Syllabus

«Introduction into general theory of relativity»

for the educational program “Materials. Devices. Nanotechnologies”

Approved by

The Academic Council

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Author	Renat Ikhsanov, Ph.D.
Credits	3
Contact work (hours)	0
Individual work (hours)	114
Course	1
The format of the discipline	on-line

I. Purpose, Learning Outcomes and Prerequisites

Purpose

Objectives of mastering the discipline "Introduction into general theory of relativity":

- give students an introduction into general theory of relativity. The introduction is based on the consideration of many practical generic examples in various scopes of the General Relativity.

Learning Input

The study of this discipline is based on the following disciplines:

- classical mechanics
- tensor analysis
- special theory of relativity
- classical electrodynamics

Learning Outcomes

On completion of the course, the student should solve basic standard problems of the general theory of relativity.

II. Topic-wise Course Content

Topic 1. General Covariance

This lecture covers the basic notions of the Special Theory of Relativity and Minkwoskian coordinates in flat space-time. This module ends up with the derivation of the geodesic equation for a general metric from the least action principle. In this equation the Christoffel symbols are defined.

Topic 2. Covariant differential and Riemann tensor

This lecture starts with the definition of what is tensor in a general curved space-time. Then the connection, parallel transport and covariant differential are we defined. The lecture shows that for Riemannian manifolds connection coincides with the Christoffel symbols and geodesic equations acquire a clear geometric meaning. The lectures ends up with the definition of the Riemann tensor and the description of its properties.

Topic 3. Einstein-Hilbert action and Einstein equations

This lecture starts with the explanation of how one can define Einstein equations from fundamental principles such as general covariance, least action principle and the proper choice of dynamical variables. Namely, the role of the latter in the General Theory of Relativity is played by the metric tensor of space-time. Then the Einstein equations are derived from the least action principle applied to the Einstein-Hilbert action.

Topic 4. Schwarzschild solution

This lecture starts the study of the black hole type solutions. It explains how to solve the Einstein equations in the simplest settings. In this lecture one can also find perhaps the most famous solution of these equations, which is referred to as the Schwarzschild black hole. Lecture ends with the description of some properties of this Schwarzschild solution.

Topic 5. Penrose-Carter diagrams

This lecture starts with the definition of the Penrose-Carter diagram for flat space-time. On this example the lecturer explains the uses of such diagrams. Then he continues with the definition of the Kruskal-Szekeres coordinates which cover the entire black hole space-time. With the use of these coordinates he defines Penrose-Carter diagram for the Schwarzschild black hole. This diagram allows one to qualitatively understand the fundamental properties of the black hole.

Topic 6. Classical tests of General Theory of Relativity

The lecture starts with the definition of Killing vectors and integrals of motion, which allows one to provide conserving quantities for a particle motion in Schwarzschild space-time. The lecturer derives the explicit geodesic equation for this space-time. This equation provides a quantitative explanation of some basic properties of black holes.

Topic 7. Interior solution and Kerr's solution

The lecture starts with the definition of the so called perfect fluid energy-momentum tensor and with the description of its properties. This tensor is used to derive the so called interior solution of the Einstein equations, which provides a simple model of a star in the General Theory of Relativity. The lecture ends up with a brief description of the Cosmic Censorship hypothesis and of the black hole No Hair Theorem.

Topic 8. Collapse into black hole

The lecture starts with the derivation of the Oppenheimer-Snyder solution of the Einstein equations, which describes the collapse of a star into black hole. The lecturer starts derivation the Penrose-Carter diagram for this solution. This module ends up with a brief description of the origin of the Hawking radiation and of the basic properties of the black hole formation.

Topic 9. Gravitational waves

The lecture explains the important difference between energy-momentum conservation laws in the absence and in the presence of the dynamical gravity. The lecturer defines the gravitational energy-momentum pseudo-tensor. Then he continues with the linearized approximation to the Einstein equations which allows us to clarify the meaning of the pseudo-tensor. He ends up this module with the derivation of the free monochromatic gravitational waves and of their energy-momentum pseudo-tensor. These waves are solutions of the Einstein equations in the linearized approximation.

Topic 10. Gravitational radiation

The lecture shows how moving massive bodies create gravitational waves in the linearized approximation. Then it continues with the derivation of the exact shock gravitational wave solutions of the Einstein equations.

Topic 11. Friedman-Robertson-Walker cosmology

This module starts our discussion of the cosmological solutions. The lecturer defines constant curvature three-dimensional homogeneous spaces. Then he derives Friedman-Robertson-Walker cosmological solutions of the Einstein equations. He describes their properties. He ends up this module with the derivation of the vacuum homogeneous but anisotropic cosmological Kasner solution.

Topic 12. Cosmological solutions with non-zero cosmological constant

In this module the lecturer derives constant curvature de Sitter and anti de Sitter solutions of the Einstein equations with non-zero cosmological constant. He describes the geometric and causal properties of such space-times and provides their Penrose-Carter diagrams. He provides coordinate systems which cover various patches of these space-times.

III. Assessment

An exam at the end of the course involves practical work for all students enrolled in the course. Topics covered by the test embraces all course material. If a student misses the exam because of some valid reason, s/he receives «absence» grade. The exam is assessed on usual 10-point scale. Formula for the final grade G is the following:

$$G=0.6Ex+0.4C,$$

where *Ex* is the grade for the exam, *C* is the cumulated grade for the course. If the cumulated grade for the course is bigger (or equal) than 40%, then the exam is cancelled and *Ex* is set equal to *C*.

IV. Evaluation tools for student certification assessment

Current control of knowledge is not provided.

The knowledge gained in the course is evaluated on the exam.

100% scale	10-point scale
80-100 %	10
76-80 %	9
71-75 %	8
60-70 %	7
51-59 %	6
45-50 %	5
40-45 %	4
20-39 %	3
5-19 %	2
0-4 %	1

V. Reading list

5.1 Optional

1. *Robert B. Scott* A Student's Manual for A First Course in General Relativity. Cambridge, UK: Cambridge University Press, (2016)

2. *Yvonne Choquet-Bruhat* Introduction to General Relativity, Black Holes and Cosmology. Oxford University Press; 1 edition (2015)

5.2 Software

not required

5.3. Professional databases, information reference systems, Internet resources (electronic educational resources)

not required

5.4. Material and technical support of the discipline

not required