Approved by the Academic council

of the Educational Program:

Economics –research Program

Protocol No.____from___________2019

Mathematics for Economists

2019, fall semester (1st module)

3 ECTS credits

Lecturers and authors: M. Levin, K. Bukin

Department of Theoretical Economics

Course description

The objective of the course is to equip the students with some of the theoretical foundations of the modern mathematics and what is more important with the analytical methods of solving problems posed by the micro and macro analysis.

Prerequisites

Undergraduate level mathematics that includes: Calculus (both single and multi-dimensional), Linear Algebra, Ordinary Differential Equations.

Course type

This course is compulsory for the students of the Research track master degree program of the Faculty of Economic Sciences and is elective for the students of the Applied Economics master degree program of the same Faculty.

Learning objectives

The course has been designed to convey to the students how mathematics can be used in the modern micro and macro economic analysis.

Emphasis is placed on the model-building techniques, methods of solution and economic interpretations.

Topics studied comprise the following: methods of optimization, dynamic programming, optimal control theory.

Learning outcomes

Upon completion an individual will:

- have the ability to solve optimization problems in the case of numerous inequality constraints,
- have acquired the knowledge of the methods of the optimal control theory and dynamic programming and its applicability for solving problems in economics,

Course plan

1. Basics of optimization, elements of convex analysis and Kuhn-Tucker method (taught by Prof. M. Levin). There will be 10 hours of classes on this topic accompanied with 24 hours of self-study.

- Optimization in many variables. Unconstrained optimization at first followed by constrained optimization
- concept of extrema
- necessary and sufficient conditions of extrema
- bordered Hessians
- convexity
- convex and concave functions, their properties
- separability theorem, separating hyperplane
- saddle point
- necessary and sufficient conditions of concave functions
- strict convexity/concavity of a function
- problem modification for the nonnegative variables
- differential characteristics of Kuhn-Tucker conditions
- the meaning of Lagrange multipliers

2. Dynamic Optimization in Continuous Time (taught by K. Bukin and M. Levin). There will be 14 hours of classes on this topic accompanied with 24 hours of self-study.

- systems of linear ordinary differential equations
- phase diagrams.
- analytical solutions of linear, homogeneous systems.
- stability.
- linearization of nonlinear systems
- typical problem of a calculus of variations.
- derivation of the first-order conditions.
- transversality conditions.
- behavior of the Hamiltonian over time.
- sufficient conditions.
- infinite horizons. Example: the neoclassical growth model.
- summary of the procedure to find the first-order conditions.
- present-value and current-value Hamiltonians.

3. Finite-Horizon Dynamic Programming (taught by K. Bukin and M. Levin). There will be 8 hours of classes on this topic accompanied with 14 hours of self-study.

- examples of the dynamic programming problems
- histories, strategies and the value function
- existence of an optimal strategy
- the Bellman equation
- stationary strategies
- example: the optimal growth strategy

Reading list (required)

- Carl P. Simon, Lawrence Blume, Mathematics for economists, W.W. Norton company Inc., 1994 or latest edition
- Kamien, M.I., Schwartz, N.L. Dynamic optimization: the calculus of variations and optimal control in economics and management, 2nd ed. New York: North-Holland, 1991.

- Rangarajan K. Sundaram. A first course in optimization theory, Cambridge University Press, 1996, 11th printing in 2007

- Coursera course "Mathematics for Economists" can be found at https://www.coursera.org/learn/mathematics-for-economists/home/

Assessment and grading system

A student will be awarded with 3 credit hours upon successful completion of the course.

There will be one open-questioned final test in the end of the course. This test lasts for 120 minutes.

Students absent from final test get unsatisfactory mark unless the absence is excused documentary. The written documentation for a missed test must be presented to the Student's Office no later than three days following return to class.

The test will be given back with a score on it. Students can also check with a lecturer about their scores for home assignments anytime during the semester.

Homework Assignments

Homework will be assigned every third week (two home assignments in total). Homework will be collected, marked and returned to the students.

Attendance Policy

Attendance is strongly encouraged. Attendance on final test is mandatory.

The final grade will be calculated as a weighted sum in accordance with the formula

Grade=0.2HW1+0.2HW2+0.6Final test

All estimates are based on 100 points, and are subject to arithmetic rounding to integer values. Further transfer from 100-point to 10 - and 5-point evaluation system is carried out by the method published on the resource https://icef-info.hse.ru/goto_icef_file_29833_download.html
In the case of a failure to show up at the final for a valid reason, as well as getting an unsatisfactory grade for the whole course will lead to retake(s). The first retake will be in the form of a written test covering all topics from the current Syllabus. Samples of retake problems for the first and second retakes are given below.
The second retake is in the form of a Commission covering all topics of the Syllabus, with the accumulated grade not taken into account. For students who missed final for a valid reason, the first retake is considered to be a zero retake.

Test problems for retakes

1. Consider a bounded control problem.

$\int_0^1 xdt \to \max$ subject to $\dot{x} = u + x,\ x(0) = 0,\ x(1)$ is free, and $-1 \le u \le 1$. What guarantees that a maximizer would be found?

2. Consider the following dynamic programming problem:

maximize $\sum_0^\infty \beta^t \ln(c_t)$, subject to $W_{t+1} = W_t - c_t$, $W_t \le W, W > 0$, $0 < \beta < 1$. Let the state variable be W and denote the next period value of W as W'.

a) Write down the Bellman equation for the value function $V(W)$.

b) Using method of undetermined coefficients find $V(W)$.

c) Find the optimal policy function $c = h(W)$.

3. Consider the optimal control problem

$\int_0^\infty e^{-rt}(2\sqrt{E(t)} - \frac{2}{3}M^{\frac{3}{2}}(t))dt \to \max$, subject to $\dot{M} = E - \delta M,\ M(0) = M_0$.

Derive the system of differential equations in $E - M$ plane, using current value Hamiltonian. Prove that the steady-state solution exists and is unique. Sketch the phase portrait of the system in the neighborhood of the equilibrium.

Constants $r, \delta > 0$.

4. Consider autonomous system of differential equations

$$\dot{x}_1 = F_1(x_1, x_2)$$
$$\dot{x}_2 = F_2(x_1, x_2)$$

Curves on the x_1x_2 - plane defined by equations $F_1 = 0$ and $F_2 = 0$ are called *demarcation loci.* Both functions belong to C^1.

Prove that the slope of $F_1 = 0$ locus in the neighborhood of the steady state is equal to the negative of the ratio of the first column entry to the second column entry of the row 1 of the Jacobian matrix evaluated at the steady state.

Teaching methods

Lectures and problem-solving sessions (classwork), intensive self-study, working on home assignments, on-line course "Mathematics for Economists" offered by Coursera, Visiting office hours.

Special Equipment not required