Dual degree program
"Applied Data Analysis"
between NRU HSE and
University of London

Syllabus for DIFFERENTIAL EQUATIONS (5 ECTS credits)

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Course description

"Differential Equations" is a spring semester course for the second year students studying at Faculty of Computer Sciences. It is designed to suit the requirements of the Faculty of Computer Sciences curriculum as well as UoL where DE is a part of the mathematical curriculum. Besides the course on differential equations is included as a topic in "Mathematical methods for economists" external exam.
This course is an important part of the bachelor stage in education of the future applied mathematicians and computer scientists. It has to give students skills for implementation of the mathematical knowledge and expertise. Its prerequisite is the knowledge of the single variable calculus.
In the spring semester the course is split into two unequal parts: it is taught from January through the end of April and after the students finish with their UoL exams in May it will resume and will continue till mid-June.
The assessment of the students will be done by setting mock exam by the end of the 3rd module, then later by the University of London (UoL) examinations in May and final exam will be set in late June. But the final grade will depend solely on mock, final exams performance and home assignments grades.

Teaching objectives.

Students are expected to develop an understanding of basic concepts of the differential and difference equations. On the practical side, among other skills, students are expected to be able to solve linear equations with the constant coefficients as well as the systems of such equations. This knowledge will allow them to progress with the further complicated topics such as optimal control theory and like.
Students are supposed to develop ability to apply the knowledge of the differential and difference equations which will enable them to analyze dynamics of the processes.

Teaching Methods

The course program consists of:
- lectures,
- classes,
- regular self-study and working on home assignments.

Assessment and control

- Home assignments (every second week)
- Mid-term test (mock exam) which lasts 80 minutes
- Final exam which lasts 120 minutes

Grade determination of DE course, missing test policy

The home assignments constitute **15%** of the final grade. The mid-term test contributes **25%** to the final grade. The final exam is **60%** of the final grade. If a student missed the mid-term test without a valid excuse (see school's schedule for valid excuses), he or she would be given zero grade. If a student missed the mid-term test with a valid excuse, the calculation of the final grade will be based on other types of assessment, using a formula that partially compensates for the lost points. In this case, the weights of all other components of the final grade are multiplied by (1+0,5a), where "a" is the weight of a grade for the missed midterm test in the final grade.

The **final** grade on DE course will be determined according to the formula:

Grade for DE=0.15*average grade for HW+0.25*grade for mock+0.6*grade for final

All grades are given initially out of 100. The final grades are also transferred to 10- and 5-points grades in accordance with the ICEF Grading Regulations (par.3) available at

https://icef-info.hse.ru/goto_icef_file_29833_download.html

Retakes are organized in accordance with the HSE Interim and Ongoing Assessment Regulations (incl. Annex 8 for ICEF). Grade determination after retakes is done in accordance with ICEF Grading Regulations (par. 5) available at

https://icef-info.hse.ru/goto_icef_file_29833_download.html

Warning

As a general policy, personal computing devices such as laptops, calculators etc. are not supposed to be used in the course. They are *absolutely prohibited* in all exams. Students are expected to do all necessary arithmetic computations by hand.

Main Reading

1.Carl P. Simon and Lawrence Blume. Mathematics for Economists, W.W. Norton and Co, 1994.

Additional Reading

1. A.F. Fillipov. Collection of problems on differential equations. Moscow, "Nauka", 1973 and later editions.

2. V.K. Romanko, Course on Differential Equations and Calculus of Variations. Moscow and Saint-Petersburg, "Fizmatlit", 2001 and later editions.

Internet resources

University of London Exam papers and Examiners reports for the last three years
http://www.londonexternal.ac.uk/current_students/programme_resources/lse/index.shtml .

Course outline

Part I. Differential equations

1. Dynamics in economics and natural sciences. Simple first-order equations. Separable equations. Concept of stability of the solution of ODE. Exact equations. General solution as a sum of a general solution of homogeneous equation and a particular solution of a nonhomogeneous equation. Bernoulli equation. Fundamental theorem on existence and uniqueness.
(SB Sections 24.1-24.2)
2. Qualitative theory of differential equations. Solow's growth model from macroeconomics.
(Section 24.)
3. Second-order linear differential equations with constant coefficients.
(Section 24)
4. Refresher on complex numbers and operations on them. Representation of a number. De Moivre and Euler formulas.
(Appendix A)
5. Higher-order linear differential equation with constant coefficients. Characteristic equation. Method of undetermined coefficients for the search of a particular solution. Stability of solutions. Routh theorem (without proof). Systems of DE (linear equations case). Variation of parameters method.
(Section 24.3)

Part II. Difference equations

6. Discrete time economic systems. Difference equations. Method of solving first-order equations. Convergence and oscillations of a solution. Cobweb model. Partial equilibrium model with the inventory.
(Section 23.2)
7. Second-order difference equations.
(Sections 23.2)
8. Higher-order difference equations. Characteristic equation. Undetermined coefficients method. Conditions for the stability of solutions. Markov processes *(Section 23.6)*

Part III. Steady states and their stability

9. Stability of linear systems via eigenvalues. Stability of nonlinear systems *(Section 25.4)*
10. Phase portraits of planar systems *(Section 25.5)*
11. First integrals *(Sections 17.1-17.3)*

12. Liapunov functions *(Sections 25.7-25.8)*

Distribution of hours

№	Topic	Total	Lectures	Classes	Self study
	Part I. Differential equations				
1	Dynamics in economics and natural sciences. Simple first-order equations. Separable equations. Concept of stability of the solution of ODE. Exact equations. General solution as a sum of a general solution of homogeneous equation and a particular solution of a nonhomogeneous equation. Bernoulli equation. Fundamental theorem.	22	6	4	12
2	Qualitative theory of differential equations. Solow's growth model.	6	2	2	2
3	Second-order linear differential equations with constant coefficients	12	2	2	8
4	Complex numbers and operations on them. Representation of a number. De Moivre and Euler formulae	12	2	2	8
5	Higher-order linear differential equation with constant coefficients. Characteristic equation. Method of undetermined coefficients for the search of a particular solution. Stability of solutions. Routh theorem (without proof). Systems of linear DE. Variation of parameters method.	26	6	6	14
	Part II. Difference equations				
6	Discrete time economic systems. Difference equations. Method of solving first-order equations. Convergence and oscillations of a solution. Cobweb model. Partial equilibrium model with the inventory	16	4	4	8
7	Second-order difference equations	14	2	2	10
8	Higher-order difference equations. Characteristic equation. Undetermined coefficients method. Conditions for the stability of solutions	18	4	4	10
	Part III. Steady states and their stability				
9	Steady state solutions. Stability of Linear Systems via eigenvalues. Stability of nonlinear systems	24	6	4	14
10	Phase portraits of planar systems	14	2	2	8
11	First integrals	14	2	2	8
12	Liapunov functions	12	4	2	6
	Total:	***190***	***40***	***40***	***110***