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Protocol No. ___ from __. __. 20

Syllabus
CALCULUS
(8 ECTS)

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1. Course Description

a) Pre-requisites

Students are expected to have a strong background in elementary mathematics, algebra, trigonometry, and geometry on the coordinate plane, the properties and graphs of elementary functions at the level of Russian high school.

b) Abstract

This course is designed to introduce students to the basic ideas and methods of mathematical analysis and their application to mathematical modeling. This course serves as a basis for the entire block of quantitative disciplines studied at HSE, and it also provides some of the analytical tools that are required by advanced courses in information technologies. This course provides students with experience in the methods and applications of calculus to a wide range of theoretical and practical problems. The course is taught in English.

2. Learning Objectives

The main objectives are as follows:

- acquisition by students of basic knowledge in calculus and ordinary differential equations;
- formation of skills for working with abstract concepts of higher mathematics;
- familiarity with the applied problems of calculus;
- development of skills to solve typical problems of calculus.

3. Learning Outcomes

By the end of this course the students should be able to:

- analyze functions represented in a variety of ways: graphical, numerical, analytical, or verbal, and understand the relationships between these various representations.
- understand the meaning of the derivative in terms of a rate of change and local linear approximation, and be able to use derivatives to solve a variety of problems.
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and be able to use integrals to solve a variety of problems.
- understand the relationship between the derivative and the definite integral, as expressed by the Fundamental Theorem of Calculus.
- communicate mathematics in well-written sentences and to explain the solutions to problems.

– model a written description of a simple situation with a function, differential equation, or an integral.
– mathematical analysis to solve problems, interpret results, and verify conclusions.
– determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.

4. Course Plan

Differential Calculus, Module 1-2.

1. Introduction
Different forms of representation of functions. Elementary concepts: domain and range of a function, even and odd functions, periodic functions. Graphs of elementary functions. Implicit functions.

2. Sequences. Limit of a sequence
Sequences: bounded and unbounded, infinitely small and infinitely large. Limit of a sequence. Limit theorems for sequences: arithmetic operations, sandwich theorem. Monotone sequences. Convergence of a monotone increasing sequence. The number e.

3. Limit of a function
The limit of a function at infinity. Asymptotes of a function at infinity. The limit of a function at a point. Limit theorems for functions. Functions that tend to zero, functions that tend to infinity. First and Second Special Limits. Types of indeterminate forms. Finding limits. Left and right limits.

4. Continuity
Definition of continuity of a function at point and on an interval. Continuity of elementary functions. Properties of continuous functions. Points of discontinuity. Classification of points of discontinuity. Vertical asymptotes.

5. The derivative
Definition of the derivative. Tangent lines and normal lines. Geometric and physical interpretations of the derivative. Right and left derivatives. Differentiability at a point. Differentiability and continuity. Differentiation. Rules of differentiation. Derivatives of elementary functions. Differentiation of inverse functions. Logarithmic differentiation. Differentiation of implicit functions. Existence of a differentiable implicit function. Definition and geometric interpretation of differentials. Approximate calculations using differentials. The second derivative. The geometric meaning of the second derivative. Higher-order derivatives and differentials. Properties of differentiable functions: Rolle's

theorem, the Mean Value theorem, Cauchy's theorem, and their geometric interpretation.

6. Applications of the derivative
L'Hospital's rule. Necessary and sufficient conditions for increasing/decreasing functions. Related rates. Concave and convex functions. Different ways of expressing concavity. Economic interpretation of concave and convex functions. Points of inflection. Local extrema. First-order necessary and sufficient conditions for a local extremum. Second-order necessary and sufficient conditions for a local extremum. Maximum and minimum values of a function on an interval. Curve sketching.

7. Number series, power series, and Taylor expansions
Necessary condition for convergence of a series. Harmonic series and power series. The ratio test. Comparing series to test for convergence. Alternating series. Sufficient condition for convergence of an alternating series. Absolute convergence. Radius and interval of convergence of a power series. Abel's theorems. Taylor's formula. Taylor and Maclaurin series. Taylor and Maclaurin expansions for elementary functions. Application of Taylor series for analyzing the behavior of a function at a point and for conducting approximate calculations.

8. Complex numbers and introduction of functions of complex variables.
Complex numbers: real and imaginary numbers, polar form - modulus and argument, Euler's formula. Arithmetic of complex numbers. Complex conjugate and its properties. Powers and roots of complex numbers. Complex polynomials. Elementary functions of complex variables. Cauchy-Riemann conditions.

Integral Calculus, Module 3

9. Antiderivatives and the indefinite integral
Antiderivatives. The indefinite integral and its properties. Table of indefinite integrals. Basic methods of integration: direct integration, substitution and integration by parts. Integration of rational functions.

10. The definite integral
Problems that require the definite integral. Definition of the definite integral using Riemann sums. Sufficient condition for the existence of the definite integral. Approximate calculation of definite integrals using rectangles and trapezoids. Simpson's rule. Properties of the definite integral. Differentiation of a definite integral with variable upper bound. The fundamental theorem of calculus. Substitution and integration by parts.

11. Applications of the definite integral
Applications of the definite integral in geometry and physics. Area of a flat region, volume of a solid of revolution, volume of a solid with known cross-sections.

12. The double integral
Definition of double integrals. Reduction of double integrals to iterated integrals. Changing the order of integration in iterated integrals. The geometric interpretation and main properties of double integrals

13. Improper Integrals
Integrals with infinite bounds. Improper integrals of the first kind. Integration of unbounded functions. Improper integrals of the second kind. Principle value. Convergence tests for improper integrals. Absolute and relative convergence of improper integrals

Differential Calculus of functions of several variables, Module 3-4

14. Functions of several variables
Graphical presentation of functions of two variables. The limit of functions of two (and more) variables. Finding limits. Continuity at a point. The main properties of continuous functions of several variables.

15. Partial derivatives and related topics
Definition of the partial derivatives of the first order. The differential, its invariance. Geometrical meaning in 2D case. Directional derivative and gradient. Partial derivatives of higher order. Properties of mixed derivatives. Implicit functions determined from a system of non-linear equations. Solvability of non-linear systems, Jacobian.

16. Optimisation problems
Local extrema of a function of several variables. The necessary and sufficient conditions for a local extremum. Application to optimisation problems for functions of two variables. Conditional extremum. The method of undetermined Lagrange multipliers. Sufficient conditions. Examples of multi-parametric optimisation under constraints.

Differential equations, Module 4

17. Differential equations and slope fields
Definition of first order differential equations. General and particular solutions. Existence and uniqueness theorem. Isoclines and direction fields. Solution of separable

differential equations. Solution of homogeneous differential equations and first-order linear equations. Application of differential equations to physics and economics.

18. Cauchy initial value problem
Initial value problem for ordinary differential equations (ODE) of the first order. Existence and uniqueness of the solution. Linear systems of ODE of the first order. Systems of ODE with constant coefficients. Fundamental solutions and general solution of the system of ODE. Examples from economics.

19. Introduction to differential equations of the *n*-th order
Linear homogeneous and non-homogeneous ODE of the *n*-th order with constant coefficients. The methods of their solution. Boundary value problems for ODE of the second and higher orders.

20. Introduction of integral transforms
Fourier Transform and Laplace Transform. Inverse transforms. General properties. Use the tables of Fourier and Laplace transforms. Convolution theorem. Application to solving ODE.

5. Reading List

a) Required

1. Dowling E.T. Schaum's outline of theory and problems of introduction to mathematical economics. McGraw-Hill, 2001.
2. Stewart J. Calculus. Early Transcendentals. 6 edition. Thomson Brooks/Cole, 2012.
3. Ильин, В. А. Основы математического анализа: учебник для вузов / В. А. Ильин, Э. Г. Позняк. – Изд. 6-е, стер. – М.: Физматлит, 2008. – (Сер. "Курс высшей математики и математической физики") .
4. Фихтенгольц Г.М. Курс дифференциального и интегрального исчисления. М., 1966 (и др. более поздние издания).

b) Optional

1. Simon, C. P. Mathematics for economists / C. P. Simon, L. Blume. – New York: W.W.Norton & Company, 1994.
2. Chiang, A. C. Fundamental methods of mathematical economics / A. C. Chiang. – 3rd ed. – Auckland: McGraw-Hill, 1984.
3. Anthony, M. Mathematics for economics and finance: methods and modelling / M. Anthony, N. Biggs. – Cambridge: Cambridge University Press, 1997.

4. Демидович, Б. П. Сборник задач и упражнений по математическому анализу: Учеб. пособие для вузов / Б. П. Демидович. – М.: АСТ: Астрель, 2003. (и др. издания).
5. Кудрявцев, Л. Д. Курс математического анализа: учебник для вузов / Л. Д. Кудрявцев. – Изд. 5-е, перераб. и доп. – М.: Дрофа, 2006. (и др. издания).
6. Зорич В.П. Математический анализ (в 2-х) томах. Фазис, 1997-1998 (и др. издания).
7. Романко, В. К. Сборник задач по дифференциальным уравнениям и вариационному исчислению / В. К. Романко, Н. Х. Агаханов, В. В. Власов, Л. И. Коваленко; Под ред. В. К. Романко. – М.: Лаборатория Базовых Знаний: Юнимедиастайл: Физматлит, 2002. (и др. издания).
8. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям. М.: НИЦ РХД, 2000 (и др. издания).
9. Эльсгольц, Л. Э. Дифференциальные уравнения и вариационное исчисление: Учебник для вузов / Л. Э. Эльсгольц. – 4-е изд. – М.: Едиториал УРСС, 2000. (и др. издания).

6. Grading System

A student obtains 8 credits for the whole course provided that his/her final mark in the 10-point rating scheme is greater than 3.

The final mark is formed as follows

- 50% of the final exam mark (*Gfinal*);
- 25% of mid-term exam mark (*Gmidterm*); and
- 25% of class-work mark.

The latter is composed of

- up to 40 pts for cumulative test mark (*Gtest*);
- up to 40 pts for regular quizzes (*Gquiz*)
- up to 20 pts for lecture quizzes (*Gminiquiz*).

Extra points may be awarded for class activity.

The formula for the final mark calculation is given below

$G = \text{round}(0.05*Gfinal+0.025*Gmidterm+0.01*Gtest+0.01*Gquiz+0.005*Gminiquiz)$

Here round(X) is the function that rounds X to the nearest integer from 0 to 10; and all terms *Gfinal, Gmidterm*, *Gtest*, *Gquiz*, *Gminiquiz* are integers in the range 0-100.

The first term mark, $G1$, is calculated as

$G1 = \text{round}(0.05*Gmidterm+0.02*G1test+0.02*G1qui1z+0.01*G1miniquiz)$

where *G1test*, *G1qui1z*, *G1miniquiz* are points (integers in the range 0-100) gained for Module 1 test and quizzes respectively by the end of the first term.

7. Examination Type

Final exam (written) – end of Module 4
Mid-term exam (written) – end of Module 2
Tests (written) - end of Module 1 and end of Module 3

8. Methods of Instruction

Lectures – 64 hr, seminars – 64 hr, self-study – 176 hr, consultations (when required). Total – 308 hr.

9. Special Equipment and Software Support (if required)

None