Syllabus

1. Course Description
    a. Title of a Course : Research Seminar "Introduction to Combinatorial Theory" (Yu. M. Burman).
    b. Pre-requisites : none.
    c. Course Type : optional
    d. Abstract : Combinatorics is a part of mathematics studying finite sets. The question to answer is usualy «how many»: how many are there connected graphs with n numbered vertices containing no cycles? how many are there ways to draw diagonals of a convex *n*-gon so as to cut it into triangles? etc. This question is answered by a multitude of methods from real and complex analysis, number theory, geometry, and more. We do not expect, however, that the students are familiar with all these areas: the necessary techniques will be explained in the course. Combinatorics is very rich in applications, ranging from mathematical physics to algebraic geometry to finance, including topology and dynamical systems on the way. Very often questions from various sciences eventually turn to be combinatorial problems. Combinatorics is an indispensable part of every mathematician's education.
2. Learning Objectives

    The seminar is intended to give a broad background in modern combinatorial theory.
3. Learning Outcomes

    Successful participants will be able to apply combinatorial methods to various enumerative problems arising in mathematics, physical and social sciences, engineering.
4. Course Plan: In the course, we study various combinatorial objects and various methods of solution of combinatorial problems. For convenience we list methods and objects separately in the program; in the actual course we alternate them.
    A. Methods
        1) Formal power series.
        2) Linear recurrence.

3) Lagrange inversion theorem.
4) Transfer matrix.

B. Objects

1) Binomial coefficients.
2) Lattice paths.
3) Catalan numbers.
4) Partitions and compositions.
5) Trees.
6) Parking functions.
7) Hurwitz numbers.
8) Tutte polynomial.

5. Reading List

a. Required

Edward A. Bender and S. Gill Williamson, Foundations of Combinatorics with Applications, Dover Publications, 2006, http://www.math.ucsd.edu/~ebender/CombText/index.html

b. Optional

S. Chmutov, S. Duzhin, Y. Mostovoy. CDBook. CUP, 2012. http://www.pdmi.ras.ru/~duzhin/papers/cdbook/cdbook.pdf

6. Grading System

The formula for marking is 0.3 cumulative + 0.7 final exam, where cumulative is proportional to number of tasks solved.

7. Guidelines for Knowledge Assessment

Sample exercises:

- Write down the generating function for the Catalan numbers.
- How the exponential generating function can be obtained knowing the ordinary generating function?
- Give a simple upper bound on the partition function.

8. Methods of Instruction

Students are individually assigned papers and textbook excerpts to give a seminar talk.

9. Special Equipment and Software Support : no requirements