

Syllabus

1. Course Description

- a. Title of Course: Discrete Mathematics 1
- b. Pre-requisites: basic school-math knowledge
- c. Course Type: compulsory
- d. Abstract: Galileo Galilei told that, Mathematics is a language. And the book of Nature is written in this language. Discrete Mathematics is a language of Computer science. If you are going to develop good complicated programs or do machine learning you will need to learn this language first. During this course we will study basics concepts, such as set theory, graphs, functions and binary relations. These things are basics for the general mathematics as well.

2. Learning Objectives

Students who complete this course successfully will learn or acquire:

- Basic concepts and methods of discrete mathematics, which are necessary for further learning at the programme and for future professional skills
- Skills in using discrete math methods to formalize and solve applied problems

3. Learning Outcomes

- Able to find mathematical structures in the object you work with
- Learn basic math notions that are needed to work in broad areas of computer science
- Basic math skills for learning advanced math and computer science courses

4. Course Plan

1. Basics of logic and set theory

Boolean logic and Boolean functions. Disjunctive and conjunctive normal forms. Definitions, theorems and proofs. Types of proof. Set-theoretic identities and connection with logic (via characteristic function).

2. Enumerative combinatorics

Rule of sum. Inclusion–exclusion principle. Recursive formulas. Mathematical induction, applications in enumerative combinatorics. Rule of product. Binomial coefficients, number of subsets. Binomial theorem, Pascal's triangle.

3. Functions and binary relations

Functions, injections, surjections, bijections. Images, preimages. Binary relations, composition, equivalence relations, equivalence classes.

4. Graphs

Basic Definitions. Vertex degrees. Subgraphs. Bipartite graph, 2-colorable graphs. Paths. Connected components. Cycles. Trees. Number of tree's edges and vertices. Spanning tree. Connected graphs. Criterion for the existence of Eulerian path. Distance and diameter. Oriented graphs. Directed acyclic graphs. Graph isomorphism.

5. Basics of number theory

Arithmetic of residuals, divisibility. LCM, GCD. Chinese remainder theorem (example of bijection). Fermat's little theorem. Euclidean algorithm. Diophantine equations. Fundamental theorem of arithmetic.

6. Cardinality of sets

Comparison of sets. Finite and infinite cardinality of sets. Countable sets, countability of sets of integer and rational numbers. Uncountable sets. Continuum. Schröder–Bernstein theorem.

7. Partial order relations

Partial order relations. Well-founded relations. Recursion and induction. Order isomorphism.

8. Basics of probability theory

Finite probability space, event, combinatorial formulas, probability. Inclusion–exclusion principle. Conditional probability, independent events. Bayes' theorem. Random variable. Expected value, linearity. Law of large numbers. Probabilistic method for proving combinatorial existence theorems.

9. Generating functions

Formal power series. Examples of generating functions for simple sequences. Operations on generating functions. Newton's generalized binomial theorem. Multinomial theorem. Generating functions for Fibonacci and Catalan numbers. Linear recurrence sequences.

10. Boolean circuits and decision trees

Decision trees. Method of proof for lower bounds. Boolean circuits and formulas. Basis and functionally complete basis.

Learners will also use video materials from Coursera's course Introduction to Discrete Mathematics for Computer Science <https://www.coursera.org/specializations/discrete-mathematics> developed by V. Podolskiy, M. Levin and A. Kulikov (HSE and University of California San Diego).

5. Reading List

a. Required

1. L. Lovasz, K. Vesztergombi. Discrete Mathematics. Lecture Notes, Yale University, 1999. <http://www.cs.elte.hu/~lovasz/dmbook.ps>
2. J. Anderson. Discrete Mathematics With Combinatorics. Prentice Hall; 2 edition 2003
3. Shen. Mathematical induction (C1) 3rd ed., Moscow: MCCME, 2007, 32 p. <http://www.mccme.ru/free-books/shen/shen-induction.pdf> (in Russian)
4. N. K. Vereshchagin, A. Shen. Introduction to the set theory. 4th ed., Moscow: MCCME, 2012, 112 c. <http://www.mccme.ru/free-books/shen/shen-logic-part1-2.pdf> (in Russian)

b. Optional

1. R. Hammack. Book of Proof, Virginia Commonwealth University 2013, <https://www.people.vcu.edu/~rhammack/BookOfProof/BookOfProof.pdf>

6. Grading System

Intermediate grade 1

Cumulative grade-1 = 50% colloquium-1 + 50% homework

Intermediate grade (module 2) = 60% cumulative grade-1 + 40% intermediate exam (module 2)

Intermediate grade 2

Cumulative grade = (3/14) colloquium-1 + (3/14) colloquium-2 + (3/14) intermediate exam + (15/42) homework

Intermediate grade = 70% cumulative grade + 30% final exam

Grade "homework assignments" is an average grade of all the homework assignments in the course.

Intermediate grade 2 is the final grade for the course included in a diploma supplement.

7. Guidelines for Knowledge Assessment

There are weekly homework assignments, colloquium, midterm (2d module) and final (3d module) exams. Students are encouraged to work together to help each other in understanding the course material and tackling the homework problems. However, it is essential that every student must write up his/ her own solution. Homework submitted after the general deadline will not be accepted.

The common mistakes made in the homework will be discussed during the seminars. Homework and exams consist of tasks that are equivalent or similar to those which have been studied at lectures and seminars. Any fact of cheating or breach of academic integrity will result in receiving a "0" (zero) for this work.

Rounding of the cumulative grade, intermediate and final grades must be performed according to the following rules. Rounding down for marks between 1 and 5, rounding by the rules of arithmetic for marks between 5 and 6, and rounding up for all the other marks.

There is no possibility to get an extra point to compensate the low cumulative grade.

8. Methods of Instruction

The discipline is delivered through lectures seminars, including computer classes.