = 1)$, let

$$C(\nu):=\Pr(\omega=0|\sigma=0)\cdot w(\nu)+\Pr(\omega=1|\sigma=0)\cdot v(\nu).$$

For each profile of advisors' truthfully reported signals s, let

$$A(s) := \Pr(s|\omega = 0)\Pr(\omega = 0|\sigma = 0)w + \Pr(s|\omega = 1)\Pr(\omega = 1|\sigma = 0)v;$$

$$B(s) := \Pr(s|\omega = 0)\Pr(\omega = 0|\sigma = 0)x + \Pr(s|\omega = 1)\Pr(\omega = 1|\sigma = 0)y.$$

First, we show that Part (i) holds for $m = m^1$. The expected reputation of signal-type 0 after asking is:

$$_{s\in\widehat{S}}A(s) +_{s\in\overline{S}} B(s).$$

Since $_{s\in\overline{S}}B(s)$ does not depend on ν, we can focus on $_{s\in\widehat{S}}A(s)$. As shown in Preliminaries, $\widehat{S}$ can be partitioned into pairs s, s' with $o(s') = n - o(s)$ and unpaired vectors s'' with $o(s'') \geq n/2$. Thus, $_{\widehat{s}\in\widehat{S}}A(\widehat{s})$ is increasing in ν when both $A(s) + A(s')$ for any such pair s, s' and $A(s'')$ for any such s'' are increasing in ν. This is what we show next.

Since $\Pr(s_i = \omega|\omega)$ depends neither on ω, nor on i, we have $\Pr(s'|\omega = 1) = \Pr(s|\omega = 0)$ and $\Pr(s|\omega = 1) = \Pr(s'|\omega = 0)$. Thus,

$$A(s) + A(s') = [\Pr(s|\omega = 0) + \Pr(s'|\omega = 0)] \cdot C(\nu). \tag{2}$$

Now we show that $C(\nu)$ is increasing in ν. Fix $\nu_0 < \nu_1$. For brevity, let

$\overline{p} := 1 - p$, $\overline{\gamma} := 1 - \gamma$, $\overline{g} = 1 - g$. We have

$$C(\nu_0) = \Pr(\omega = 0|\sigma = 0) \cdot w(\nu_0) + \Pr(\omega = 1|\sigma = 0) \cdot v(\nu_0) <$$

$$\Pr(\omega = 0|\sigma = 0) \cdot w(\nu_1) + \Pr(\omega = 1|\sigma = 0) \cdot v(\nu_1) = C(\nu_1) \Leftrightarrow$$

$$\frac{p\gamma}{p\gamma + (1-p)(1-\gamma)} \left(\frac{\nu_0 gq + (1-g)q}{\nu_0\gamma + 1 - \gamma} - \frac{\nu_1 gq + (1-g)q}{\nu_1\gamma + 1 - \gamma} \right) <$$

$$\frac{(1-p)(1-\gamma)}{p\gamma + (1-p)(1-\gamma)} \left(\frac{gq + \nu_1(1-g)q}{\gamma + \nu_1(1-\gamma)} - \frac{gq + \nu_0(1-g)q}{\gamma + \nu_0(1-\gamma)} \right) \Leftrightarrow$$

$$p\gamma \frac{\nu_0 g \nu_1 \gamma + \nu_0 g \overline{\gamma} + \overline{g}\nu_1\gamma + \overline{g}\overline{\gamma} - \nu_1 g \nu_0 \gamma - \nu_1 g \overline{\gamma} - \overline{g}\nu_0\gamma - \overline{g}\overline{\gamma}}{\nu_0\nu_1\gamma^2 + \nu_0\gamma\overline{\gamma} + \nu_1\gamma\overline{\gamma} + \overline{\gamma}^2} <$$

$$\overline{p\gamma} \frac{g\gamma + g\nu_0\overline{\gamma} + \nu_1\overline{g}\gamma + \nu_1\overline{g}\nu_0\overline{\gamma} - g\gamma - g\nu_1\overline{\gamma} - \nu_0\overline{g}\gamma - \nu_0\overline{g}\nu_1\overline{\gamma}}{\gamma^2 + \nu_1\gamma\overline{\gamma} + \nu_0\gamma\overline{\gamma} + \nu_0\nu_1\overline{\gamma}^2} \Leftrightarrow$$

$$p\gamma \frac{(\nu_1 - \nu_0)(\overline{g}\gamma - g\overline{\gamma})}{\nu_0\nu_1\gamma^2 + \nu_0\gamma\overline{\gamma} + \nu_1\gamma\overline{\gamma} + \overline{\gamma}^2} < \overline{p\gamma} \frac{(\nu_1 - \nu_0)(\overline{g}\gamma - g\overline{\gamma})}{\gamma^2 + \nu_1\gamma\overline{\gamma} + \nu_0\gamma\overline{\gamma} + \nu_0\nu_1\overline{\gamma}^2} \Leftrightarrow$$

$$\frac{p\gamma}{\overline{p\gamma}} > \frac{\nu_0\nu_1\gamma^2 + \overline{\gamma}^2 + \nu_0\gamma\overline{\gamma} + \nu_1\gamma\overline{\gamma}}{\nu_0\nu_1\overline{\gamma}^2 + \gamma^2 + \nu_1\gamma\overline{\gamma} + \nu_0\gamma\overline{\gamma}}, \tag{3}$$

where the last line uses $\nu_0 < \nu_1$ and $\overline{g}\gamma - g\overline{\gamma} = \gamma - g < 0$. The last inequality is always true if $\nu_0, \nu_1 \leq 1$ because then, by $\gamma^2 > \overline{\gamma}^2$, the RHS is smaller than 1, whereas the LHS is always bigger than 1. Moreover, if $p \geq \gamma$, the inequality is satisfied for all $\nu_0 < \nu_1$ because then

$$\frac{p\gamma}{\overline{p\gamma}} \geq \frac{\gamma^2}{\overline{\gamma}^2} = \frac{\nu_0\nu_1\gamma^2}{\nu_0\nu_1\overline{\gamma}^2} > \frac{\nu_0\nu_1\gamma^2 + \nu_0\gamma\overline{\gamma} + \nu_1\gamma\overline{\gamma} + \overline{\gamma}^2}{\nu_0\nu_1\overline{\gamma}^2 + \nu_0\gamma\overline{\gamma} + \nu_1\gamma\overline{\gamma} + \gamma^2}.$$

Finally, whenever $C(\nu) = \Pr(\omega = 0|\sigma = 0) \cdot w(\nu) + \Pr(\omega = 1|\sigma = 0) \cdot v(\nu)$ increases with ν,

$$A(s'') = \Pr(s''|\omega = 0) \Pr(\omega = 0|\sigma = 0) \cdot w(\nu) +$$

$$+ \Pr(s''|\omega = 1) \Pr(\omega = 1|\sigma = 0) \cdot v(\nu) = \Pr(s''|\omega = 1) C(\nu) +$$

$$+ \Pr(\omega = 0|\sigma = 0) \cdot [\Pr(s''|\omega = 0) - \Pr(s''|\omega = 1)] \cdot w(\nu)$$

does too, because $w(\nu)$ increases with ν, and $\Pr(s''|\omega = 0) \geq \Pr(s''|\omega = 1)$ (recall that $o(s'') \geq n/2$).

Note that Part (i) holds also for $m = m^0$ and $p > \gamma$ because $C(\nu)$ represents precisely the expected reputation of signal-type 0 after not asking.

For Part (ii), write the expected reputation of signal-type 0 after not asking when signal-type 1 always asks as ${}_{s\in\overline{S}\cup\widehat{S}}B(s)$.[16] Given Part (i), the expected reputation of signal-type 0 after asking when signal-type 1 always asks is maximal for $\nu = 1$. So, the difference in expected reputation between not asking and asking for signal-type 0 is bounded below by

$${}_{s\in\overline{S}\cup\widehat{S}}B(s) - \big({}_{s\in\overline{S}}B(s) +{}_{s\in\widehat{S}}\, A(s)|_{\nu=1}\big) ={}_{s\in\widehat{S}}\, (B(s) - A(s)|_{\nu=1}).$$

Similarly to the proof of Part (i), we can use the fact that $\widehat{S}$ can be partitioned into pairs s, s' with $o(s') = n - o(s)$ and unpaired s'' with $o(s'') \geq n/2$. Then it is enough to show that

$$B(s) + B(s') \geq (A(s) + A(s'))|_{\nu=1};$$
$$B(s'') \geq A(s'')|_{\nu=1}.$$

for any such pair s, s' and any such s'' respectively.

By Equation (2),

$$(A(s) + A(s'))|_{\nu=1} = [\Pr(s|\omega = 0) + \Pr(s'|\omega = 0)] \cdot C(1),$$

and, analogously to the derivation of (2), we can derive

$$B(s)+B(s') = [\Pr(s|\omega = 0)+\Pr(s'|\omega = 0)]\cdot[\Pr(\omega = 0|\sigma = 0)\cdot x+\Pr(\omega = 1|\sigma = 0)\cdot y].$$

[16]Under A3, this is the expected reputation of signal-type 0 after not asking also when she always asks too.

Note that

$$[\Pr(\omega = 0|\sigma = 0) \cdot x + \Pr(\omega = 1|\sigma = 0) \cdot y] = \lim_{\nu \to \infty} C(\nu),$$

and thus $B(s) + B(s') \geq (A(s) + A(s'))|_{\nu=1}$ is equivalent to $\lim_{\nu\to\infty} C(\nu) \geq C(1)$. Using Equation (3) with the weak inequality sign, for $\nu_0 = 1$ and $\nu_1 = \infty$ we get

$$\frac{p\gamma}{\overline{p}\gamma} \geq \frac{\gamma^2 + \gamma\overline{\gamma}}{\overline{\gamma}^2 + \gamma\overline{\gamma}} \Leftrightarrow \frac{p}{\overline{p}} \geq \frac{\gamma + \overline{\gamma}}{\overline{\gamma} + \gamma} = 1,$$

which is always true, and holds as a strict inequality unless $p = 1/2$.

Thus,

$$[\Pr(\omega = 0|\sigma = 0) \cdot x + \Pr(\omega = 1|\sigma = 0) \cdot y] \geq C(1) = q,$$

and together with $\Pr(s''|\omega = 0) \geq \Pr(s''|\omega = 1)$, and $x > q > y$,

$$B(s'') = \Pr(s''|\omega = 0) \Pr(\omega = 0|\sigma = 0) \cdot x + \Pr(s''|\omega = 1) \Pr(\omega = 1|\sigma = 0) \cdot y \geq$$
$$\geq \Pr(s''|\omega = 0) \Pr(\omega = 0|\sigma = 0) \cdot q + \Pr(s''|\omega = 1) \Pr(\omega = 1|\sigma = 0) \cdot q =$$
$$= A(s'')|_{\nu=1}.$$

■

Proof of Lemma 4. Recall first that in Preliminaries we defined j and j' as the critical numbers of 0's in s such that the decision after s is 1 if and only if $o(s) < j$ when $\sigma = 0$ and $o(s) < j'$ when $\sigma = 1$.

Then, since signal-type 0 takes $d = 0$ after not asking, the difference in expected instrumental utility between asking and not asking for this signal-type is:

$$\Delta IU_0 :=_{s:o(s)<j} [\Pr(\omega = 1, s|\sigma = 0) - \Pr(\omega = 0, s|\sigma = 0)] =$$
$$= \Pr(\omega = 1|\sigma = 0)_{s:o(s)<j} \Pr(s|\omega = 1) - \Pr(\omega = 0|\sigma = 0)_{s:o(s)<j} \Pr(s|\omega = 0).$$

Analogously, for signal-type 1, if $\Pr(\omega = 0|\sigma = 1) > 1/2$ it is:

$$\Delta IU_1 :=_{s:o(s)<j'} [\Pr(\omega = 1, s|\sigma = 1) - \Pr(\omega = 0, s|\sigma = 1)] \geq$$

$$\geq_{s:o(s)<j} [\Pr(\omega = 1, s|\sigma = 1) - \Pr(\omega = 0, s|\sigma = 1)] =$$

$$= \Pr(\omega = 1|\sigma = 1)_{s:o(s)<j} \Pr(s|\omega = 1) - \Pr(\omega = 0|\sigma = 1)_{s:o(s)<j} \Pr(s|\omega = 0)$$

where the inequality holds because $j' \geq j$ and, for every s with $o(s) < j'$,

$$\Pr(\omega = 1, s|\sigma = 1) - \Pr(\omega = 0, s|\sigma = 1) =$$

$$= [\Pr(\omega = 1|s, \sigma = 1) - \Pr(\omega = 0|s, \sigma = 1)] \cdot \Pr(s|\sigma = 1) \geq 0. \qquad (4)$$

If $\Pr(\omega = 1|\sigma = 1) \geq 1/2$, it is:

$$\Delta IU_1' :=_{s:o(s)\geq j'} [\Pr(\omega = 0, s|\sigma = 1) - \Pr(\omega = 1, s|\sigma = 1)] \geq$$

$$\geq_{s:o(s)\geq n-j+1} [\Pr(\omega = 0, s|\sigma = 1) - \Pr(\omega = 1, s|\sigma = 1)] =$$

$$= \Pr(\omega = 0|\sigma = 1)_{s:o(s)>n-j} \Pr(s|\omega = 0) - \Pr(\omega = 1|\sigma = 1)_{s:o(s)>n-j} \Pr(s|\omega = 1),$$

where the inequality holds because $j' \leq n - j + 1$ (due to $p \geq 1/2$) and, for every s with $o(s) \geq j'$,

$$\Pr(\omega = 0, s|\sigma = 1) - \Pr(\omega = 1, s|\sigma = 1) =$$

$$= [\Pr(\omega = 0|s, \sigma = 1) - \Pr(\omega = 1|s, \sigma = 1)] \cdot \Pr(s|\sigma = 1) > 0.$$

It follows immediately from $\Pr(\omega = 0|\sigma = 0) > \Pr(\omega = 0|\sigma = 1)$ that ΔIU_1 is bigger than ΔIU_0. Note furthermore that since $\Pr(s_i = \omega|\omega)$ depends neither on ω, nor on i, we have:

$$_{s:o(s)<j} \Pr(s|\omega = 1) =_{s:o(s)>n-j} \Pr(s|\omega = 0).$$

Then it follows immediately from $\Pr(\omega = 0|\sigma = 0) \geq \Pr(\omega = 1|\sigma = 1)$ that $\Delta IU_1'$ is weakly bigger than ΔIU_0.

When signal-type 1 always asks (Condition 2 of the lemma), the difference in expected reputation between asking and not asking for signal type 0 is:

$$\Delta R_0 :=_{s\in\widehat{S}} [\Pr(\omega = 0, s|\sigma = 0)(w - x) + \Pr(\omega = 1, s|\sigma = 0)(v - y)]+$$
$$+_{s\in\overline{S}}[\Pr(\omega = 0, s|\sigma = 0)(x - x) + (\Pr(\omega = 1, s|\sigma = 0)(y - y)].$$

For signal-type 1, if $\Pr(\omega = 0|\sigma = 1) > 1/2$ it is:

$$\Delta R_1 :=_{s\in\widehat{S}} [\Pr(\omega = 0, s|\sigma = 1)(w - x) + \Pr(\omega = 1, s|\sigma = 1)(v - y)]+$$
$$+_{s\in\overline{S}}[\Pr(\omega = 0, s|\sigma = 1)(y - x) + \Pr(\omega = 1, s|\sigma = 1)(x - y)],$$

and if $\Pr(\omega = 1|\sigma = 1) \geq 1/2$ it is:

$$\Delta R_1' :=_{s\in\widehat{S}} [\Pr(\omega = 0, s|\sigma = 1)(w - y) + \Pr(\omega = 1, s|\sigma = 1)(v - x)]+$$
$$+_{s\in\overline{S}}[\Pr(\omega = 0, s|\sigma = 1)(y - y) + \Pr(\omega = 1, s|\sigma = 1)(x - x)].$$

The second terms of ΔR_0 and $\Delta R_1'$ are zero, whereas the second term of ΔR_1 is non negative because for every $s \in \overline{S}$, Equation (4) holds and $x > y$. The first term of ΔR_1 is strictly bigger than the first term of ΔR_0 because $w - x < 0$, $v - y > 0$, $\widehat{S} \neq \varnothing$ (by A1), and

$$\Pr(\omega = 0, s|\sigma = 0) = \Pr(s|\omega = 0) \cdot \Pr(\omega = 0|\sigma = 0) >$$
$$> \Pr(s|\omega = 0) \cdot \Pr(\omega = 0|\sigma = 1) = \Pr(\omega = 0, s|\sigma = 1).$$

So, if $\Pr(\omega = 0|\sigma = 1) > 1/2$, signal-type 1 strictly prefers to ask, given that signal-type 0 weakly prefers to ask (Condition 3 of the lemma).

If $\Pr(\omega = 1|\sigma = 1) \geq 1/2$, suppose by contraposition that signal-type 1 weakly prefers not to ask. Then, since by A1 $\Delta IU_1'$ is positive, $\Delta R_1'$ must be negative. Then, since (as we have shown in Preliminaries) $w - y > x - v$, it must be that

$$\Pr(\{\omega = 0\} \times \widehat{S}|\sigma = 1) < \Pr(\{\omega = 1\} \times \widehat{S}|\sigma = 1). \tag{5}$$

Rewrite the first term of $\Delta R_1'$ as:

$$(w - x)_{s \in \widehat{S}} \Pr(\omega = 1, s|\sigma = 1) + (v - y)_{s \in \widehat{S}} \Pr(\omega = 0, s|\sigma = 1) +$$
$$(v - w)_{s \in \widehat{S}} \Pr(\omega = 1, s|\sigma = 1) + (w - v)_{s \in \widehat{S}} \Pr(\omega = 0, s|\sigma = 1),$$

Due to Condition 2 of the Lemma, $\nu < 1$, implying $v > w$. Hence, together with inequality (5), we obtain that the second line is positive. The first line is weakly bigger than ΔR_0, because $w - x < 0$, $v - y > 0$, and, by $\Pr(\omega = 0|\sigma = 0) \geq \Pr(\omega = 1|\sigma = 1)$ and Equation (1) from Preliminaries,

$$_{s \in \widehat{S}} \Pr(\omega = 1, s|\sigma = 1) = \Pr(\widehat{S}|\omega = 1) \Pr(\omega = 1|\sigma = 1) \leq$$
$$\leq \Pr(\widehat{S}|\omega = 0) \Pr(\omega = 0|\sigma = 0) =_{s \in \widehat{S}} \Pr(\omega = 0, s|\sigma = 0);$$
$$_{s \in \widehat{S}} \Pr(\omega = 0, s|\sigma = 1) = \Pr(\widehat{S}|\omega = 0) \Pr(\omega = 0|\sigma = 1) \geq$$
$$\geq \Pr(\widehat{S}|\omega = 1) \Pr(\omega = 1|\sigma = 0) =_{s \in \widehat{S}} \Pr(\omega = 1, s|\sigma = 0).$$

So, $\Delta R_1' > \Delta R_0$ and (as we have shown above) $\Delta IU_1' \geq \Delta IU_0$. Thus signal-type 0 strictly prefers not to ask, contradicting Condition 3 of the Lemma. ■

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Supplemental Appendix

I. Other equilibria with information aggregation

Beside the separating, partially separating, and pooling equilibria analyzed in the main body, there may exist other equilibria with truthful reporting by the advisors and positive probability of asking for advice by the decision-maker. Here we analyze alternative equilibrium behavior at the asking/not asking stage, while maintaining the solutions of the decision and advising stages pinned down in Sections 3.1 and 3.2 of the main text.

Note preliminarily that an equilibrium in which only signal-type 0 asks with positive probability and the advisors report truthfully does not exist under A2. There may exist, however, the following equilibria:

- "Bad" partially separating I: signal-type 0 never asks for advice, signal-type 1 randomizes between asking and not asking;
- "Bad" partially separating II: signal-type 0 always asks for advice, signal-type 1 randomizes between asking and not asking;
- "Fully mixed" equilibrium: both signal-types randomize between asking and not asking.

Now we argue that, for any given $\rho \in \left[\underline{\rho}, \widehat{\rho}\right]$ in Cases 1 and 2 or $\rho \in [\widehat{\underline{\rho}}, \widehat{\rho}]$ in Case 3, each of these equilibria (if it exists) it is ex-ante worse than the pooling-on-asking equilibrium (if it exists) or the “good” partially separating equilibrium for the same ρ.

First, take a “bad” partially separating equilibrium of type I. With respect to this equilibrium, both signal-types ask with non lower probability in the “good” partially separating equilibrium (as well as in the separating equilibrium with truthful reporting in Cases 1 and 2 if $\rho \leq \overline{\rho}$).

Second, take a “bad” partially separating equilibrium of type II. Signal-type 0 asks with higher probability than signal-type 1. If this still triggers truthful reporting by the advisors, then pooling on asking triggers truthful reporting by the advisors too ((TR) is a fortiori satisfied), and it is an equilibrium by Proposition 3.

Finally, suppose that there exists a “fully mixed” equilibrium. If signal-type 0 asks more frequently than signal-type 1, then pooling on asking must trigger truthful reporting too, and it is an equilibrium. If signal-type 0 asks less frequently than signal-type 1, the “fully mixed” equilibrium is worse than the “good” partially separating equilibrium for the following reason. In order not to be inferior to the “good” partially separating equilibrium, the “fully mixed” equilibrium must yield a higher probability of asking by signal-type 0. This, coupled with a lower than 1 probability of asking by signal-type 1, implies by Lemma 3 (part (i)) that the expected reputation of signal-type 0 after asking is higher than in the “good” partially separating equilibrium. After not asking, if signal-type 1 considers state 1 weakly more likely and hence decides 1, the expected reputation of signal-type 0 is the same in the two equilibria. Else, we have $p > qg + (1-q)b$, so by Lemma 3 (part (i)) the expected reputation of signal-type 0 after not asking is higher in the "good" partially separating equilibrium. Hence, in both cases, in the "fully mixed equilibrium" signal-type 0 would strictly prefer to ask, a contradiction.

II. Proofs for Section 4

Proof of Proposition 5. By inspection of ΔIU's in the proof of Lemma 4, it is easy to observe that the difference in expected instrumental utility between asking and not asking increases when p decreases.

For reputation, suppose first that, as p decreases, $\widehat{S}$ does not change. The difference in expected reputation between asking and not asking for signal-type 0, ΔR_0, reads:

$$\Pr(\widehat{S}|\omega=0)\Pr(\omega=0|\sigma=0)(w-x)+\Pr(\widehat{S}|\omega=1)\Pr(\omega=1|\sigma=0)(v-y).$$

As (i) only $\Pr(\omega|\sigma=0)$ depends on p, (ii) $\Pr(\omega=0|\sigma=0)$ decreases as p decreases, and (iii) $w-x<0<v-y$, for a given μ, ΔR_0 increases as p decreases. Moreover, it is straightforward to observe that $\underline{\mu}$ and $\overline{\mu}$ weakly increase as p decreases. By Lemma 3, part (i), an increase in $\nu=\mu$ when signal-type 1 always asks induces an increase in expected reputation of signal-type 0 after asking. Thus, the difference in the overall expected payoff of signal-type 0 between asking and not asking under $\underline{\mu}$, $\overline{\mu}$, and $\mu=0$ increases as p decreases. Then, since the difference in expected instrumental utility is

positive by A1,[17] for signal-type 0 to remain indifferent between asking and not asking as p decreases, $\underline{\widehat{\rho}}$, $\widehat{\rho}$, and $\underline{\rho}$ must increase.

Consider now a change in $\widehat{S}$ as p marginally decreases. Namely, suppose that for some $k \leq n$ and each vector of advices s with $o(s) = k$, some signal-type σ switches from considering $\omega = 0$ to considering $\omega = 1$ more likely. When $\sigma = 0$, were she to still decide 0, the reasoning for the case in which $\widehat{S}$ does not change would hold. By switching to $d = 1$, she improves her expected payoff after asking. When $\sigma = 1$, this means that, after s, signal-type 1 considers $\omega = 0$ and $\omega = 1$ equally likely. Then, if the prior is updated with s but not with $\sigma = 1$, $\omega = 0$ results more likely than $\omega = 1$. Thus, given s, signal-type 0 prefers to be perceived as such rather than pooling with signal-type 1 on $d = 0$. This observation is equivalent to Lemma 3, part (ii), as the probability of $\omega = 0$ conditional on s is higher than 1/2 like the prior p. Hence, the switch of signal-type 1 to $d = 1$ increases the expected reputation of signal-type 0 after s. Thus, a change in $\widehat{S}$ may only increase the difference in the expected payoff of signal-type 0 between asking and not asking, and this makes $\underline{\rho}$, $\widehat{\rho}$ and $\underline{\widehat{\rho}}$ increase even further.

Finally, consider a switch from $\underline{\rho}$ to $\underline{\widehat{\rho}}$. In Case 3, as $\Pr(\omega = 1|\sigma = 1)$ approaches $hr + l(1-r)$, $\underline{\mu}$ approaches 0. Thus, $\underline{\rho} = \underline{\widehat{\rho}}$ when $\Pr(\omega = 1|\sigma = 1) = hr + l(1-r)$, i.e., as we switch from Case 2 to Case 3. ■

Proof of Proposition 6. By (TR), $\overline{\mu}$, if it is not 1, is defined implicitly

[17] Hence, when signal-type 0 is indifferent between asking and not asking, the difference in expected reputation is negative.

by

$$\Pr(\omega = 0|m^1)|_{\mu=\overline{\mu}} = rh + (1-r)l. \tag{6}$$

An increase in $rh + (1-r)l$ allows an increase in $\Pr(\omega = 0|m^1)|_{\mu=\overline{\mu}}$, hence an increase in $\overline{\mu}$.

To see the effect of an increase in the competence of the decision-maker ($\gamma = qg + (1-q)b$), write $\Pr(\omega = 0|m^1)$ as

$$\frac{[\Pr(m^1|\sigma=0)\Pr(\sigma=0|\omega=0) + \Pr(m^1|\sigma=1)\Pr(\sigma=1|\omega=0)]\Pr(\omega=0)}{num. + [\Pr(m^1|\sigma=0)\Pr(\sigma=0|\omega=1) + \Pr(m^1|\sigma=1)\Pr(\sigma=1|\omega=1)]\Pr(\omega=1)}.$$

So we get

$$\Pr(\omega = 0|m^1)|_{\mu=\overline{\mu}} = \frac{(\overline{\mu}\gamma + 1 - \gamma)p}{(\overline{\mu}\gamma + 1 - \gamma)p + (\overline{\mu}(1-\gamma) + \gamma)(1-p)}.$$

As γ goes up, when $\overline{\mu} < 1$, $(\overline{\mu}\gamma + 1 - \gamma)$ goes down and $(\overline{\mu}(1-\gamma) + \gamma)$ goes up. (Thus, as expected, $\Pr(\omega = 0|m^1)|_{\mu=\overline{\mu}}$ goes down). Then, to restore equality (6), $\overline{\mu}$ must go up, so that, by $\gamma > 1/2$ (informative signals), $(\overline{\mu}\gamma + 1 - \gamma)$ increases more than $(\overline{\mu}(1-\gamma) + \gamma)$. ■

Proof of Proposition 7.

1) Greater advisors' competence.

Higher prior competence of the advisors impacts on the decision-maker's asking/not asking incentives in two ways. The most straightforward effect is a higher incentive to ask for advice due to more valuable advisors' information.

The less obvious effect is a possible discontinuous decrease in the expected reputation of signal-type 0 from asking (hence, a lower incentive to ask). It can arise because, with higher advisors' competence, there is a lower chance for signal-type 0 to separate and reveal her signal *after* asking (for instance, in the extreme case of the advisors receiving perfect signals, both signal-types will always take the same decision after asking). Suppose we are in Case 2 and consider a situation in which a certain profile of advices makes signal-type 1 believe that $\omega = 1$ is just marginally more likely than $\omega = 0$. Then, under this profile of advices, the two signal-types separate with the decision, but a marginal increase in the prior quality of the advisors will make signal-type 1 switch to $d = 0$. This induces a discrete fall in signal-type 0's expected reputation after asking, by the same argument as in the proof of Proposition 5.

Consider now $\rho = \underline{\rho}$ and suppose that a marginal improvement in the prior quality of advisors does not cause the second effect. Then such an improvement makes signal-type 0 strictly prefer to ask, which is going to destroy the advisors' truthtelling.[18] Effectively, $\underline{\rho}$ moves up and there is no equilibrium with information aggregation at the initial value of $\underline{\rho}$. In this case, higher advisors' competence harms through provoking excessive advice-seeking.

Consider now $\rho = \widehat{\rho}$ and suppose we are exactly at the point where a marginal increase in the advisors' competence is going to cause the second effect. Then, such an increase leads to a discrete fall in $\widehat{\rho}$. At the initial value of $\widehat{\rho}$, this results in the failure of not only the second- or first-best

[18] As argued, better advisors' competence also widens the set of beliefs about the state for which they report truthfully, but since we consider a marginal improvement in competence, this effect will be marginal, whereas the change in the asking/not asking behavior of signal-type 0 is discrete (and actually extreme).

equilibrium, but also of any hypothetical equilibrium sufficiently close to the second- or first best (in terms of the probabilities of asking). Thus, the fall in information aggregation will be discontinuous. In such a case, a higher advisors' competence harms through provoking excessive advice-avoidance.

In both cases, while the improvement of advisors' competence is marginal, the fall in information aggregation is discrete, meaning a reduction in the efficiency of decisions.

2) Greater decision-maker's competence.

Consider the pooling-on-asking equilibrium at $\widehat{\rho}$. In this equilibrium, signal-type 0 is indifferent between asking and not asking. Consider a marginal increase in the competence of the decision-maker. The increase in signal-type 0's confidence reduces her expected instrumental utility benefit from asking and can obviously increase her expected reputational gain from not asking. Then, signal-type 0 will strictly prefer not to ask when both signal-types are always expected to ask. Therefore, the pooling equilibrium cannot be sustained anymore at the old value of $\widehat{\rho}$. The same applies to any hypothetical equilibrium in the neighborhood of the pooling equilibrium, whenever signal-type 1 strictly prefers to ask in the pooling equilibrium under the initial level of competence. Indeed, by continuity, signal-type 1 would strictly prefer to ask in such an equilibrium, which implies $\nu \equiv \Pr(m^1|\sigma = 0)/\Pr(m^1|\sigma = 1) \leq 1$. But since, due to Lemma 3 (part (i)), the expected reputation of signal type 0 from asking is increasing in ν, deviation for $\nu = 1$ implies deviation for any $\nu \leq 1$.

Thus, the fall in information aggregation will be discrete. Since the increase in the decision-maker's competence is marginal, this implies a reduction in the efficiency of decisions. ■

III. Robustness of results to different modeling assumptions

This section complements Section 5 of the main body of the text.

III.A. Asking a subset of advisors

In this section, we consider equilibria in which only a proper subset of advisors is asked and argue that they are qualitatively the same as the equilibria of the baseline model.

First, suppose there is an equilibrium in which signal-type 0, with positive probability, asks a subset of advisors Ω_0 different from any subset approached by signal-type 1. Then, due to A2, asking Ω_0 leads to herding by the advisors and, thus, it is equivalent to not asking at all.

Pooling equilibria with a proper subset of advisors being asked are possible, although they are obviously dominated by the pooling equilibrium in which all advisors are asked. In any case, an analogue of Proposition 3 clearly holds with respect to any such pooling equilibrium: Once ρ becomes too high, signal-type 0 would want to deviate to not asking.[19]

[19] In addition, such equilibria, even though they formally exist, look implausible for sufficiently low ρ, in the sense of Grossman and Perry (1986). As any signal-type would be happy to ask the full set of advisors to improve her instrumental payoff, such a deviation should naturally keep the advisors' belief about the decision-maker's signal unchanged,

It is also clear that separating and partially-separating equilibria in which signal-type 1 asks a proper subset of advisors are similar to their counterparts of the baseline model: The trade-offs and, hence, the incentive compatibility constraints of both signal-types remain qualitatively the same. Essentially, these equilibria are just equilibria of the baseline game with a reduced number of advisors, with deviations to asking more advisors being ruled out by picking appropriate off-the-path beliefs.[20] Therefore, the analysis of Section 3.4.1 of the main text holds for any given subset of advisors being asked.

In principle, asking a proper subset of advisors can extend the set of ρ where some information aggregation is possible: Lowering the number of advisors that are asked can reduce the incentive of signal-type 0 to ask and, thus, lower $\underline{\rho}$. Thus, once we go below $\underline{\rho}$ of the baseline model, we can still sustain *some* information aggregation by reducing the equilibrium subset of asked advisors. However, it is rather obvious that, as ρ moves down, information aggregation eventually deteriorates due a lower and lower number of advisors being asked. Hence, our qualitative result that a too low weight on reputation is detrimental to information aggregation still holds.

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thus making the deviation indeed profitable for both signal-types.

[20] For example, we can impose that asking more than the equilibrium subset makes the advisors believe that $\sigma = 0$, which results in herding.

III.B. Decision-maker's statements after asking for advice

Suppose that the decision-maker can make a non-verifiable statement about her signal after asking for advice. We argue that this additional cheap talk stage does not change substantially the results of the model.

Consider an equilibrium of the modified game in which both signal-types ask with positive probability, make different and informative statements τ' and τ'', and both statements trigger truthful reporting (otherwise one would be clearly equivalent to not asking)

First, suppose that $\Pr(m^1, \tau|\sigma = 0)/\Pr(m^1, \tau|\sigma = 1) < 1$ for $\tau = \tau', \tau''$. Hence, signal-type 0 does not always ask. Since the two statements are different and signal-type 0 makes both less frequently than signal-type 1, by Lemma 3 (part (i)) signal-type 0 strictly prefers and makes only one of the two statements, say τ'. Then, signal-type 1 would strictly prefer τ' to τ'' if she would consider state 0 more likely. Since sometimes she states τ'', it must be that she considers state 1 more likely. Thus, signal-type 0 is perceived as such after not asking (for signal-type 1 decides 1). Moreover, since she plays both τ' and not asking, she must be indifferent between the two. But then, since expected reputation depends only on relative probabilities, there also exists (and aggregates more information) our "good" partially separating equilibrium, where signal-type 0 asks with probability $\Pr(m^1, \tau'|\sigma = 0)/\Pr(m^1, \tau'|\sigma = 1)$ (instead of $\Pr(m^1, \tau'|\sigma = 0)$ like here).

Second, suppose that after one statement, say τ',

$$\Pr(m^1, \tau' | \sigma = 0) / \Pr(m^1, \tau' | \sigma = 1) \geq 1.$$

Then, also pooling on asking triggers truthful reporting and can be implemented in equilibrium without statements.

When the pooling equilibrium of our baseline model exists, an equilibrium where signal-type 0 always plays (m^1, τ') and signal-type 1 randomizes between (m^1, τ') and (m^1, τ'') may exist above $\widehat{\rho}$. So, it is true that under some restrictive conditions on the parameters the additional cheap talk stage extends the implementation of the first best above $\widehat{\rho}$. However, as shown, the introduction of the non-verifiable statements does not affect at all our results for the intermediate values of ρ we are interested in, and it only confirms the message that intermediate values ρ are generally optimal, while too high or too low values of ρ harm information aggregation.

III.C. Impossibility of not asking for advice

In some real-life contexts, it could be impossible to prevent an advisor from expressing his opinion by not asking. In such cases, "not asking" essentially becomes unfeasible, and the decision-maker can only make one of two non-verifiable statements about her signal prior to receiving advice. Such a modification does not affect substantially the results. First, the separating, partially separating, and pooling equilibria exist and have the same characteristics as in the baseline model for the same values of ρ. To see this, simply note that in any of these equilibria not asking is played only by signal-type 0. Then, we can substitute not asking with statement τ' without any effect, because, due to A2, the advisors will herd after τ'. Second, any novel equilibrium of the modified game has exactly the same features as pooling on asking with subsequent statements τ' and τ'' in the game with statements after asking. So, the argument and the conclusions of Section III.B apply here as well.

III.D. Possibility of unobserved advice-seeking

Suppose now that the decision-maker was given the additional opportunity to ask for advice without being observed by the observer. Obviously, this can happen only when the observer is not the advisors. Both when the decision-maker "secretly" asks for advice and when she does not ask for advice at all, the observer observes only the final decision. Then, the reputation of the decision-maker must be the same in the two situations, given the same decision.

It is straightforward to note that the separating, partially separating, pooling-on-asking equilibria of the baseline model have equivalent counterparts in the modified game. If only signal-type 0 asks for advice secretly, she will not receive truthful advice, and this sustains the separating and partially separating equilibria. To sustain the pooling equilibrium with public asking, it is enough that when the advisors are asked for advice secretly, they assign probability 1 to signal-type 0 of the decision-maker (and so does the observer, when no advice-seeking is observed).

Suppose now instead that both signal-types ask for advice secretly, with probabilities that induce the advisors to report truthfully. Then, by A1, the decision-maker strictly prefers to ask secretly rather than not asking. So, whenever the decision-maker does not ask for advice publicly, it is clear to the observer that the decision-maker is asking for advice secretly. Thus, the situation is analogous to the one where asking and not asking are substituted by two different statements: asking publicly and asking "secretly". The only difference is the following: After asking secretly, the observer does not learn the advice that the decision-maker has received. Thus, for each of the two decisions and states of the world, the reputation of the decision-maker will be the same regardless of the unobserved vector of advices. This may eliminate separation at the decision stage in some contingencies where the two signal-types of the decision-maker do consider different states more likely (because a deviation does not entail being perceived as the opposite signal-type anymore). However, as we have already mentioned in Section 3.1 of the main text, different equilibrium choices at the decision stage do not affect qualitatively the results. Hence, all the observations of Section III.C apply here too.

III.E. Privately known decision-maker's type. Effect of p.

Assume the decision-maker knows her competence-type. There will be now four privately known competence-signal-types (call them just "types"), as each of the competence-types $\{G, B\}$ can receive either $\sigma = 0$ or $\sigma = 1$: $G0$, $G1$, $B0$, $B1$.

For $p \leq rh + (1-r)l$, pooling on asking generates truthful reporting by the advisors. Hence reputation concerns do not matter as long as they are not so high that $G0$ prefers to signal her competence-type by not asking[21].

When $p > rh + (1-r)l$, the first best cannot be achieved and, similarly to the baseline model, all informative equilibria will, roughly speaking, have the following feature: signal-types 0 will refrain from asking more often than signal-types 1.

Let us focus, for simplicity, on equilibria in pure strategies. As an example, consider the following equilibrium: $G0$ does not ask for advice, while $G1$, $B0$ and $B1$ ask, and the advisors report truthfully.[22] Such an equilibrium must exist for a range of parameters. Provided that p it is neither too high nor too low relative to the precision of the good competence-type, g, the advisors' belief after being asked will be sufficiently close to $1/2$ so that

[21]Assuming the natural off-the-path belief that not asking followed by $d = 0$ makes the observer believe that $\theta = G$.

[22]Another possible equilibrium is the one in which $G0$ and $B0$ always refrain from asking, while $G1$ and $B1$ always ask and receive truthful advice. From the point of view of the advisors, the strategy of the decision-maker conveys the same information as in the separating equilibrium of the baseline model. From the point of view of the decision-maker, since asking and not asking are unable to signal the competence-type directly, the trade-off is qualitatively the same as in the baseline model, and it is solved through a similar single-crossing argument. (Of course, since the competence-types are privately known, the relevant incentive compatibility constraints will not be exactly the same as in the baseline model). Provided that $G1$ does not have a too strong belief in $\omega = 1$, she will prefer pooling with $B1$ instead of not asking and choosing $d = 1$.

Finally, when p is sufficiently close to $1/2$, there can potentially exist equilibria in which G-types never ask and take the decisions corresponding to their signals (the behavior of B-types is likely to vary depending on the equilibrium).

(TR) holds (if the proportion of bad competence-types is high enough, then p should just not be too high).

Thus we will have the familiar trade-off between having a higher instrumental utility from asking and higher reputational payoff from not asking. Since $G0$ is most confident about the state of all types, her expected instrumental utility from asking is smaller compared to the other three. For simplicity (not crucial), we can assume that not asking followed by $d = 1$ yields an (off-the-path) belief that the decision-maker is $G1$ (a version of A3 for the privately known types setup). Then not asking always yields the belief that $\theta = G$. Then, naturally, there will be thresholds $\underline{\rho}'$ and $\overline{\rho}'$ such the that the equilibrium under consideration exists if and only if $\rho \in [\underline{\rho}', \overline{\rho}']$ Threshold $\underline{\rho}'$ will be determined by the incentive compatibility of $G0$: when $\rho < \underline{\rho}'$, the reputation concerns are so low that $G0$ will want to deviate to asking for advice. Threshold $\overline{\rho}'$ will be determined by the incentive compatibility of either $B0$ or $G1$ (the deviation incentive of $B1$ is obviously weaker than that of $B0$): when $\rho > \overline{\rho}'$, high reputation concerns will make either of these types deviate to not asking.

As p grows, type $G0$ becomes more confident about the state and, thus, less tempted to ask for advice. Therefore, a lower level of reputation concerns becomes enough for her to refrain from asking, i.e., $\underline{\rho}'$ decreases. Type $B0$ also becomes more confident that $\omega = 0$, which makes her less willing to ask. Consequently, a lower level of reputation concerns is needed to keep $B0$ asking. If $\overline{\rho}'$ is determined by the incentive compatibility of $B0$, this means that $\overline{\rho}'$ goes down. If $\overline{\rho}'$ is determined by the incentive compatibility of $G1$, it must be that $G1$ believes that $\omega = 1$ is more likely. Then, a higher p

results in higher willingness to ask by $G1$, meaning an increase in $\overline{\rho}'$. It is clear, however, that at some point $\overline{\rho}'$ becomes determined by the incentive compatibility of $B0$, and, thus, eventually goes down.

IV. Numerical Example

We conclude the Supplemental Appendix with a numerical example, which shows how the ρ-thresholds for equilibria are determined and change with the prior uncertainty.

Fix the following values of the parameters:

$$q = r = \frac{1}{2};\ g = h = \frac{7}{9};\ b = l = \frac{5}{9};\ n = 3.$$

We leave the prior uncertainty p free, to study how it influences the effect of reputation concerns on information aggregation. Note that the average signal precision, i.e. the ex-ante probability that a state generates the corresponding signal, is the same for the decision-maker and for the advisors (2/3). This has two implications. First, if both signal-types of the decision-maker always ask, each signal-type of the advisor has the same posterior over the state of the world as the decision-maker of the same signal-type. Second, the posterior over the state of the world of the decision-maker depends only on the total number of signals of each kind that she learns, *including her own.* Note that this is not a knife-edge case, in the sense that whether the decision-maker is on average better informed than the advisors or not does not determine per se any qualitative difference in the results.

First, we compute the decision-maker and the advisors' beliefs as a function of p. By Bayes rule, we can use the average signals precision (2/3) as a deterministic signal precision. Denote by $o(s)$ the number of 0's in a profile of truthfully revealed signals s. Then we have:

$$\Pr(\omega = 0|\sigma = 0) = \frac{2p}{p+1} = \Pr(\omega = 0|s_i = 0);$$
$$\Pr(\omega = 0|\sigma = 1) = \frac{p}{2-p} = \Pr(\omega = 0|s_i = 1);$$
$$\Pr(\omega = 0|\sigma = 0, s) = \frac{(\frac{2}{3})^{o(s)+1}(\frac{1}{3})^{3-o(s)}p}{(\frac{2}{3})^{o(s)+1}(\frac{1}{3})^{3-o(s)}p + (\frac{1}{3})^{o(s)+1}(\frac{2}{3})^{3-o(s)}(1-p)} =$$
$$= \begin{cases} \frac{16p}{1+15p} \text{ if } o(s) = 3 \\ \frac{4p}{1+3p} \text{ if } o(s) = 2 \\ p \quad \text{ if } o(s) = 1 \\ \frac{p}{4-3p} \text{ if } o(s) = 0 \end{cases}$$
$$\Pr(\omega = 0|\sigma = 1, s) = \begin{cases} \frac{4p}{1+3p} \text{ if } o(s) = 3 \\ p \quad \text{ if } o(s) = 2 \\ \frac{p}{4-3p} \text{ if } o(s) = 1 \\ \frac{p}{16-15p} \text{ if } o(s) = 0 \end{cases}$$
$$\Pr(\omega = 0|m^1) =$$
$$= \frac{(2\Pr(m^1|\sigma = 0) + \Pr(m^1|\sigma = 1))p}{(2\Pr(m^1|\sigma = 0) + \Pr(m^1|\sigma = 1))p + (2\Pr(m^1|\sigma = 1) + \Pr(m^1|\sigma = 0))(1-p)}.$$

As p changes, we have the following situations.

- $p \geq \frac{4}{5}$. Then $\Pr(\omega = 0|\sigma = 0, s) \geq \frac{1}{2}$ for $o(s) = 0$. This case contradicts A1.and thus it is not analyzed.

- $\frac{2}{3} < p < \frac{4}{5}$. Then $\Pr(\omega = 0|\sigma = 1) = \Pr(\omega = 0|s_i = 1) > \frac{1}{2}$. This is Case 1; moreover the advisors herd in case of pooling on asking. Signal-type 0 changes her mind only if all the advisors suggest 1. Signal-type 1, instead, follows the majority of the advisors.

- $\frac{1}{2} < p \leq \frac{2}{3}$. Then $\Pr(\omega = 0|\sigma = 1) = \Pr(\omega = 0|s_i = 1) \leq \frac{1}{2}$. This is Case 2; moreover the advisors report truthfully in case of pooling on asking. The reactions of the decision-maker to the advices are the same as in the previous case.

- $p = \frac{1}{2}$. We are still in Case 2, but also signal-type 1 now changes her mind only if all the advisors suggest 0. The analysis of this case is left to the reader

For no value of p we fall in Case 3, for which it is necessary (but not sufficient) that the advisors' signals have worse average precision than the decision-maker's one.

So, we call $\frac{2}{3} < p < \frac{4}{5}$ Case 1 and $\frac{1}{2} < p \leq \frac{2}{3}$ Case 2.

Both signal-types of the decision-maker react to the advisors' suggestions in the same way in the two cases. Moreover, signal-type 0 always decides 0 after not asking. Thus, we can compute all values of instrumental utility and reputation in the same way for both cases, except for signal-type 1 when she does not ask.

The expected instrumental utility for signal-type 0 after not asking is $\Pr(\omega = 0|\sigma = 0) = \frac{2p}{p+1}$ and for signal-type 1 it is $\Pr(\omega = 0|\sigma = 1) = \frac{p}{2-p}$ in Case 1 and $\Pr(\omega = 1|\sigma = 1) = \frac{2-2p}{2-p}$ in Case 2. After asking, the expected instrumental utility for signal-type 0 is

$$_{s:o(s)\geq 1} \Pr(\omega = 0, s|\sigma = 0) + \Pr(\omega = 1, s = (1,1,1)|\sigma = 0) =$$

$$=_{s:o(s)\geq 1} \Pr(s|\omega = 0) \Pr(\omega = 0|\sigma = 0) +$$

$$+ \Pr(s = (1,1,1)|\omega = 1) \Pr(\omega = 1|\sigma = 0) =$$

$$= (1 - \frac{1}{3^3})\frac{2p}{p+1} + \frac{2^3}{3^3}(1 - \frac{2p}{p+1}) = \frac{2}{3}\frac{2p}{p+1} + \frac{8}{27} = \frac{44p+8}{27p+27};$$

and for signal-type 1 it is

$$_{s:o(s)\geq 2} \Pr(\omega = 0, s|\sigma = 1) +_{s:o(s)<2} \Pr(\omega = 1, s|\sigma = 1) =$$

$$=_{s:o(s)\geq 2} \Pr(s|\omega = 0) \Pr(\omega = 0|\sigma = 1) +$$

$$+_{s:o(s)<2} \Pr(s|\omega = 1) \Pr(\omega = 1|\sigma = 1) = (\frac{2^3}{3^3} + 3 \cdot \frac{1}{3} \cdot \frac{2^2}{3^2})\frac{p}{2-p} +$$

$$+(\frac{2^3}{3^3} + 3 \cdot \frac{1}{3} \cdot \frac{2^2}{3^2})(1 - \frac{p}{2-p}) = (\frac{2^3}{3^3} + 3 \cdot \frac{1}{3} \cdot \frac{2^2}{3^2}) = \frac{20}{27}.$$

Suppose now that signal-type 1 always asks and signal-type 0 asks with probability μ. Then, after not asking, the advisors believe that the decision-maker has received signal 0 after decision 0 (by equilibrium strategy or A3) and signal 1 after decision 1 (by A3). Using the same notation as in the Appendix ($x := \Pr(G|\sigma = \omega),\ y := \Pr(G|\sigma \neq \omega)$), the expected reputation for signal-type 0 after not asking is:

$$\Pr(\omega = 0|\sigma = 0)x + \Pr(\omega = 1|\sigma = 0)y =$$
$$= \frac{2p}{p+1} \cdot \frac{7}{12} + (1 - \frac{2p}{p+1}) \cdot \frac{1}{3} = \frac{1}{4}\frac{2p}{p+1} + \frac{1}{3} = \frac{5p+2}{6p+6};$$

and for signal-type 1, in Case 1 (where she optimally decides 0), it is:

$$\Pr(\omega = 0|\sigma = 1)x + \Pr(\omega = 1|\sigma = 1)y =$$
$$= \frac{p}{2-p} \cdot \frac{7}{12} + (1 - \frac{p}{2-p}) \cdot \frac{1}{3} = \frac{1}{4}\frac{p}{2-p} + \frac{1}{3} = \frac{8-p}{24-12p}.$$

and in Case 2 (where she optimally decides 1), it is:

$$\Pr(\omega = 0|\sigma = 1)y + \Pr(\omega = 1|\sigma = 1)x = \\ = \frac{p}{2-p} \cdot \frac{1}{3} + (1 - \frac{p}{2-p}) \cdot \frac{7}{12} = \frac{7}{12} - \frac{1}{4}\frac{p}{2-p} = \frac{7-5p}{12-6p}.$$

After asking, the expected reputation of the two signal-types is different since they decide differently if $o(s) = 1$. Using v and w as defined in the Appendix, for signal-type 0 it is:

$$\Pr(\omega{=}1, o(s) \neq 1|0)v + \Pr(\omega{=}0, o(s) \neq 1|0)w + \Pr(\omega{=}1, o(s){=}1|0)y + \\ + \Pr(\omega{=}0, o(s){=}1|0)x == (1 - 3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{2p}{p+1})v + \\ + (1 - 3\frac{2}{3}\frac{1}{3^2})\frac{2p}{p+1}w + (3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{2p}{p+1})y + (3\frac{2}{3}\frac{1}{3^2})\frac{2p}{p+1}x = \\ = \frac{5}{9}(1 - \frac{2p}{p+1})\frac{7+2\mu}{12+6\mu} + \frac{7}{9}\frac{2p}{p+1}\frac{7\mu+2}{12\mu+6} + \frac{4}{9}(1 - \frac{2p}{p+1})\frac{1}{3} + \\ + \frac{2}{9}\frac{2p}{p+1}\frac{7}{12} = \frac{35+10\mu}{108+54\mu} + \frac{2p}{p+1}(\frac{49\mu+14}{108\mu+54} - \frac{35+10\mu}{108+54\mu} - \frac{1}{54}) + \frac{4}{27};$$

and for signal-type 1:

$$\Pr(\omega{=}1, o(s) \neq 1|1)v + \Pr(\omega{=}0, o(s) \neq 1|1)w + \Pr(\omega{=}1, o(s){=}1|1)x + \\ + \Pr(\omega{=}0, o(s){=}1|1)y = (1 - 3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{p}{2-p})v + (1 - 3\frac{2}{3}\frac{1}{3^2})\frac{p}{2-p}w + \\ + (3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{p}{2-p})x + (3\frac{2}{3}\frac{1}{3^2})\frac{p}{2-p}y = \frac{5}{9}(1 - \frac{p}{2-p})\frac{7+2\mu}{12+6\mu} + \\ + \frac{7}{9}\frac{p}{2-p}\frac{7\mu+2}{12\mu+6} + \frac{4}{9}(1 - \frac{p}{2-p})\frac{7}{12} + \frac{2}{9}\frac{p}{2-p}\frac{1}{3} = \\ = \frac{35+10\mu}{108+54\mu} + \frac{p}{2-p}(\frac{49\mu+14}{108\mu+54} - \frac{35+10\mu}{108+54\mu} - \frac{5}{27}) + \frac{7}{27}.$$

To look for $\underline{\rho}$ and $\overline{\rho}$ we need to compute the values of reputation for $\mu = 0$. Then, for signal-type 0 the expected reputation after asking is:

$$\frac{35}{108} + \frac{2p}{p+1}(\frac{28}{108} - \frac{35}{108} - \frac{2}{108}) + \frac{16}{108} = \frac{51}{108} - \frac{9}{108}\frac{2p}{p+1} = \frac{33p+51}{108p+108}$$

For signal-type 1 it is:

$$\frac{35}{108} - \frac{27}{108}\frac{p}{2-p} + \frac{28}{108} = \frac{126-90p}{108(2-p)} = \frac{7-5p}{12-6p}.$$

The difference in expected payoff between asking and not asking for signal-type 0 is zero for $\underline{\rho}$ such that:

$$(1-\underline{\rho})(\frac{44p+8}{27p+27} - \frac{2p}{p+1}) + \underline{\rho}(\frac{33p+51}{108p+108} - \frac{5p+2}{6p+6}) = 0$$
$$(1-\underline{\rho})(32-40p) + \underline{\rho}(15-65p) = 0$$
$$\underline{\rho} = \frac{32-40p}{17+25p}.$$

As expected, for $p \uparrow 4/5$, $\underline{\rho} \downarrow 0$: signal-type 0 has no strict incentive to follow three 1 suggestions, so there is no gain from asking. For $p \downarrow 2/3$, $\underline{\rho} \uparrow \frac{16}{101}$. So, in Case 1 $\underline{\rho} \in (0, \frac{16}{101})$. Recall that in Case 1 pooling on asking does not trigger truthful reporting. Thus, there is no information aggregation up to $\underline{\rho}$, i.e. there is no information aggregation for too low reputation concerns. As uncertainty increases, i.e. as p decreases, $\underline{\rho}$ increases. That is, higher reputation concerns are needed to obtain some degree of information aggregation. For $p \downarrow 1/2$, $\underline{\rho} \uparrow \frac{24}{59}$, so in Case 2, $\underline{\rho} \in [\frac{16}{101}, \frac{24}{59})$. However in Case 2, pooling on asking triggers truthful reporting and can be implemented from $\rho = 0$.

The difference in expected payoff between asking and not asking for signal-type 1 in Case 1 is zero for $\overline{\rho}$ such that:

$$(1-\overline{\rho})(\frac{20}{27} - \frac{p}{2-p}) + \overline{\rho}(\frac{7-5p}{12-6p} - \frac{8-p}{24-12p}) = 0$$
$$(1-\overline{\rho})(\frac{160-188p}{108(2-p)}) + \overline{\rho}(\frac{54-81p}{108(2-p)}) = 0$$
$$\overline{\rho} = \frac{160-188p}{106-107p}.$$

As expected, for $p \downarrow 2/3$, $\overline{\rho} \uparrow 1$. Note indeed that in Case 2, as suggested by Proposition 1, $\overline{\rho} = 1$: The expected reputation after asking and not asking is the same, so no value of ρ makes signal-type 1 indifferent between asking and not asking. For $p = 4/5$, $\overline{\rho} = \frac{24}{51}$. Thus, also $\overline{\rho}$ increases as p decreases.

To compute $\widehat{\rho}$, we need to compute the highest value of μ such that the advisors report truthfully. It solves:

$$\Pr(\omega = 0|m^1) = \frac{2\overline{\mu}p + p}{\overline{\mu}p + 2 + \overline{\mu} - p} = \frac{2}{3}.$$
$$\overline{\mu} = \frac{4 - 5p}{4p - 2}.$$

For $p \uparrow 4/5$, $\overline{\mu} \downarrow 0$, so the "second best" partially separating equilibrium approaches the separating equilibrium with weak incentive for signal-type 0. As anticipated, for $p = 2/3$, $\overline{\mu} = 1$ so for every $p > 2/3$ there is no pooling-on-asking equilibrium with truthful reporting. Now we look for the value of ρ such that signal-type 0 is indifferent between asking and not asking for $p = 2/3$ and $\mu = 1$: this is the upper bound for $\widehat{\rho}$ in Case 1. Substituting $\mu = 1$ in the reputation after asking, the difference in expected payoff for signal-type 0 between asking and not asking is zero for $\widehat{\rho}$ such that:

$$(1 - \widehat{\rho})(\frac{44p + 8}{27p + 27} - \frac{2p}{p + 1}) + \widehat{\rho}(\frac{45}{162} + \frac{2p}{p + 1}(\frac{15}{162}) + \frac{4}{27} - \frac{5p + 2}{6p + 6}) = 0$$
$$(1 - \widehat{\rho})(48 - 60p) + \widehat{\rho}(15 - 36p) = 0$$
$$\widehat{\rho} = \frac{48 - 60p}{33 - 24p}.$$

For $p \downarrow 2/3$, $\widehat{\rho} \uparrow \frac{8}{17}$. So, in Case 1, $\widehat{\rho} \in (0, \frac{8}{17})$. Note that for every p, as expected, $\widehat{\rho} > \underline{\rho}$.

In case 2, pooling on asking triggers truthful reporting. So, we are interested in $\widehat{\rho}$ as the maximum weight on reputation such that the pooling-on-asking equilibrium exists under A3. For $p \downarrow 1/2$, $\widehat{\rho} \uparrow \frac{6}{7}$. Thus, in Case 2, the pooling-on-asking equilibrium exists up to $\widehat{\rho} \in \left[\frac{8}{17}, \frac{6}{7}\right)$.

Also $\widehat{\rho}$ increases as p decreases. That is, more uncertainty requires (in Case 1) or allows (in Case 2) higher reputation concerns to achieve the best feasible level of information aggregation.

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