

Syllabus
Discrete optimization

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1. Course Description

a. Pre-requisites

The Course is to be based on the acquisition of the following Courses:

- Mathematical analysis;
- Linear algebra;
- Discrete mathematic;

b. Abstract

This course teaches how to solve complex search problems with discrete optimization concepts and algorithms, including constraint programming, local search, and mixed-integer programming. Optimization technology is ubiquitous in our society. It schedules planes and their crews, coordinates the production of steel, and organizes the transportation of iron ore from the mines to the ports. Optimization clears the day-ahead and real-time markets to deliver electricity to millions of people. It organizes kidney exchanges and cancer treatments and helps scientists understand the fundamental fabric of life, control complex chemical reactions, and design drugs that may benefit billions of individuals. This class is an introduction to discrete optimization and exposes students to some of the most fundamental concepts and algorithms in the field. It covers constraint programming, local search, and mixed-integer programming from their foundations to their applications for complex practical problems in areas such as scheduling, vehicle routing, supply-chain optimization, and resource allocation.

c. Course Type

Blended learning

2. Learning Objectives

The objective of this course is to form a foundation of discrete optimization.

3. Learning Outcomes

On completion of the course, the student should know the fundamental concepts and algorithms in discrete optimization.

4. Course Plan

Topic 1. Introduction.

These lectures and readings give you an introduction to this course: its philosophy, organization, and load. They also tell you how the assignments are a significant part of the class. This topic covers the common input/output organization of the assignments, how they are graded, and how to succeed in this class.

Topic 2. Knapsack.

These lectures introduce optimization problems and some optimization techniques through the knapsack problem, one of the most well-known problem in the field. It discusses how to formalize and model optimization problems using knapsack as an example. It then reviews how to apply dynamic programming and branch and bound to the knapsack problem, providing intuition behind these two fundamental optimization techniques. The concept of relaxation and search are also discussed.

Topic 3. Constraint Programming.

Constraint programming is an optimization technique that emerged from the field of artificial intelligence. It is characterized by two key ideas: to express the optimization problem at a high level to reveal its structure and to use constraints to reduce the search space by removing, from the variable domains, values that cannot appear in solutions. These topic cover constraint programming in detail, describing the language of constraint programming, its underlying computational paradigm and how it can be applied in practice.

Topic 4. Local Search.

Local search is probably the oldest and most intuitive optimization technique. It consists in starting from a solution and improving it by performing (typically) local perturbations (often called moves). Local search has evolved substantially in the last decades with a lot of attention being devoted on which moves to explore. These lectures explore the theory and practice of local search, from the concept of neighborhood and connectivity to meta-heuristics such as tabu search and simulated annealing.

Topic 5. Linear Programming.

Linear programming has been, and remains, a workhorse of optimization. It consists in optimizing a linear objective subject to linear constraints, admits efficient algorithmic solutions, and is often an important building block for other optimization techniques. These lectures review fundamental concepts in linear programming, including the infamous simplex algorithm, simplex tableau, and duality.

Topic 6. Mixed Integer Programming.

Mixed Integer Programming generalizes linear programming by allowing integer variables, which dramatically changes the complexity of the problems but also broadens the potential applications significantly. These lectures review how to model problems in mixed-integer programming and how to solve mixed-integer programs using branch and bound. Advanced techniques such as cutting planes and polyhedral cuts are also covered.

Topic 7. Advanced Topics: Part I.

These lectures cover some more advanced concepts in optimization. They introduce constraint-programming techniques for scheduling and routing.

Topic 8. Advanced Topics: Part II.

These lectures continue to cover some more advanced concepts in optimization. They introduce large neighborhood search, which often combines constraint programming and

local search, and column generation which decomposes an optimization model into a master and pricing problem, using more complex variables.

5. Reading List

All materials in <https://www.coursera.org/learn/discrete-optimization>

6. Grading System

The student's final assessment consists of the assessment of the exam A_{exam} and the accumulated assessment A_{acc} obtained on the platform <https://www.coursera.org/learn/discrete-optimization> as follows: $A_{\text{final}} = (A_{\text{acc}} + A_{\text{exam}})/2$

7. Examination Type

Oral examination. Control elements are not blocking.

8. Methods of Instruction

The course is being studied on the online platform <https://www.coursera.org/learn/discrete-optimization>

9. Special Equipment and Software Support (if required)

Not required