Syllabus

Analysis of Algorithms

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Approved by The Academic Council
Protocol No 3 June 20, 2019

1. Course Description

a. Pre-requisites

The Course is to be based on the acquisition of the following Courses:

- Mathematical analysis;
- Linear algebra.
- Discrete mathematics

b. Abstract

This course teaches a calculus that enables precise quantitative predictions of large combinatorial structures. In addition, this course covers generating functions and real asymptotics and then introduces the symbolic method in the context of applications in the analysis of algorithms and basic structures such as permutations, trees, strings, words, and mappings.

c. Course Type

Blended learning

2. Learning Objectives

The objective of this course is to form a foundation of Analysis of Algorithms.

3. Learning Outcomes

On completion of the course, the student should know the basic tools of analysis of algorithms such as generating functions and the symbolic method.

4. Course Plan

Topic 1. Analysis of Algorithms.

We will start from considering historical context and motivation for the scientific study of algorithm performance. Then we consider a classic example that illustrates the key ingredients of the process: the analysis of Quicksort. The lecture concludes with a discussion of some resources that you might find useful during this course.

Topic 2. Recurrences.

We begin this topic with an overview of recurrence relations, which provides us with a direct mathematical model for the analysis of algorithms. We finish by examining the fascinating oscillatory behavior of the divide-and-conquer recurrence corresponding to the mergesort algorithm and the general "master theorem" for related recurrences.

Topic 3. Generating Functions.

Since the 17th century, scientists have been using generating functions to solve recurrences, so we continue with an overview of generating functions, emphasizing their utility in solving problems like counting the number of binary trees with N nodes.

Topic 4. Asymptotics.

Exact answers are often cumbersome, so we next consider a scientific approach to developing approximate answers that, again, mathematicians and scientists have used for centuries.

Topic 5. Analytic Combinatorics.

With a basic knowledge of recurrences, generating functions, and asymptotics, you are ready to learn and appreciate the basic features of analytic combinatorics, a systematic approach that avoids much of the detail of the classical methods that we have been considering. We introduce unlabeled and labelled combinatorial classes and motivate our basic approach to studying them, with numerous examples.

Topic 6. Trees.

The quintessential recursive structure, trees of various sorts are ubiquitous in scientific enquiry, and they arise explicitly in countless computing applications. You can find broad coverage in the textbook, but the lecture focuses on the use of analytic combinatorics to enumerate various types of trees and study parameters.

Topic 7. Permutations.

The study of sorting algorithms is the study of properties of permutations. We introduce analytic-combinatoric approaches to studying permutations in the context of this relationship.

Topic 8. Strings and Tries.

From DNA sequences to web indices, strings (sequences of characters) are ubiquitous in modern computing applications, so we use analytic combinatorics to study their basic properties and then introduce the trie, an essential and fundamental structure not found in classical combinatorics.

Topic 9. Words and Mappings.

We view strings as sets of characters or as functions from [1..N] to [1..M] to study classical occupancy problems and their application to fundamental hashing algorithms. Functions from [1..N] to [1..N] are mappings, which have an interesting and intricate structure that we can study with analytic combinatorics.

5. Reading List

All materials in https://www.coursera.org/learn/analysis-of-algorithms

Grading System

The student's final assessment consists of the assessment of the exam A_{exam} and the accumulated assessment A_{acc} obtained on the platform https://www.coursera.org/learn/analysis-of-algorithms as follows: $A_{final} = (A_{acc} + A_{exam})/2$.

6. Examination Type

Oral examination. Control elements are not blocking.

7. Methods of Instruction

The course is being studied on the online platform https://www.coursera.org/learn/analysis-of-algorithms

8. Special Equipment and Software Support (if required)

Not required