Syllabus for the course
"Computational methods of stochastics"

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ECTS	6
Contact hours	64
Self-study hours	164
Study year	2
Format	Full-time

Prerequisites

Basics of probability theory

Course description

This is course on Monte Carlo and Markov Chain Monte Carlo methods. It is intended for mathematicians, computer scientists, scientists, statisticians, and others interested in learning about and using modern Monte Carlo methods in their research. The course covers basic sampling methods including mappings, rejection, and Markov chain Monte Carlo (MCMC). We will study in detail variuos variance reduction methods, such as control variates, systematic sampling, multilevel methods and importance sampling. Advanced topics should include recent improvements in MCMC samplers, stochastic approximation and optimization, evaluation of evidence and partition function integrals, rare event sampling strategies, mathematical analysis of MCMC - spectral gaps, burn-in time, etc. Applications in physical sciences, Bayesian statistics, and machine learning will be discussed.

Objectives

- Making students familiar with the most important elements of the Monte Carlo method and of the Markov Chain Monte Carlo
- The students should be able to apply the method to solve problems in physics and other sciences such as economics and social sciences.

Outcomes

After finishing the course the students will be able to

- generate random numbers
- use various variance reduction methods in concrete problems
- use various Markov Chain Monte Carlo methods to evaluate integrals
- solve otimization problems via MC and MCMC
- perform rare events simulation

Topics

1. Direct (or simple) Monte Carlo

(a) generating uniform random numbers
(b) generating continuous and discrete random variables,
(c) generating multidimensional Gaussian processes (Cholesky),
(d) rejection sampling
(e) importance sampling
(f) review of CLT and confidence intervals as background for next topic
(g) error estimates for simple monte carlo
(h) statistical tests relevant to MC, e.g., Kolmogorov-Smirnov goodness-of-fit

2. Markov chain Monte Carlo (MCMC)
(a) the requisite Markov chain theory and the key idea of MCMC
(b) the Gibbs sampler
(c) the Metropolis-Hasting algorithm
(d) autocorrelation times and error bars
(e) error bars via batched means
(f) burn-in time
(g) examples from various fields including Bayesian statistics

3. Further topics:

(a) variance reduction
(b) sequential monte carlo
(c) adaptive monte carlo
(d) simulated annealing
(e) stratified sampling
(f) perfect sampling
(g) optimizing functions evaluated by Monte Carlo
(h) rare event simulation
(i) other topics to be chosen based on the students interests

Grading will be based on several assignments, a midterm exam and a final project (exact weighting yet to be decided). Assignments will include programming in Python as well as theoretical exercises. Assignments will emphasize elements of programming methodology relevant to Monte Carlo methods, including verification protocols and visualization methods. There will be a long term project done individually or in small groups.

Grading System

Grading Task1: Assignments
Weighting 20%

Grading Task 2: Mid-Semester test
Weighting 20%

Grading Task 3: Project
Weighting 20%

Grading 4: Final Exam
Weighting 40%

Reading List

Required

- Christian P. Robert and George Casella, Monte Carlo Statistical Methods, Springer, 2nd edition

Optional

- Jean-Michel Marin and Christian P. Robert, Bayesian Core: A Practical Approach to Computational Bayesian Statistics, Springer, to appear.
- Denis G.T. Denison, Chris C. Holmes, Bani K. Mallick and Adrian F.M. Smith, Bayesian Methods for Nonlinear Classification and Regression, Wiley.
- Arnaud Doucet, Nando De Freitas and Neil J. Gordon (eds), Sequential Monte Carlo in Practice, Springer.
- Andrew Gelman, John B. Carlin, Hal Stern and Donald B. Rubin, Bayesian Data Analysis, Chapman&Hall/CRC, 2nd edition.
- Christian P. Robert, The Bayesian Choice, Springer, 2nd edition.