State University Higher School of Economics

International College of Economics and Finance

Abstract Mathematics Syllabus

1. Course description

Abstract Mathematics is a two-semester course for the third year bachelor's programme students who selected specialization Economics and Mathematics. The course is based on the Introduction to Abstract Mathematics course of the University of London (UoL) with further expansions into selected topics from algebra, real analysis and topology. The course is taught in English.

1.1. Course pre-requisites

For the theory itself there are no pre-requisites except for an aptitude for logical reasoning. However, many examples will reference concepts from Calculus, Statistics, Mathematics for Economists and Linear Algebra courses from the 1st and 2nd years of the ICEF bachelor's programme.

1.2. Abstract

The emphasis of the course is on the theory rather than on the method. One central topic of the course is formal mathematical reasoning. The students will practice formulating precise mathematical statements and proving them rigorously. These skills are essential for the current specialization, they often remain in shadows in other math courses where the focus is on solving problems through calculation.

The second central topic of the course is the abstract mathematical structures from algebra (groups, fields, etc.), analysis, topology (topological spaces, manifolds) and functional analysis.

We will develop some of these theories roughly to the extent of standard 1^{st} and 2^{nd}-year courses of the mathematical departments. The awareness of the theoretical foundations of these classical theories is key in understanding the contemporary theoretical research and the synergies between different areas of mathematics and its applications.

2. Learning Objectives & Outcomes

At the conclusion of the course, students should:

- understand the main mathematical concepts in discrete mathematics, algebra, real analysis, functional analysis and topology;
- be able to illustrate the concepts by specific examples and counter-examples;
- be able to use formal notations correctly and in connection with precise statements in English;
- be able to give definitions, formulate statements of the key theorems and present their proofs;
- be able to find and formulate proofs of problems based on the main definitions and theorems;
- be able to critically analyze a proposed proof of a given statement and make a conclusion on the completeness and accurateness of the proof;
- have a generic understanding of the applications of the discussed classical theories.

3. Methods of Instruction

Throughout the academic year there will be 1 lecture and 1 seminar per week. In addition, the lector and the class teacher will held regular office hours.

Almost each week a homework assignment will be posted following the lecture. The homework will have two parts: a part to be submitted in writing on the next lecture, and a self-study part. The first part will follow the format of the UoL exam. The self-study part will include either a reference or a quote from a textbook or a research paper as well as extra problems for further discussion. Students are encouraged to do both parts.

4. Reading List

The homework assignments will include brief lecture notes referencing all the definitions and theorems discussed. These notes should be sufficient to solve all the main homework problems and all the exam problems. However, it is recommended that the students read the relevant chapters from the textbooks listed below. The textbooks differ not only in the content, but also in their exposition style, the choice of the latter is left to the students.

4.1. Recommended by UoL Abstract Mathematics syllabus

- Biggs, Norman L. Discrete mathematic, 2nd edition. (Oxford University Press, 2002).
- Eccles, P.J., An Introduction to Mathematical Reasoning: numbers, sets and functions. (Cambridge University Press, 1997).
- Binmore, K.G., Mathematical Analysis: A Straightforward Approach. (Cambridge University Press, 1982).
- Bryant, V., Yet Another Introduction to Analysis (Cambridge University Press, 1982).
- Halmosh, P.R., Finite Dimensional Vector Spaces. (Springer, 1996).
- Rudin, W., Principles of Mathematical Analysis, 3rd edition, (McGraw-Hill, 1976).

4.2. Supplementary reading

- Vinberg E.B., A Course in Algebra. (Factorial Press, 2001).
- Kolmogorov A.N., Fomin S.V., Elements of the Theory of Functions and Functional Analysis (any Russian or English edition).
- Warner, Frank W., Foundations of Differentiable Manifolds and Lie Groups. (Springer, 1983).
- Vassiliev V.A., Introduction to Topology. (MCCME 2014).

In addition, some quotes from other textbooks and some research papers will be suggested as supplementary reading for particular topics.

5. Special Equipment and Software Support

The lecture notes, the homework assignments and some other supplementary materials are posted on the ICEF information portal. Thus, the students are required to have some means of accessing that information. The requirements are minimalistic and can be met by any contemporary computer or a smartphone.

No special equipment is required for the Abstract Mathematics lectures, classes or the exams.

While working on their homework the students are free to use contemporary math software packages. In many cases using such software helps to develop the deeper understanding of the material. However, since the focus of this course is on the theory rather than on computations, most of the software use cases will provide just some illustrations to the concepts under discussion.

6. Grading System and Examination Type

The course grades are based on the 4 exams and on the homework assignments.

6.1. Exam Schedule

The academic year 2019 – 2020 at ICEF is divided into 4 modules as follows:

Fall Semester

- Module I: from 2nd September 2019 till 26th October 2019;
- Module II: from 4th November 2019 till 21st December 2019;

Spring Semester

- Module III: from 13th January 2020 till 21st March 2020;
- Module IV: from 30th March 2020 till 30th April 2020.

During the week following the end of modules I, II and III students sit an ICEF internal written exam. The exact dates of these exams are decided during the year and announced in form of the cross-subject exam schedule. In May the students sit a UoL written exam. The date of this final exam is determined in accordance with the UoL exam schedule.

The ICEF exams format mimics that of the UoL final exam. The ICEF exams include all the topics covered from the beginning of the year up to the date of the exam. Thus, the exam in December covers the topics from modules I and II, the exam in March covers all the topics from modules I, II and III.

6.2. Assessment Rules

Each of the three ICEF exams and the UoL exam are marked out of 100 points. The grades for the first semester and for the whole course are also initially computed out of 100 points using the following formulas:

Fall100 = 0.35 * Exam-I + 0.45 * Exam-II + 0.2 * Fall_Hwk_Quiz

Course100 = 0.325 * Fall100 + 0.175 * Exam-III + 0.45 * UoL + 0.05 * Spring_Hwk_Quiz

Here Exam-I, Exam-II and Exam-III are the grades out of 100 points for the ICEF exams following the Modules I, II and III respectively, UoL is the grade out of 100 points for the final exam in May, Fall_Hwk_Quiz and Spring_Hwk_Quiz are the marks out of 100 points for the homework and in-class activities in the corresponding semesters. It is expected that the latter scores will be based on at least 5 homework assignments and at least 2 in-class quizzes in each semester.

The grades Fall100 and Course100 are converted to 10-point scale according to the ICEF guidelines. The resulting grades out of 10 points are the official HSE grades for the first semester and the course. The standard Russian grades out of 5 points are also computed based on the grades out of 10 points and the HSE generic conversion rules.

In case a student misses one of the exams the ICEF generic rules will apply in accordance with the HSE Interim and Ongoing Assessment Regulations (incl. Annex 8 for ICEF). Grade determination after retakes is done in accordance with ICEF Grading Regulations (par. 5).

7. Course Plan

The following table gives an approximate plan of the topics to be discussed in each of the modules. The actual schedule may vary depending on the pace of the lectures.

7.1. Module I

- Introduction to Set Theory
    a. Definitions and notations
    b. Set operations, maps between sets
    c. Discussion of the proper foundations of the set theory*
- Algebraic Structures: Groups, Modules, Vector Spaces
    a. Definition and basic properties of groups

b. Subgroups, quotient groups and homomorphisms, cosets & Lagrange's theorem.

c. Homomorphisms, group isomorphism theorems

d. Automorphisms and semi-direct products

e. Group actions

f. Sylow theorems*

g. Classification of Abelian groups*

h. Introduction to the representation theory of finite groups*

7.2. Module II

- Algebraic Structures: Rings and Fields

a. Divisibility of integers

b. Congruence and modular arithmetic.

c. Definition and basic properties of rings

d. Ideals, ring homomorphisms

e. The Chinese Remainder Theorem

f. Definition and basic properties of fields

g. Field of complex numbers

h. Finite fields

i. Rings of polynomials

j. Field Theory and Galois theory*

7.3. Module III

- Elements of Real Analysis, Functional Analysis and Topology

a. Real numbers as a complete ordered field

b. The Archimedian property of real numbers

c. Equivalent definitions of completeness in an archimedian field

d. Representation of real numbers via decimal fractions

e. Topological spaces and operations with them

f. Continuity, properties of continuous functions

g. Metric spaces

h. Norm and normed spaces

i. Contracting maps, contracting map theorem for complete metric spaces

j. Homotopy groups, homotopy equivalence.*

k. Coverings, cell spaces (CW-complexes).*

l. Differentiable manifolds, diffeomorphisms*

m. Tangent vectors and differentials, differential forms*.

n. Lie Groups*

7.4 Module IV

- Elements of mathematical logic and formal reasoning
- Course Review

Note: topics marked with * will be only touched in the self-study part of the homework assignments. They are not included in the internal exams or in the UoL exam.

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Updated by A. Akhmetshin, 2019.