Syllabus

Title of a Course : Algebraic Geometry. Language of Schemes
(6 ECTS)

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Faculty of Mathematics

Meeting Minute # ___dated _____ 20_

1. Course Description

a) Pre-requisites

PREREQUISITES: Commutative algebra: it is expected that students have studied the material from the book «Introduction To Commutative Algebra» by Atiyah and MacDonald, though some of the results will be reviewed and even reproved. Homological Algebra and sheaf theory. Cohomology of sheaves. I recommend the book «Methods of homological algebra» by Gelfand and Manin though it has more material then we will actually use. If you can reproduce a proof that the cohomology of the constant sheaf $\mathbb{R}$ on a smooth manifold can be computed by the de Rham complex you should not worry.

b) Abstract

DESCRIPTION: The course will cover most of «Algebraic Geometry» by Hartshorne. Additional topics may include: general Riemann – Roch Theorem, the Hilbert scheme and its application to the existence theorems, a proof the Weil conjectures for curves over finite fields, rational curves on Fano varieties («bend-and-break trick»).

2. Learning Objectives: Master the language of modern Algebraic Geometry

3. Learning Outcomes: Get prepared for research in Algebraic Geometry and related areas such as Geometric Representation Theory and Number Theory.

4. Course Plan (each of the topics below will take approximately 6 hours)

- Review of commutative algebra
- Schemes, fiber products
- Proper morphisms, valuation criteria
- Coherent sheaves
- Divisors, the Picard group
- The case of curves
- Differentials, smooth morphisms
- Cohomology of coherent sheaves
- Serre duality
- Riemann - Roch theorem
- Applications to counting points over finite field
- Introduction to the deformation theory with applications to rational curves on Fano varieties

5. **Reading List**

a) Required:

1) Foundations of Algebraic Geometry, by R. Vakil; available electronically at http://math.stanford.edu/~vakil/216blog/FOAGaug2610public.pdf,

2) Theory of schemes, a course by D. Gaitsgory; available electronically at http://www.math.harvard.edu/~gaitsgde/Schemes_2009/

b) Optional:

Introduction to Algebraic Geometry, by I. Dolgachev; available electronically at http://www.math.lsa.umich.edu/~idolga/631.pdf

6. **Grading System**

GRADING RULES: course grade $= 0.5 \times$ (homework grade) $+ 0.5 \times$ (final exam grade) rounded to the nearest integer (and, for an integer n, the number $n + 0.5$ is rounded to $n + 1$).

7. **Examination Type:** written exam

8. **Methods of Instruction:** lectures and problem solving sessions

9. **Special Equipment and Software Support** : no requirements

10. **Further reading:**

Algebraic Geometry, by R. Hartshorne, Springer 1997