Syllabus

1. Course Description

a. Title of a Course : Algebraic Geometry: A First Geometric Look (V. S. Zhgoon)

b. Pre-requisites : linear and multilinear algebra, basic ideas of polynomials, commutative rings and their ideals, tensor products, affine and projective spaces, topological spaces and their open, closed and compact subsets. No deep knowledge is assumed, all essential definitions and technique will be recalled during the course.

c. Course Type : optional

d. Abstract : Algebraic geometry studies geometric loci looking locally as a solution set for a system of polynomial equations on an affine space. It gives an explicit algebraic explanation for various geometric properties of figures, and in the same time, brings up a geometric intuition underlying abstract purely algebraic constructions. It plays an important role in many areas of mathematics and theoretical physics, and provides the most visual and elegant tools to express all aspects of the interaction between different branches of mathematical knowledge. The course gives the geometric flavor of the subject by presenting examples and applications of the ideas of algebraic geometry, as well as a first discussion of its technical tools.

2. Learning Objectives

The seminar is intended to introduce the subject area to the students, and to offer them an opportunity to prepare and give a talk.

3. Learning Outcomes

Successful participants imporve their presentation skills and prepare for participation in research projects in the subject area.

4. Course Plan

1) Projective spaces. Geometry of projective quadrics. Spaces of quadrics.

2) Lines, conics. Rational curves and Veronese curves. Plane cubic curves. Additive law on the points of cubic curve.
3) Grassmannians, Veronese's, and Segre's varieties. Examples of projective maps coming from tensor algebra.
4) Integer elements in ring extensions, finitely generated algebras over a field, transcendence generators, Hilbert's theorems on basis and on the set of zeros.
5) Affine Algebraic Geometry from the viewpoint of Commutative Algebra. Maximal spectrum, pullback morphisms, Zariski topology, geometry of ring homomorphisms.
6) Agebraic manifolds, separateness. Irreducible decomposition. Projective manifolds, properness. Rational functions and maps.
7) Dimension. Dimensions of subvarieties and fibers of regular maps. Dimensions of projective varieties.
8) Linear spaces on quadrics. Lines on cubic surface. Chow varieties.
9) Vector bundles and their sheaves of sections. Vector bundles on the projective line. Linear systems, invertible sheaves, and divisors. The Picard group.
10) Tangent and normal spaces and cones, smoothness, blowup. The Euler exact sequence on a projective space and Grassmannian.

5. Reading List

a. Required

Igor V. Dolgachev, Classical Algebraic Geometry: A Modern View. Cambridge University Press, 2012 http://www.math.lsa.umich.edu/~idolga/CAG.pdf

b. Optional

W. Fulton, Algebraic Topology: A First Course (Graduate Texts in Mathematics), The Benjamin/Cummings Publishing Co. (1969), Addison-Wesley (1989) http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf

6. Grading System

½ in class written exam based on problem sheets+ ½ oral exam on the theoretical part.

7. Guidelines for Knowledge Assessment

- Dimension of affine space. Theorem on the dimension of fibers of morphism. Semi-continuity of dimension.
- Krull theorem on the dimension of hypersurface.
- Intersection of varieties of complementary dimension in a projective space.
- Linear spaces on quadrics.

8. Methods of Instruction

Students are individually assigned papers and textbook excerpts to give a seminar talk.

9. Special Equipment and Software Support : no requirements