Discipline Syllabus

«Conflict and Cooperation»

1. Course Description

a. Title of a Course: Conflict and Cooperation

b. Lecturer: Fuad Aleskerov, Lyudmila Egorova, Andrey Subochev

c. Pre-requisites: basic courses in Calculus, Theory of Games.

d. Course Type (compulsory, elective, optional): elective

e. Abstract

This course presents an introduction to cooperative games, solutions and applications to conflict situations.

2. Learning Objectives: To familiarize students with the concepts, models and statements of the cooperative games in application to the theory of conflict.

3. Learning Outcomes:

- Know basics of cooperative games and theory of conflict;
- Be able to choose adequate models in practical problems;
- Have skills in model construction and solving problems of cooperative games and theory of conflict.

4. Course Plan:

Topic 1. Cooperative games with transferable utility (TU games) and their interpretation.

The characteristic function, relation between TU games and noncooperative normal form games, saving games and cost games. Types of TU games (essential and inessential games, superadditive games, convex games, monotonic games, simple games, constant sum games. Strategic equivalence, normalization. Basis of unanimity games, Harsanyi dividends.

Examples of TU games: glove games, a game “landlord and peasants”, bankruptcy games, airport games, weighted majority games, market games, veto rich games, assignments games, big boss games.

Topic 2. Main solution concepts, their properties and axiomatic characterization.

The imputation set. Domination. Stable sets (von Neumann and Morgenstern solutions). The core, balanced games, necessary and sufficient conditions for the nonemptyness of the core. Totally balanced games, the $\varepsilon-$core and the least core. Core catchers, in particular the Weber set.

The nucleolus, existence and uniqueness, relation to the core and the kernel, characterization via balancedness (the Kohlberg’s theorem).

The Shapley value, different formula representations and their interpretation. Axiomatic characterization: axiomatic of Shapley and axiomatic of Young. The potential of the Shapley value. Simple games, the Shapley-Shubik power index. Properties, in particular the null-player out property.

Asymmetric extensions of the Shapley value—probabilistic values, random-order values, the weighted Shapley value. Peculiarities of different solution concepts in particular classes of applied TU games.

Topic 3. Classes of games with a nonempty core: convex games and 1-convex/1-concave games.

Necessary and sufficient conditions for the convexity of a game, the Shapley's lemma and the Ichiishi's theorem. 1-convex and 1-concave games and their properties, 1- concave basis in the space of all TU games. Applied models of 1-convex/1-concave games: library games, data games, co-insurance games.

Topic 4. TU games with limited cooperation and their solutions.

Games with coalition structures. The Aumann-Drèze value and Owen value. Games with undirected graph communication structures. The Myerson value and its efficient modification, the average tree solution. Games with directed communication structures and their solutions for particular case of forest games. TU games endowed with both coalition and communication structures.

Applications: the social capital index; the water distribution problem of a river with multiple sources, a delta and possible islands along the river bed, and a river with multiple users.

Topic 5. Solution Concepts in Social choice models

Social choice models. Classic and non classic concepts of solutions: Condorcet winner, core, different versions of uncovered set, minimal weakly stable set, untrapped set, minimal dominant and undominating sets, k-stable alternatives and k-stable sets. Their matrix-vector representation.

5. Reading List

a. Required

1. Aleskerov F., Subochev A. 'On Stable Solutions to the Ordinal Social Choice Problem', Doklady Mathematics, 2009, v.79, #3, 437-439
2. Fuad Aleskerov, Andrey Subochev 'Modeling optimal social choice: matrix- vector representation of various solution concepts based on majority rule', Journal of Global Optimization, v.56, no.2, 2013, p.737-756
3. Schelling, Thomas C. (1980). The Strategy of Conflict (Reprint, illustrated and revised. ed.). Harvard University Press. p. 309. ISBN 978-0-674-84031-7. Retrieved September 21, 2010.
4. Thomas C. Schelling (1978) Micromotives and Macrobehavior, Norton. Description, preview. (2006), "Some Fun, Thirty-Five Years Ago," ch. 37, in Handbook of Computational Economics, Elsevier, v. 2, pp. 1639–1644. doi:10.1016/S1574-0021(05)02037-X.
5. M. Maschler, E. Solan, and S. Zamir, Game theory, Cambridge University Press, 2013.
6. B. Peleg and P. Sudhölter, Introduction to the theory of cooperative games, Springer, 2003 (1st ed.), 2007 (2nd ed.).

7. H. Peters, Game theory. A multi-leveled approach, Springer, 2008.
8. H. Moulin, Axioms of cooperative decision making, 1988
9. Subochev A. Dominant, weakly stable and uncovered sets in the ordinal problem of choice. Automation and Remote Control, 2010, #1

b. Optional

1. Aleskerov F., Subochev A. Matrix-vector representation of various solution concepts. Working paper WP7/2009/03. Moscow: SU - Higher School of Economics. 2009.
2. Ambec, S., Y. Sprumont (2002), Sharing a river, Journal of Economic Theory, 107, 453–462.
3. Aumann, R.J., M. Maschler (1985), Game theoretic analysis of a bankruptcy problem from the Talmud, Journal of Economic Theory, 36, 195-213.
4. Brink, R. van den, G. van der Laan, and V. Vasil’ev (2007), Component efficient solutions in line-graph games with applications, Economic Theory, 33, 349–364.
5. Brink, R. van den, A. Khmelnitskaya, and G. van der Laan (2012), An efficient and fair solution for communication graph games, Economic Letters, 117, 786-789.
6. Driessen T.S.H., V. Fragnelli, I.V. Katsev, and A.B. Khmelnitskaya (2011), On 1-convexity and nucleolus of co-insurance games, Insurance: Mathematics & Economics, 48, 217-225. Driessen T.S.H.,
7. Khmelnitskaya A.B., and Sales J. (2012), 1-concave basis for TU games and the library game, TOP, 20, 578-591.
8. Gonzalez-Aranguena, E., A. Khmelnitskaya, C. Manuel, and M. del Pozo (2011), A social capital index, mimeo.
9. Herings, P. J. J., G. van der Laan, and A. J. J. Talman (2008), The average tree solution for cycle-free graph games, Games and Economic Behavior, 62, 77–92.
10. Herings, P. J. J., G. van der Laan, A. J. J. Talman, and Z. Yang (2010), The average tree solution for cooperative games with communication structure, Games and Economic Behavior, 68, 626–633.
11. Khmelnitskaya, A. (2010), Values for rooted-tree and sink-tree digraph games and sharing a river, Theory and Decision, 69, 657–669.
12. Khmelnitskaya, A. (2013), Values for games with two-level communication structures, Discrete Applied Mathematics, doi:10.1016/j.dam.2013.10.019.
13. Khmelnitskaya A. and E. Yanovskaya (2007), Owen coalitional value without additivity axiom, Mathematical Methods of Operations Research, 66, 255-261.
14. Myerson, R. B. (1977), Graphs and cooperation in games, Mathematics of Operations Research, 2, 225–229.
15. Owen, G. (1977), Values of games with a priori unions, in: Essays in mathematical economics and game theory (Henn R, Moeschlin O, eds.), Springer-Verlag, Berlin, pp. 76–88.
16. Vazquez-Brage, M., I. Garcia-Jurado, and F. Carreras (1996), The Owen value applied to games with graph-restricted communication, Games and Economic Behavior, 12, 42–53.

17. Young, H.P. (1985), Monotonic solutions of cooperative games, International Journal of Game Theory, 14, 65-72.

6. Grading System

30% homework + 30% mid-term exam + 40% final exam

7. Guidelines for Knowledge Assessment

8. Methods of Instruction

The discipline is delivered through lectures seminars, including computer classes.

9. Special Equipment and Software Support (if required): Computer classes