Syllabus

1. Course Description

   a. Title of a Course : Introduction to Galois Theory (C. Brav),

   b. Pre-requisites : Basic algebra: groups, rings, linear algebra over a field.

   c. Course Type : optional

   d. Abstract : Galois theory is the study of roots of polynomials and their symmetries in terms of Galois groups. As the algebraic counterpart of the fundamental group of topology, the Galois group is an essential object in algebraic geometry and number theory.

2. Learning Objectives

The seminar is intended to introduce the subject area to the students, and to offer them an opportunity to prepare and give a talk.

3. Learning Outcomes

Successful participants imporve their presentation skills and prepare for participation in research projects in the subject area.

4. Course Plan

   a. Review of polynomial rings and more general principal ideal domains.
   b. Extensions of fields, algebraic and transcendental.
   c. Splitting fields of polynomials and Galois groups.
   d. The fundamental theorem of Galois theory.
   e. Computing Galois groups.
   f. Applications.

5. Reading List

   a. Required

      1. Anthony W. Knapp, Basic Algebra, Birkhäuser, 2006 http://www.math.stonybrook.edu/~aknapp/download/b2-alg-inside.pdf

      2. Robert B. Ash , Basic Abstract Algebra: For Graduate Students and Advanced Undergraduates, Dover Publications , 2013 https://faculty.math.illinois.edu/~r-ash/Algebra.html

   b. Optional

      1. James Milne. Fields and Galois Theory,

http://www.jmilne.org/math/CourseNotes/FT.pdf

2. David S. Dummit, Richard M. Foote, Abstract Algebra, Wiley, 2003

6. Grading System

The formula for marking is 0.3 cumulative + 0.7 final exam, where cumulative is proportional to number of tasks solved.

7. Guidelines for Knowledge Assessment

1. Let $F \subset K, L \subset \overline{F}$ be finite extensions, $KL \subset \overline{F}$ the smallest subfield which contains K and L. Is it true that $[KL : F] = \frac{[K:F][L:F]}{[K \cap L:F]}$? Give a proof or provide a counterexample.

2. Does the cyclotomic polynomial $\Phi_9(x)$ have a root in $\mathbb{F}_{125}$?

3. Construct a finite extension $\mathbb{Q} \subset K$ whose Galois group is isomorphic to the dihedral group D_5 (*i.e.*, the group of symmetries of a regular pentagon).

8. Methods of Instruction

Students are individually assigned papers and textbook excerpts to give a seminar talk. Blended courses

9. Special Equipment and Software Support : no requirements