Syllabus

High Performance Finite Element Modeling

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1. Course Description

a. Pre-requisites

The Course is to be based on the acquisition of the following Courses:

- Mathematical analysis;
- Linear algebra;
- Discrete mathematics.

b. Abstract

Engineering simulations are rapidly becoming fundamental in virtually all industrial sectors, from medicine to energy, aerospace and beyond. In this course, you will learn the breakthrough general adaptive finite element methods (AFEM) and open source FEniCS software that will position you to take lead to effectively solve the grand challenges in science and engineering.

c. Course Type

Blended learning

2. Learning Objectives

The objective of this course is to form a foundation of the finite element methods, including error estimation, adaptivity, non-linear problems and automated software frameworks, with FEniCS as a reference example.

3. Learning Outcomes

On completion of the course, the student should be able to:

- Explain how physical phenomena in different fields can be described by the same mathematical model.
- Connect fields of science to the study of particular partial differential equations.
- Explain the concept of a Galerkin method to compute approximate solutions to a general PDE.
- Explain the concept of a finite element method

4. Course Plan

Topic 1. FEM in Science and Industry.

After this part, students will be able to explain how physical phenomena in different fields can be described by the same mathematical model and will be able to connect some fields of science to the study of particular partial differential equations. We will explain the concept of a Galerkin method to compute approximate solutions to a general PDE. And we will explain the concept of a finite element method.

Topic 2. Galerkin 1: Introducing FEM in an easy to understand setting without differential equations.

After this session students should be able to:

- Explain the piecewise linear approximation of a function, such as the solution in FEM, including mesh and cell size in the mesh.
- Explain the piecewise linear approximation in terms of basis functions in a function space, conceptually and in FEniCS.
- Explain the L2 inner product, and the associated L2 norm of functions, conceptually and in FEniCS.
- Formulate L2 orthogonal projection using the L2 inner product, compute examples in FEniCS and compare against interpolation.

Topic 3. Galerkin 2: Expanding on FEM, now in the setting of partial differential equations.

After this session students should be able to:

- Recall integration by parts in 1D and 2D/3D.
- Formulate the Galerkin orthogonality of a general stationary PDE in weak form in FEM, conceptually and in FEniCS.
- Explain the linear system associated with a general FEM discretization, conceptually and in FEniCS.
- Formulate the weak form of a given general stationary PDE given in strong form.

Manipulate the basic components of FEM: weak form of PDE and piecewise linear approximation, conceptually and in FEniCS.

Topic 4. Assemble: An automated methodology to construct the linear systems arising from FEM.

After this session students should be able to:

- Learn about the mapping from a reference cell, conceptually and in FEniCS/FEniCS-mini.
- Formulate the general assembly algorithm, conceptually and in FEniCS-mini.
- How to enforce boundary conditions - the general Robin penalty formulation, conceptually and in FEniCS.

Topic 5. Non-linear 1: How to extend FEM to time-dependent and non-linear problems automatically. In part 1, time stepping methods will be introduced.

After this session students should be able to:

- Recall time-stepping for ODE.
- Formulate the Implicit Euler and Trapezoid method for a general PDE in weak form, both conceptually and in FEniCS.

Topic 6. Non-linear 2: In part 2, the fixed-point iteration methodology for non-linear problems will be introduced.

After this session students should be able to:

- Recall fixed-point iteration and Newton's method.

- Formulate Newton's method for a non-linear general PDE, both conceptually and in FEniCS.

Topic 7. Adaptivity: A goal-oriented duality-based adaptive error control methodology.

After this session students should be able to:

- Formulate a posteriori error estimate of FEM for linear functional output - error representation.
- Formulate a do-nothing duality-based adaptive error control algorithm.

5. Reading List

All materials in https://www.edx.org/course/high-performance-finite-element-modeling-0

6. Grading System

The student's final assessment consists of the assessment of the exam A_{exam} and the accumulated assessment A_{acc} obtained on the platform https://www.edx.org/course/high-performance-finite-element-modeling-0 as follows: $A_{final} = (A_{acc} + A_{exam})/2$.

7. Examination Type

Oral examination. Control elements are not blocking.

8. Methods of Instruction

The course is being studied on the online platform https://www.edx.org/course/high-performance-finite-element-modeling-0

9. Special Equipment and Software Support (if required)

Not required