Syllabus
Mathematics

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1. Course Description

a) Pre-requisites

To learn the course, students should have the following knowledge and skills:

- knowledge of algebra, calculus and probability theory at the level of the school curriculum;
- ability to navigate the Internet resources and to know the basics of bibliography;
- sufficient proficiency in English.

b) Abstract

In the process of studying the discipline, students will become familiar with theoretical foundations and basic methods of solving tasks on the following topics

- Derivative and its applications;
- combinatorial analysis, definition of probability, random events;
- independent events, expected value and variance of random variable; main discrete distributions of random variables;
- Normal distribution. Limit theorems;
- Sample. Descriptive statistics: sample mean, median, sample variance, quintiles, quartiles.

2. Learning Objectives

The course aims to provide students with understanding of key concepts and methods of calculus and probability theory for understanding other practical courses, related to data analysis and programming and economics.

3. Learning Outcomes

As a result of this course a student will:

- know main definitions and results of probability theory and statistics to be essential for understanding further practical courses;
- be able to formalize the problem from subject area, choose the adequate methods of solutions, perform calculations and to interpret the results;
- have skills of solving problems to be important in professional activity.

4. Course Plan

№	Theme	Total Hours			Student Work
			Lecture hours	Workshop hours	
1.	Derivative and its properties	16	2	2	12
2.	Combinatorial analysis. Axioms of probability.	16	2	2	12
3.	Classical definition of probability. Conditional probability and independence	16	2	2	12
4.	Random variables. Main characteristics. Main types of discrete random variables	16	2	2	12
5.	Expected value and Variance of random variable	16	2	2	12
6.	Continuous random variables. Normal distribution. Limits theorem	16	2	2	12
7.	Basic definitions of statistics.	18	2	2	14
	In total	114	14	14	86

5. Reading list

a) Required

1. S. Ross. A first course in probability (1997), Prentice hall, Upper saddle river, New Jersey.

2. F.M. Dekking, C. Kraaikamp, H.P. Lopuhaa, L.E. Meester. A Modern Introduction to Probability and Statistics. Understanding Why and How. London: Springer-Verlag, 2005.

3. Stewart J. Calculus. Early Transcendentals. 6th edition. Thomson Brooks/Cole, 2008.

b) Optional

1. Кремер. Н. Ш. Теория вероятностей и математическая статистика. М.: Юнити-Дана, 2010.

2. Ю.Н. Тюрин, А.А. Макаров, Г.А. Симонова. Теория вероятностей. Учебник для экономических и гуманитарных специальностей. М.: МЦНМО, 2009.

3. S. Ross. Introduction to probability models (1997), Academic press.

6. Grading System

The *final grade* can be obtained by rounding the score S obtained by the following formula:

$$S=0,25*C+0,25*W+0,5*E,$$

where

- E is a mark for the final exam on the course held at the end of the first module (its duration is 120 minutes).
- C is the grade for the control work held at the last seminar. Control work is not allowed to be rewritten at extra time.
- W is a score obtained for the regular quizzes held at seminars and the seminar activity. W is calculated as average of all marks obtained for quizzes (each of them is 10 mark max) and seminar activity.

7. Guidelines for Knowledge Assessment

Part 1. Derivative and its properties ([3], Ch 2, 2.7-2.8; Ch 3, 3.1-3.7).

Part 2. Combinatorial analysis. Combinations. Permutations. Axioms of probability. Sample space. Event. Main properties of probability ([1], Ch 1, 1.2-1.4, Ch 2, 2.2 – 2.4).

Part 3. Classical definition of probability. Conditional probabilities. Bayes' formula. Independent Events. ([1], Ch 2, 2.5, Ch 3, 3.2 – 3.4).

Part 4. Random variables. Distribution function. Discrete random variables ([1], Ch 4, 4.1 – 4.3).

Part 5. Expected value of discrete random variable. Expectation of a Function of a Random Variable. Variance and standard deviation of random variable ([1], Ch 4, 4.4 – 4.6).

Part 6. Continuous random variables. Normal distribution. Limits theorem ([1], Ch 5, 5.2 – 5.5, Ch 8, 8.2 - 8.3).

Part 7. Basic definitions of statistics. Exploratory data analysis: graphical summaries. Histograms. Kernel density estimates. The empirical distribution function. Scatter plot. The center of a dataset. Empirical quintiles, quartiles, and the IQR. ([2], Ch 15, 16).

Typical problems for final exam

1. A hotel has spare one-seated rooms numerated from 1 to 30. The first guest prefers rooms numerated from 1 to 20 and the second one prefers rooms numerated from 10 to 30. How many ways are there to settle both of them?
2. A retail establishment accepts either the American Express or the VISA credit card/ A total of 24 percent of its customers carry an American Express card, 61 percent carry a VISA card and 11 percent carry both. What percentage of irs customers carry a credit card that the establishment will accept?
3. 60% of the students at the certain school wear neither a ring nor a necklace. 20% wear a ring and 30% wear a necklace. If one of the students is chosen randomly what is the probability that this student is wearing a) a ring or a necklace; b) a ring and a necklace?
4. A total of 28 percent of American males smoke cigarettes, 7% smoke cigars and 5% smoke both cigars and cigarettes. a) What percentage of males smoke neither cigars nor cigarettes? b) What percentage of males smoke cigars but not cigarettes?
5. The king comes from a family of 2 children. What is the probability that the other child is his sister?

6. In a certain community, 36 percent of the families own a dog, and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is (a) the probability that a randomly selected family owns both a dog and a cat; (b) the conditional probability that a randomly selected family owns a dog given that it owns a cat?

7. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 24 percent as conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the conservatives voted. A voter is chosen at random. Given that this person voted in the local election, what is the probability that he or she is (a) an Independent; (b) a Liberal; (c) a Conservative? (d) What fraction of voters participated in the local election?

8. A total of 48 percent of the women and 37 percent of the men that took a certain "quit smoking" class remained nonsmokers for at least опе year after completing the class. These people then attended a success party at the end of a year. If 62 percent of the original class were male, (a) what percentage of those attending the party were women? (b) what percentage of the original class attended the party?

9. Suppose that 5% of men and 0.25 percent of women are color blind. A colorblind person is chosen at random. What is the probability of this person being mail? Assume that there are an equal number of males and females. What if the population consisted of twice as many males as females?

10. English and American spellings "rigour" and "rigor", respectively. A man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel. If 40 percent of the English-speaking *men* at the hotel are English and 60 percent are Americans, what is the probability that the writer is an Englishman?

11. Stores *A, B and C* have 50, 75, and 100 employees and, respectively, 50, 60, and 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns, and this is a woman. What is the probability that she works in store C?

12. Find $E(X)$, $Var(X)$, $E(Y)$, $Var(Y)$, $E(XY)$, $Cov(X,Y)$ and $\rho(X,Y)$ for random vector (X,Y) with distribution

	$X=0$	$X=1$	$X=2$
$Y=0$	0.2	0.1	0.1
$Y=1$	0.1	0.3	0.2

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13. Independent RVs X,Y take integer values: X – from 1 to 12 with the same probability, Y – from 1 to 7 with probability $\frac{1}{7}$. Find $P(X+Y<6)$.

14. On the plane two circles are drawn whose radiuses are 5 and 25 respectively. The smaller circle is contained in larger circle. In big circle 5 points are thrown at random. Let X be the number of points fell in the small circle. Calculate $E(X)$ and $D(X)$.

15. For RV X having normal distribution with $E(X)=5$ and $Var(X)=16$ find $P(1<X<7)$.

8. Methods of Instruction

Delivery of seminar homework can be done remotely by e-mails. The result of essential homework and final control work are sent by e-mail. Every week students get brief summary of the lectures to help them to do homework and literature study.