Syllabus

1. Course Description

   a. Title of a Course: Differential Geometry (O. V. Schwarzman).

   b. Pre-requisites: 2nd year Calculus

   c. Course Type: optional

   d. Abstract: The course will serve as an introductory guide to basic topics of Differential and Riemann geometry: the theory of Riemannian and Loretzian manifolds, the theory of affine connections on manifolds, The Gauss-Bonne-Chern-Weil theory.

2. Learning Objectives

The seminar is intended to introduce the subject of differential geometry to the students.

3. Learning Outcomes

Successful participants will master fundamental notions of differential geometry (most importantly: the concepts of a connection and a curvature), be able to apply them to solving problems in mathematics and mathematical physics.

4. Course Plan

Differential and Riemann geometry of smooth hypersurfaces in the Euclidean space.

- Parallel transport. The Gauss Map. The Shape operator.
- The metric connection. Covariant derivatives. Parallel transport.
- Completeness and geodesics. The Exponential Map. The Hopf - Rinow theorem.
- Curvature. Geodesics.

Riemann manifolds: Riemannian metric and Levi - Chivita connection. Curvature.

- Calculations with curvature tensor. The Gauss curvature.
- The Ricci tensor.
- Spaces of constant curvature.

Variational theory of geodesics.

- First and second variation of arc length.
- Jacobi's equation and conjugate points.

- The Gauss lemma and polar coordinates.

Connections in vector bundles.

- Parallel transport and Covariant derivatives.
- Introduction to the Chern - Weil theory.

5. Reading List

   a. Required

      1) Lynn H. Loomis, Shlomo Sternberg, Advanced Calculus, Jones & Bartlett Publishers, 1989 http://www.math.harvard.edu/~shlomo/docs/Advanced_Calculus.pdf

   b. Optional

      1) Shlomo Sternberg, Semi-Riemann Geometry and General Relativity, Orange Grove Texts Plus, 2009 http://www.math.harvard.edu/~shlomo/docs/semi_riemannian_geometry.pdf

6. Grading System

The formula for marking is 0.3 cumulative + 0.7 final exam, where cumulative is proportional to number of tasks solved.

7. Guidelines for Knowledge Assessment

Sample problems for exercises and tests (including the final exam)

Write down a projective transformation that maps the unit sphere of R^3 to the paraboloid $x_3={x_1}^2+={x_2}^2$

Calculate the curvature of a surface of revolution generating by rotating a plane curve $X(s),Y(s)$,where s is arc length

Show that a Hyperbolic manifold has flat sphere bundle

8. Methods of Instruction

Students are individually assigned papers and textbook excerpts to give a seminar talk.

9. Special Equipment and Software Support : no requirements