On discrete WKB methods for resonance electromagnetic traps

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The WKB asymptotics are well known and widely used to obtain semiclassical estimates of eigenstates and the spectrum for the one-dimensional Schrödinger operator, that is a second-order differential operator. On the other hand, many one-dimensional quantum systems can be naturally represented in terms of difference equations. Namely, if one considers a quantum system in the spectral representation of a coordinate operator, where the coordinate is an action operator with a discrete spectrum, then the corresponding spectral equation takes the form of a difference equation. The theory of semiclassical asymptotics for a difference equation, or discrete WKB methods, can be formally obtained by considering a difference equation as a pseudodifferential equation. The rigorous WKB asymptotics for difference equations including tunneling effects are also presented [1, 2, 3] but are significantly less known than their continuous counterparts.

We consider the semiclassical asymptotics for a second-order difference equation that corresponds to resonance electromagnetic traps [4, 5]. The one-dimensional Hamiltonian is obtained by quantum averaging of small anharmonic terms in the hyperbolic resonance trap. We express the Hamiltonian in terms of integrals (symmetries) of the ideal resonance trap. These integrals form a non-Lie algebra with a creation-annihilation-neutral operator structure. Here the neutral operator is an action operator with a discrete spectrum. The one-dimensional effective Hamiltonian is a polynomial on these basis operators. Therefore, it can be presented as a difference operator on the spectral representation of neutral operator.

We discuss several basic theorems of discrete WKB methods and their application to the tunneling asymptotics for resonance electromagnetic traps Hamiltonians. This is a joint work with M.V. Karasev and E.M. Novikova. The work was supported by the Program for Fundamental Research of Higher School of Economics.

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