1. Organizational and methodological issues

- *The aim of the course.* The course "Academic writing" offered to M.Sc. and Ph.D. students of the Faculty of Mathematics is aimed at mastering basic principles of mathematical and scientific exposition, understanding the role of Mathematical Rigor, Notation and Style both in popular and specialized texts such as mathematical papers, overviews, lecture notes, and dissertation theses.
- *Objectives of the course.* Upon successful completion of the course, the students will be able to write mathematical papers, reviews, dissertations etc. satisfying the standards of international mathematical community. They will *know* which grammatical structures are used in technical writing, the main tips on the style, reference books on mathematical English. Student will be *familiar with* major mathematical typesetting tools including the LaTeX system and the BibTeX bibliography meta-language.
- *Original methodological approaches used in the course.* We do not produce templates of well-written mathematical texts. Instead, we focus on developing analytical skills by analyzing examples of different actual scientific texts of various stylistic and methodological quality. We seek ways to improve the texts based on few guiding principles and on the understanding of the objectives, subject matter and the circle of potential readers. We use a large database of excerpts from mathematical literature mostly taken from open-access online repositories.
- *The place of the course in the system of innovative qualifications that are formed in the course of study.* The course is offered to the first year Master of Science students in Mathematics and to the first year Ph.D. students in Mathematics that are enrolled in the "academic Ph.D. studies". The M.Sc. programme is an international programme conducted by the Faculty of Mathematics in English. Students enrolled in this programme have very different scientific and language background, and their overall command of English, although in any case sufficient for communicating mathematical ideas, may also be different. In particular, we do have English native speakers but, for most students, English is a second language. We had to take this into account while developing the syllabus.

  The practical skills that we can help to develop are: communicating mathematical ideas most efficiently, mainly in writing; preparing dissertations, Curriculum Vitae, statements of interests, job applications, reviewing and editing mathematical publications.

2. The content of the course

What makes this course unique (description of unique scientific and methodological features, comparison with similar courses offered by the NRU HSE and other universities in Russia and worldwide). Courses on academic writing in English are not very typical for the Russian educational system. On the other hand, universities in the UK and in the USA offer regular courses on academic writing, mostly of undergraduate level (sophomore). We mainly adopt the western tradition, however, significant modifications were done to adjust the syllabus for a graduate-level course. Parts of the usual "mathematical writing" syllabi have been removed, e.g. those which deal with principles of mathematical argumentation (composition of proofs, definitions, examples, etc.) because our graduate students will know these principles already, and will have been using them for a while. Instead, we concentrate on making mathematical ideas clear, using appropriate style, terminology, notation. The main emphasis in on how to help the reader understand the results being published. We do not need to explain what a mathematical definition is or what a mathematical proof is, but we can help students organize and present their definitions and proofs in a more comprehensible way.

The study is based on consideration of many examples (taken from real-life sources) and suggestions for their improvements. We compare many "before" and "after" fragments. There is no universal recipe of good writing. The process of writing is very personal; it requires inspiration and a certain amount of talent. Thus we do not try to state "unbreakable rules" although we do emphasize recommendations that are supported by most professional mathematicians. We will also discuss, in the form of a dispute, some personal recommendations of distinguished scholars that may sometimes be inconsistent or contradictory.

Some tips on software packages like TEX, LATEX, BibTeX will be given.

The structure of the syllabus.

Section 1: Basic principles of scientific and technical writing. Understanding of the subject and the audience; i.e. having good ideas of "what to write about" and "for whom to write". Planning: organization and arrangement of the material, choosing the notation. Avoiding notational dissonance and "freezed" letters. Making the logic clear. Beginning a new section with an outline of the setting. Making self-contained statements of results. The role of opening paragraphs.

References: [7, 3, 9].

Section 2: Writing Mathematics in general. Avoiding slang, colloquialisms, abbreviations. Fighting excessive notation and "overloaded" punctuation signs. Possible sources of mathematical ambiguity (inexplicit use of quantifiers, suppressing simple arguments, assigning the same notation to different objects, etc.). "I" versus "we" versus "one" in mathematical writing. Examples of bad writing: starting a sentence with a notation, splitting infinitives, omitting "that", "then" etc., using unnecessary special symbols, using ambiguous notation.

References: [4, 11, 5].

Section 3: Specific types of mathematical communication. Books, theses, papers, lecture notes, reviews, references, presentation slides: similarities and differences. Curriculum vitae, job applications, grant proposals. Differences and similarities of various types of mathematical communication. The amount of background information that is reasonable to put.

References: [16, 9, 3]

Section 4: Giving proper acknowledgements and avoiding plagiarism Typical forms of acknowledgements. Citation principles and copyright issues (which citations are legal to make without explicit permission of copyright holders). Using publicly available (e.g. online) resources. Most common types of copyright licences.

References: [16]

Section 5: Advice of distinguished scholars. John Littlewood and his book "A mathematician's miscellany". Paul Halmos and his classic and very influential article

"How to write mathematics": writing in spirals, a continuous organization, rewrite vs. correct. Vladimir Arnold and his style of writing.

References: [1, 2, 4].

Section 6: Principles of proof-reading Waiting time before the proof-reading. What to check for. Using spell-checkers. Inviting test-readers.

References: [4, 9]

Section 7: The structure of LaTeX. Document classes and packages. Inline and display formulas. Common LaTeXcommands in the mathematical modes. Some useful LaTeXcommands in the text mode. Styles used in LaTeXdocuments.

References: [6, 8, 10, 12].

Section 8: Bibliography standards and BibTeX. A simple way of including bibliographies into LaTeX documents. Standard abbreviations of Journal titles. Citation styles: the Modern Language Association (MLA) style, Chicago Manual of Style, bibliography styles adopted by specific Mathematical Journals. The BibTeX entries and their meaning. Benefits of using BibTeX.

References: [17].

Section 9: Other software instruments that help in mathematical writing Text editors with special markup features for TeX- or LaTeX- documents (WinEdt, Kile, TeXworks, LyX, gedit, kate).

References: online user's guides and manuals.

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3. Examples of assignments and tests

Exercise 1. Pretend that your are writing a textbook in Calculus. Give definitions of the following notions as they should appear in the book:

(1) a continuous function;

(2) a differential of a function;
(3) a uniformly continuous function;
(4) the Legesgue measure;
(5) the Riemann integral;
(6) the length of a curve.

Exercise 2. Remove unnecessary notation from the statement of the following theorem: "THEOREM. Every continuous function $f(x)$ on the interval $[0,1]$ attains its maximal value $\max_{x\in[0,1]} f(x)$ and its minimal value $\min_{x\in[0,1]} f(x)$".

Exercise 3. Remove unnecessary notation from the statement of the following theorem: "THEOREM. Let $f(z)$ be a function analytic in the ring-shaped region between two concentric circles C and C', of radii R and R' $(R' < R)$, and center a, and on the circles themselves. Then $f(z)$ can be expanded in a series of positive and negative powers of $z-a$, convergent at all points of the ring-shaped region."

Exercise 4. Suggest an improvement of the following abstract: "ABSTRACT. Let $1 \to H \to G \to \mathbb{Z} \to 1$ be an exact sequence of hyperbolic groups induced by a fully irreducible automorphism ϕ of the free group H. Let $H_1 (\subset H)$ be a finitely generated distorted subgroup of G. Then H_1 is of finite index in H. This is an analog of a Theorem of Scott and Swarup for surfaces in hyperbolic 3-manifolds."

Exercise 5. Type the character table for A_5 in LaTeX.

Exercise 6. Type the following fragment in LaTeX:

$$\begin{array}{ccccccccc}
 & \longrightarrow & \mathbb{C}P^1 & \xrightarrow{\hat f} & \mathbb{C}P^1 & \xrightarrow{\hat f} & \mathbb{C}P^1 & \xrightarrow{\hat f} & \mathbb{C}P^1 \\
\cdots & & {\scriptstyle \hat\phi_3}\big\downarrow & & {\scriptstyle \hat\phi_2}\big\downarrow & & {\scriptstyle \hat\phi_1}\big\downarrow & & \big\downarrow{\scriptstyle \hat\phi_0} \\
 & \longrightarrow & \mathbb{C}P^1 & \xrightarrow{R_2} & \mathbb{C}P^1 & \xrightarrow{R_1} & \mathbb{C}P^1 & \xrightarrow{R_0} & \mathbb{C}P^1
\end{array}$$

Exercise 7. Type the following fragment in LaTeX:

$$\left(\frac{\partial^k}{\partial z_1 \ldots \partial z_k} - \left(\frac{\partial}{\partial z_1} + \frac{\partial}{\partial z_2}\right) \cdots \left(\frac{\partial}{\partial z_{k-1}} + \frac{\partial}{\partial z_k}\right)\right) E_k = 0.$$

Exercise 8. Suggest an improvement of the following abstract: "ABSTRACT. Let X, X_1, X_2,... be a sequence of i.i.d. random variables with mean $\mu = EX$. Let $v_1^{(n)}, ..., v_n^{(n)}{}^{\infty}_{n=1}$ be vectors of non-negative random variables (weights), independent of the data sequence $X_1, ..., X_n{}_{n=1}^{\infty}$, and put $m_n = \sum_n v_i^{(n)}$. Consider $X_1^*, ..., X_{m_n}^*$, $m_n \ge 1$, a bootstrap sample, resulting from re-sampling or stochastically re-weighing a random sample $X_1, ..., X_n$, $n \ge 1$. Put $\bar X_n = \sum_n X_i/n$, the original sample mean, and define $\bar X^*{}_{m_n} = \sum_n v_i^{(n)} X_i/m_n$, the bootstrap sample mean. Thus, $\bar X^*{}_{m_n} - \bar X_n = \sum_n (v_i^{(n)}/m_n - 1/n) X_i$. Put $V_n^2 = \sum_n (v_i^{(n)}/m_n - 1/n)^2$ and let S_n^2, $S_{m_n}^{*2}$ respectively be the the original sample variance and the bootstrap sample variance. The main aim of this exposition is to study the asymptotic behavior of the bootstrapped t-statistics $T_{m_n}^* := (\bar X^*{}_{m_n} - \bar X_n)/(S_n V_n)$ and $T_{m_n}^{**} := \sqrt{m_n}(\bar X^*{}_{m_n} - \bar X_n)/S_{m_n}^*$ in terms of conditioning on the weights via assuming that, as $n, m_n \to \infty$, $\max_{1\le i\le n}(v_i^{(n)}/m_n - 1/n)^2/V_n^2 = o(1)$ almost surely or in probability on the probability space of the weights. This view of justifying the validity of the bootstrap is believed to be new. The need for it arises naturally in practice when exploring the nature of information contained in a random sample via re-sampling, for example. Conditioning on the data is also revisited for Efron's bootstrap weights under conditions on n, m_n as $n \to \infty$ that differ from requiring m_n/n to be in the interval (λ_1, λ_2) with $0 < \lambda_1 < \lambda_2 < \infty$ as in Mason and Shao. Also, the validity of the bootstrapped t-intervals for both approaches to conditioning is established."