Syllabus

1. Course Description
    a. Title of a Course: Research Seminar of Master's Programme "Open Problems of Modern Mathematics" (A. Loktev)
    b. Pre-requisites: accessible to any first year student of the master's program in mathematics, no special pre-requisite required.
    c. Course Type (compulsory, elective, optional): compulsory
    d. Abstract: each participant of the seminar give a talk about open problems in the area of his/her own research

2. Learning Objectives: the seminar is intended to introduce most popular open mathematical problems and known approaches to solve them. Also it offers the students an opportunity to prepare and give a talk.

3. Learning Outcomes: successful participants will know the current state of various branches of mathematics, which problems are open now and what is already done. Along the way the participants improve their presentation skills and ability to understand mathematics from each other.

4. Course Plan

   The list of subjects is preliminary and can be changed according to a research interests of the participants.

   Year 1

   Number theory-1

   Grand Riemann hypothesis

   Generalized Riemann hypothesis

   Hilbert's ninth problem

   Hilbert's eleventh problem

   Hilbert's twelfth problem

Lehmer's totient problem: if $\varphi(n)$ divides $n - 1$, must n be prime?

Are there infinitely many amicable numbers?

Are there any pairs of amicable numbers which have opposite parity?

Are there any pairs of relatively prime amicable numbers?

Are there infinitely many betrothed numbers?

Are there any pairs of betrothed numbers which have same parity?

Are there infinitely many perfect numbers?

Do quasiperfect numbers exist?

Do any odd weird numbers exist?

Do any Lychrel numbers exist?

Exponent pair conjecture

Is π a normal number (its digits are "random")?

Which integers can be written as the sum of three perfect cubes?

Group theory

Is every finitely presented periodic group finite?

For which positive integers m, n is the free Burnside group B(m,n) finite?

Is every group surjunctive?

Andrews–Curtis conjecture

Herzog–Schönheim conjecture

Are there an infinite number of Leinster Groups?

Partial differential equations

Regularity of solutions of Euler equations

Existence and regularity of solutions of Navier-Stokes equation

Regularity of solutions of Vlasov–Maxwell equations

Dynamical systems

Collatz conjecture (3n + 1 conjecture)

Furstenberg conjecture

Margulis conjecture

MLC conjecture – Is the Mandelbrot set locally connected?

Weinstein conjecture

Arnold–Givental conjecture

Algebra

Finite lattice representation problem

Hilbert's sixteenth problem

Hilbert's fifteenth problem

Hadamard conjecture

Jacobson's conjecture

Existence of perfect cuboids and associated cuboid conjectures

Zauner's conjecture: existence of SIC-POVMs in all dimensions

Köthe conjecture

Birch–Tate conjecture

Serre's conjecture II

Bombieri–Lang conjecture

Farrell–Jones conjecture

Bost conjecture

Rota's basis conjecture

5. Reading List
   a. Required
      1) A list of open problems in Wiki:
      https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_mathematics
      2) Clay Math Institute on millennium problems
      http://www.claymath.org/millennium-problems
      3) David Hilbert. Mathematical Problems. Lecture delivered before the International Congress of Mathematicians at Paris in 1900
      https://mathcs.clarku.edu/~djoyce/hilbert/problems.html
      http://www.math.tamu.edu/~rojas/hilbert23reprinted.pdf

   b. Optional
      1) Arxiv.org (regular look through)

6. Grading System.

Students are encouraged to give talks at research seminars. This way they learn how to communicate their knowledge to their colleagues in comprehensible and attractive way. To prepare a good talk, it is important to attend talks of senior colleagues to see the best practices. It is equally important to attend talks of other students and to learn from their mistakes. When you fail to follow the talks of fellow students you might get some ideas on how to improve your own talk. For this reason, we encourage young participants to attend talks of their classmates, ask questions and make comments.

Accordingly, there will be two separate grades O_A (the attendance grade) and O_T (the talk grade). To get 10 for attendance you have to be an active participant of at least 3/4 of the seminars. To be an active participant means that you not only listen to the talk but also understand the main statement of the talk and are able to work out the simplest meaningful application of this statement. To get the perfect grade for your own talk you have to formulate a result with all necessary definitions so that the audience understands it. You are also expected to prepare and give the audience a simple problem on applications of the main result.

The final grade for the seminar will be determined as follows: $O_F = 0.4\ O_A + 0.6\ O_T$. You O_A and O_T grade may exceed 10 if you attend more than 3/4 of the seminars and/or prepare an especially interesting talk.

7. Guidelines for Knowledge Assessment:
   - State 2-3 open problems from the Hilbert problem list
   - State 2-3 open problems from the Millenium Prize problem list
   - State some open problems in spectral geometry
   - Which problems in representation theory was solved in last 10 years?
   - Which methods are used to solve actual problems in algebraic number theory?

8. Methods of Instruction: students are individually assigned papers and textbook excerpts to give a seminar talk.

9. Special Equipment and Software Support (if required): Not required