Syllabus on the course “Economic and Mathematic Modeling”

Approved by Programme Academic Council

Process-verbal 2 from April 10, 2018

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| Credits | 3 |
| Academic Hours | 114 |
| Year of study | 1 |
| Mode of study | Full-time |

1. **Pre-requisites**
* Calculus
* Linear Algebra
* Probability Theory and Mathematical Statistics
* Economics
1. **Course Type**

Economic and Mathematic Modeling is an compulsory course for first year master students enrolled on the program “Big Data Systems”.

1. **Abstract**

Economic and Mathematical Modeling is the study of the stochastic and dynamic economic systems. Economic and Mathematical Modeling is the application of mathematical methods to represent theories and analyze problems in economics. Much of economic theory is currently presented in terms of mathematical economic models. Course is focused on understanding the role of mathematic modeling for quantitative analysis of stochastic and dynamic economic systems. Course content includes random walk theory, stochastic process theory, dynamic systems theory and dynamic chaos theory. In the lab part of the course the students can try different mathematic modeling software tools.

1. **Learning Objectives**

The main objective of the Course is to present, examine and discuss with students fundamentals and principles of economic and mathematic modeling. This course is focused on understanding the role of mathematic modeling for quantitative analysis of stochastic and dynamic economic systems. Generally, the objective of the course can be thought as a combination of the following constituents:

• familiarity with peculiarities of bifurcation theory, catastrophe theory, chaos theory, Levy random walk and minority games theory as applied areas related to economic and mathematic modeling,

• understanding of the main notions of dynamic and stochastic systems theory; the framework of dynamic and stochastic modeling as the most significant areas of economic systems studies, understanding of the main notions of dynamic and stochastic systems theory; the framework of dynamic and stochastic modeling as the most significant areas of economic systems studies,

• understanding of the role of mathematic modeling in financial and economic modeling,

• obtaining skills in utilizing nonlinear dynamic modeling in economic problem solving,

• obtaining skills in utilizing stochastic modeling in financial problem solving,

• understanding of the role of equilibrium theory and instabilities in economic modeling,

• understanding of the role of stable distributions in financial process modeling.

1. **Learning Outcomes**

While mastering the course material, the student will

• know main notions of the bifurcation theory, catastrophe theory, chaos theory, random walk theory and stochastic process theory,

• acquire skills of analyzing and solving economic and mathematic problems,

• gain experience in economic and mathematic modeling with use main notions of the bifurcation theory, catastrophe theory, chaos theory, random walk theory and stochastic process theory.

1. **Course Plan**

Lecture 1. Economical dynamics, growth and equilibrium

Lecture’s content:

• Solow-Swan model. Assumptions of the model. Mathematics of the model. Balanced-growth equilibrium. Golden rule. Production function. Dynamical system. Stability of the dynamical system: Lyapunov stability, asymptotic stability, orbital stability. Dynamic equilibrium. Koopmans theorem (existence of an equilibrium in the Solow model). Arrow-Gurwicz theorem (existence of stable equilibrium). Asymptotic stability of the model.

• Attractors and repellers.

• Walrasian and Marshallian equilibriums. Open and closed systems. Dynamic and static equilibrium. Practice 1.

Ramsey-Cass-Koopmans model. Problem of consumer choice. Pontryagin’s maximum principle. General economic equilibrium. Modified golden rule. Attractor of Ramsey-Cass-Koopmans model.

Lecture 2. Systems of first-order differential equations and economic dynamics

Lecture’s content:

• Goodwin model. Lotka-Volterra equation. Equilibrium points. Linearization. Jacobian matrix. Eigenvalues of the matrix. Equilibrium points of linear autonomous system: saddle point, stable node, unstable node, stable focus, unstable focus, centre. Phase diagram. The principle of linearized stability. Equilibrium points of Lotka-Volterra equation. Structural stability. Structural instability of Lotka-Volterra model. Conservative system. Conservative Lotka-Volterra system.

• Slow and fast variables. Tikhonov theorem on dynamical systems.

Practice 2.

The simplest model of competition between two firms. Competition model with a limited production growth. Bazykin model. Mankiw-Romer-Weil model. Accounting for external.

Lecture 3. Bifurcation theory and economic dynamics

Lecture’s content:

• Modeling regional dynamics. Local bifurcations: saddle-node (fold) bifurcation, transcritical bifurcation, Pitchfork bifurcation, period-doubling (flip) bifurcation, Hopf bifurcation, Neimark bifurcation.

• Global bifurcations: homoclinic bifurcation, heteroclinic bifurcation, infinite-period bifurcation, blue sky catastrophe. Codimension of a bifurcation. Bifurcation diagram. Flows. Hopf theorem. The Poincare-Bendixson theorem.

• Limit cycle (attractor). Stable, unstable and semi-stable limit cycle.

Practice 3.

Dynamic transportation modal choice. Oscillations in van der Ploeg’s hybrid growth model. Periodic optimal employment policy. Optimal economic growth associated with endogenous fluctuations.

Lecture 4. Catastrophe theory and economic dynamics

Lecture’s content:

• Business cycles in the Kaldor model. Structural stability. Morse lemma. Thom theorem. Morse critical points. Degenerate critical points. Thom elementary catastrophes.

• Potential functions of one active variable: fold catastrophe, cusp catastrophe, Swallowtail catastrophe, butterfly catastrophe. Potential functions of two active variables: hyperbolic umbilic catastrophe, elliptice umbilic catastrophe, parabolic umbilic catastrophe.

Practice 4.

Resource management. Multiple equilibria in Wilson’s retail model. Stock market forecasting.

Lecture 5. Chaos theory and economic dynamics

Lecture’s content:

• Chaotic dynamic price formation. Lorenz system. Properties of the Lorenz system: homogeneity, symmetry, dissipativity, bounded trajectories. Equilibrium points of the Lorenz system. Lyapunov stability of equilibrium points. Lorenz attractor. Lorenz map.

• Measures of chaos: Lyapunov exponent, correlation function, Hausdorff-Besicovitch fractal dimension, Renyi fractal dimension. Fractal. Sensitivity to initial conditions. Strange attractors.

• Chaos and economic forecasting. Deterministic systems and time series.

Practice 5.

Chaotic dynamic of cities. Chaos in an international economic model.

Lecture 6. Probability distribution function of stock market instruments. Correlation in the stock market

Lecture’s content:

• Empirical distributions of stock returns and number of shares. Stable distribution. Characteristic function. Properties of stable distribution: infinitely divisible, leptokurtotic, closure under convolution. A generalized central limit theorem.

• Levy distribution. Probability density function and characteristic function of the Levy distribution. Autocorrelation function and spectral density. Higher-order correlation: the volatility. Stationarity of price changes.

Practice 6.

Empirical distributions of share volumes. Empirical distributions of the number of transactions. Empirical distributions of time interval between transactions.

Lecture 7. Random walk in financial market models

Lecture’s content:

• Options pricing. Lattice random walk. One-dimensional random walk. Gaussian random walk. The speed of convergence. Berry-Essen theorem 1. Berry-Essen theorem 2. Basin of attraction.

• Levy flight. Mandelbrot survival function. Scale invariant. Truncated Levy flights and fluctuations of stock market instruments. Functional Levy flight.

• Holtsmark distribution.

Practice 7.

Black-Scholes equation.

Lecture 8. Minority games

Lecture’s content: Price dynamics. Formulation of the minority game. Thermal minority game. Minority game without information. Grand-canonical minority game. Analytic approach.

1. **Reading List**

**Required**

* Haldrup N. Essays in nonlinear time series econometrics. Oxford University Press. 2014.
* Shone R. Economic dynamics : phase diagrams and their economic application. Cambridge University Press. 2002.
* Richmond P. Econophysics and physical economics. Oxford University Press. 2013.

**Optional**

* Denkowski Z. An introduction to nonlinear analysis: theory. Kluwer Academic Publishers. 2003.
* Vialar T. Complex and chaotic nonlinear dynamics : advances in economics and finance, mathematics and statistics. Springer. 2009.
* King A. C.  Differential equations: linear, nonlinear, ordinary, partial. Cambridge University Press. 2003.
* Rosser J. B. Complex evolutionary dynamics in urban-regional and ecologic-economic systems: from catastrophe to chaos and beyond. Springer. 2011.
1. **Grading System**

Current and resultant grades are made up of the following components:

• 1 class assignment

implies problems solving in the end of 1st module; material to be covered by class assignment is fully determined by both course schedule and topics discussed by the corresponding date. The class assignment (CA) is assessed on the ten-point scale.

• 8 homeworks

are done by students individually, herewith each student has to prepare electronic (PDF format solely) report. All reports have to be submitted in LMS. All reports are checked and graded by the instructor on tenpoint scale by the end of the 1 st module. All homeworks (HW) is assessed on the ten-point scale summary.

• pass-final examination

implies written test (WT) and computer-based problem solving (CS). Finally, the total course grade on ten-point scale is obtained as

O(Total) = 0,2 \* O(HW) + 0,4 \* O(CA) + 0,1 \* O(WT) + 0,3 \* O(CS).

A grade of 4 or higher means successful completion of the course ("pass"), while grade of 3 or lower means unsuccessful result ("fail"). Conversion of the concluding rounded grade O(Total) to five-point scale grade.

1. **Guidelines for Knowledge Assessment**

Questions for Pass-Final Examination are as follows:

Economic Models

1. Solow-Swan model. Assumptions of the model. Mathematics of the model. Balanced-growth equilibrium. Golden rule. Production function.

2. Ramsey-Cass-Koopmans model. Problem of consumer choice. Pontryagin’s maximum principle. General economic equilibrium. Modified golden rule.

3. Goodwin model.

4. The simplest model of competition between two firms. 5. Competition model with a limited production growth.

6. Bazykin model.

7. Mankiw-Romer-Weil model. Accounting for external.

8. Modeling regional dynamics.

9. Dynamic transportation modal choice.

10. Oscillations in van der Ploeg’s hybrid growth model.

11. Periodic optimal employment policy.

12. Optimal economic growth associated with endogenous fluctuations.

13. Business cycles in the Kaldor model.

14. Resource management.

15. Multiple equilibria in Wilson’s retail model.

16. Stock market forecasting.

17. Chaotic dynamic price formation.

18. Chaotic dynamic of cities.

19. Chaos in an international economic model.

20. Empirical distributions of stock returns and number of shares.

21. Empirical distributions of share volumes.

22. Empirical distributions of the number of transactions.

23. Empirical distributions of time interval between transactions.

24. Options pricing.

25. Black-Scholes equation.

26. Price dynamics.

27. Speculative trading

Mathematical Foundations

1. Dynamical system. Stability of the dynamical system: Lyapunov stability, asymptotic stability, orbital stability.

2. Dynamic equilibrium. Koopmans theorem (existence of an equilibrium in the Solow model).

3. Arrow-Gurwicz theorem (existence of stable equilibrium). Asymptotic stability of the model.

4. Attractor of Ramsey-Cass-Koopmans model.

5. Attractors and repellers.

6. Open and closed systems. Dynamic and static equilibrium.

7. Lotka-Volterra equation. Equilibrium points. Linearization. Jacobian matrix. Eigenvalues of the matrix. Equilibrium points of linear autonomous system: saddle point, stable node, unstable node, stable focus, unstable focus, centre. Phase diagram.

8. The principle of linearized stability. Equilibrium points of Lotka-Volterra equation. Structural stability. Structural instability of Lotka-Volterra model. Conservative system. Conservative LotkaVolterra system. Attrractors. Slow and fast variables.

9. Tikhonov theorem on dynamical systems.

10. Local bifurcations: saddle-node (fold) bifurcation, transcritical bifurcation, Pitchfork bifurcation, period-doubling (flip) bifurcation, Hopf bifurcation, Neimark bifurcation.

11. Global bifurcations: homoclinic bifurcation, heteroclinic bifurcation, infinite-period bifurcation, blue sky catastrophe. Codimension of a bifurcation. Bifurcation diagram. Flows. Hopf theorem. The Poincare-Bendixson theorem.

12. Limit cycle (attractor). Stable, unstable and semi-stable limit cycle.

13. Structural stability. Morse lemma. Thom theorem. Morse critical points. Degenerate critical points. Thom elementary catastrophes.

14. Potential functions of one active variable: fold catastrophe, cusp catastrophe, Swallowtail catastrophe, butterfly catastrophe.

15. Potential functions of two active variables: hyperbolic umbilic catastrophe, elliptice umbilic catastrophe, parabolic umbilic catastrophe.

16. Lorenz system. Properties of the Lorenz system: homogeneity, symmetry, dissipativity, bounded trajectories.

17. Equilibrium points of the Lorenz system. Lyapunov stability of equilibrium points. Lorenz attractor. Lorenz map.

18. Measures of chaos: Lyapunov exponent, correlation function, Hausdorff-Besicovitch fractal dimension, Renyi fractal dimension.

19. Fractal. Sensitivity to initial conditions. Strange attractors.

20. Chaos and economic forecasting.

21. Deterministic systems and time series.

22. Stable distribution. Characteristic function. Properties of stable distribution: infinitely divisible, leptokurtotic, closure under convolution.

23. A generalized central limit theorem.

24. Levy distribution. Probability density function and characteristic function of the Levy distribution.

25. Autocorrelation function and spectral density. Higher-order correlation: the volatility.

26. Lattice random walk. One-dimensional random walk. Gaussian random walk. The speed of convergence.

27. Berry-Essen theorem 1. Berry-Essen theorem 2.

28. Basin of attraction.

29. Levy flight.

1. **Methods of Instruction**

During classes various types of active methods are used: analysis of practical problems, group work, computer simulations in computational software program Mathematica, distance learning with use LMS.

1. **Special Equipment and Software Support**

Mathematica 9.0, HSE Electronic Library access: Books24\*7, Scopus, EBSCOHost, Science Direct, Web of Knowledge HSE Learning Management System, computer, projector (for lectures or practice), computer class.