Syllabus

Title of a Course : Introduction to Commutative Algebra

(6 ECTS)

Author, lecturer A.B.Pavlov (abpavlov@hse.ru, https://www.hse.ru/org/persons/224233362)

Faculty of Mathematics

Meeting Minute # ___dated _____ 20_

1.Course Description

a) Pre-requisites

PREREQUISITES: Basic courses given at the faculty of mathematics for the first 3 semesters, including (a) basic algebra (groups, rings, fields), (b) Linear algebra (tensor products), (3) Basic geometry

b) Abstract

DESCRIPTION: At its most basic level, algebraic geometry is the study of the geometry of solution sets of polynomial systems of equations. Classically, the coefficients of the polynomial equations are assumed to lie in an algebraically closed field. Considering more general coefficient rings, in particular rings of integers in number fields, one arrives at modern algebraic geometry and algebraic number theory. Commutative algebra provides the tools for answering basic questions about solutions sets of polynomial systems, such as finite generation of the system, existence of solutions in some extension of the coefficient ring, dimension and irreducible components, and smoothness and singularities.

2.Learning Objectives

- Learn proofs of fundamental theorems of commutative algebra.
- Understand relations between commutative algebra and other areas of mathematics, such as algebraic geometry and algebraic number theory.
- Apply theorems to provide proofs of statements about commutative rings and modules over them (problem solving).
- Apply knowledge of commutative algebra to analyze examples of particular rings and modules, for example given in terms of generators and relations.

3.Learning Outcomes

At the end of the course students are expected to be able to state fundamental theorems of commutative algebra, provide proofs and apply the theorems to solve problems and analyze examples.

4.Course Plan

- Ideals and radicals
- Modules over commutative rings
- Localization
- Chain conditions for rings and modules

- Primary decomposition
- Integral extensions
- Flatness
- Completions
- Dimension theory
- Discrete valuation rings and Dedekind rings

5. Reading List

a) Required

Atiyah M. F., Macdonald I. Introduction to commutative algebra, Addison-Wesley Series in Mathematics, 1969

b) Optional

Altman A., Kleiman S. A term of commutative algebra, electronic copy available for free at http://www.centerofmathematics.com/wwcomstore/index.php/commalg.html

6. Grading System

40% Final exam
30% Midterm
15% Participation in the tutorial
15% Tests

7. Examination Type

Written midterm and final exam.

8. Methods of Instruction

One lecture and one tutorial per week.

9. Special Equipment and Software Support

No requirements.

10. Further reading

1. Eisenbud, D. - Commutative Algebra with a View Toward Algebraic Geometry. New York. Springer-Verlag, 1995.
2. Matsumura, H. - Commutative Algebra. Reading, Mass., Benjamin-Commings, 1980.