Syllabus

Title of a Course : Algebraic Geometry: A First Geometric Look

(6 ECTS)

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Faculty of Mathematics

Meeting Minute # ___dated _____ 20_

1. Course Description

a) Pre-requisites : linear and multilinear algebra, and basic ideas of polynomials, commutative rings and their ideals, tensor products, affine and projective spaces, topological spaces and their open, closed and compact subsets. No deep knowledge is assumed, all essential definitions and technique will be recalled during the course.

b) Abstract

Algebraic geometry studies geometric loci looking locally as a solution set for a system of polynomial equations on an affine space. The main feature of this subject, that it provides an algebraic explanation to various geometric properties of the figures and it the same time it give geometric intuition to purely algebraic contructions. It plays an important role in many areas of mathematics and theoretical physics, and provides the most visual and elegant tools to express all aspects of the interaction between different branches of mathematical knowledge. The course gives the flavor of the subject by presenting examples and applications of the ideas of algebraic geometry, as well as a first discussion of its technical tools.

2. Learning Objectives

3. Learning Outcomes

4. Course Plan

∘ Projective spaces. Geometry of projective quadrics. Spaces of quadrics.

∘ Lines, conics. Rational curves and Veronese curves. Plane cubic curves. Additive law on the points of cubic curve.

∘ Grassmannians, Veronese's, and Segre's varieties. Examples of projective maps coming from tensor algebra.

∘ Integer elements in ring extensions, finitely generated algebras over a field, transcendence generators, Hilbert's theorems on basis and on the set of zeros.

∘ Affine Algebraic Geometry from the viewpoint of Commutative Algebra. Maximal spectrum, pullback morphisms, Zariski topology, geometry of ring homomorphisms.

∘ Agebraic manifolds, separateness. Irreducible decomposition. Projective manifolds, properness. Rational functions and maps.

∘ Dimension. Dimensions of subvarieties and fibers of regular maps. Dimensions of projective varieties.

∘ Linear spaces on quadrics. Lines on cubic surface. Chow varieties.

∘ Vector bundles and their sheaves of sections. Vector bundles on the projective line. Linear systems, invertible sheaves, and divisors. The Picard group.

∘ Tangent and normal spaces and cones, smoothness, blowup. The Euler exact sequence on a projective space and Grassmannian.

5. Reading List

a) Required

b) Optional

6. Grading System : 1⁄3×(solution of the problems from the task sheet)+2⁄3×(final exam)

7. Examination Type

8. Methods of Instruction

9. Special Equipment and Software Support : no requirements

10. Further reading