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ELECTRODYNAMICS AND WAVE PROPAGATION

On the Possibility of Existence of a Self-Consistent Solution for an Electromagnetic Field in Vacuum

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Abstract—The paper shows the possibility of existence of a self-consistent solution in a system uniting the internal and external Sestroretskii cubes; this system simulates vacuum. An expression for estimating time for which half of the energy is radiated is presented.

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INTRODUCTION

In [1], we presented the results of numerical simulation of the frequency characteristics of the external Sestroretskii cube radiating into the free space: the standing wave ratio (SWR), the loss, the attenuation, and the gain and compared them with the frequency characteristics of the internal Sestroretskii cube [2]. When comparing the scattering matrices of the external and internal Sestroretskii cubes at the frequencies corresponding to the edge length much smaller than the wavelength, we noted the following property: the absolute values of the scattering matrices of the external and internal Sestroretskii cubes differ by a value tending to zero with an increase in the wavelength. This property suggests the possibility of existence of a solution similar to the eigensolution for lossless systems [3], which may be called a self-consistent solution for a system uniting the ports of the external and internal Sestroretskii cubes for the quasi-static case. Let us consider the process of constructing such a solution for a system consisting of the external and internal Sestroretskii cubes in the quasi-static case, i.e., at the frequencies corresponding to the to the edge length of the Sestroretskii cubes much smaller than the wavelength.

It should be noted that the internal Sestroretskii cube simulates an isolated vacuum cell onto which plane waves are incident from the external space, while the external Sestroretskii cube simulates an unbounded space onto which plane waves are incident from the internal Sestroretskii cube. As a result, after uniting the external and internal Sestroretskii cubes, we obtain a system making it possible to simulate the process of energy scattering in an unbounded space, i.e., in vacuum, from the viewpoint of both internal and external electrodynamic problems.

1. THE SELF-CONSISTENT SOLUTION OF THE SYSTEM OF THE EXTERNAL AND INTERNAL SESTRORETSKII CUBES FOR THE QUASI-STATIC CASE

Let us separate a domain in an infinite space and define in it external and internal Sestroretskii cubes. Figure 1 shows vectors of the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields and the Poynting vector $\vec{S}$ (the energy flux density of the electromagnetic field) of the incident and reflected waves for the external Sestroretskii cube in the first cycle. Solid lines represent the vectors of electric, $\vec{E}$, and magnetic, $\vec{H}$, fields and the Poynting vector $\vec{S}$ of the wave incident onto the external Sestroretskii cube. Suppose that the amplitude of these waves is unity. Then, as follows from the results of [1], the amplitudes of the reflected waves will is $\sqrt{D}/2$ for the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields and $D/4$ for the Poynting vector $\vec{S}$, where the value of D in the quasi-static case is very close to unity. The vectors of the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields and the Poynting vector $\vec{S}$ of the reflected waves in Fig. 1 are represented by the dashed lines.

In the quasi-static case, the quantity D, which characterizes the radiation loss of the external Sestroretskii cube, can be written in the form

$$D = 1 - x, \tag{1}$$

where x is the fraction of energy radiated by the external Sestroretskii cube into the free space for one cycle, which, in the quasi-static case, tends to zero.

The waves that were reflected for the external Sestroretskii cube (Fig. 1, dashed lines), in the first cycle, are incident for the internal Sestroretskii cube.

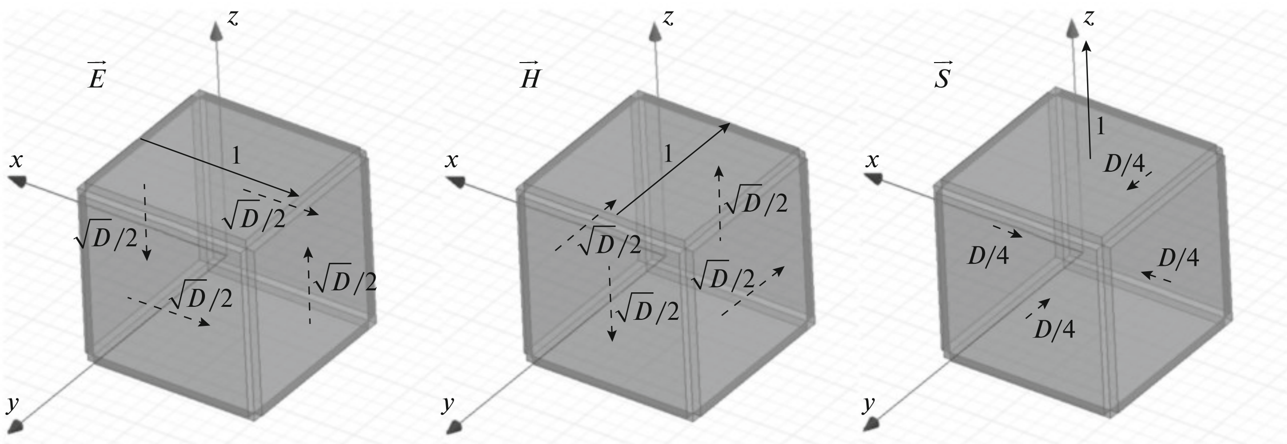

Fig. 1. Vectors of the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields and the Poynting vector $\vec{S}$ of the incident and reflected waves for the external Sestroretskii cube.

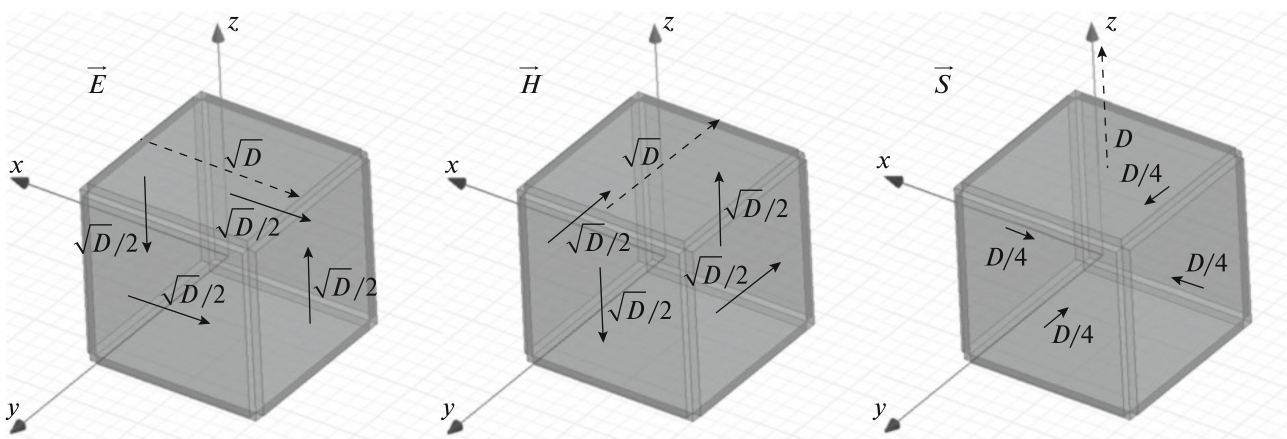

Fig. 2. Vectors of the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields and the Poynting vector $\vec{S}$ of the incident and reflected waves of the first cycle for the internal Sestroretskii cube.

Figure 2 shows the vectors of the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields and the Poynting vector $\vec{S}$ of the incident (solid lines) and reflected (dashed lines) waves of the first cycle for the internal Sestroretskii cube. The amplitudes of the four incident waves are $\sqrt{D}/2$ for the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields and $D/4$ for the Poynting vector $\vec{S}$. As follows from the results presented in [2], the amplitude of the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields of the reflected wave is $\sqrt{D}$, and the amplitude of the energy flux density (the Poynting vector) $\vec{S}$ of the reflected wave is D.

Thus, we have described the first cycle of the energy transfer through the system, which is a recomposition (combination) of the external and internal Sestroretskii cubes and simulates the energy scattering in the free space (vacuum). The time spent for the first cycles is

$$\tau = \tau_{\text{in}} + \tau_{\text{ext}}, \tag{2}$$

where τ_{in} is the delay of the electromagnetic wave passing through the internal Sestroretskii cube [2] and τ_{ext} is the delay of the electromagnetic wave passing through the external Sestroretskii cube [1].

Considering in a similar manner the second cycle of the energy transfer through the system of the external and internal Sestroretskii cubes, we obtain the amplitude of the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields of the reflected wave:

$$D = (\sqrt{D})^2. \tag{3}$$

The amplitude of the energy flux density (the Poynting vector) $\vec{S}$ for the second cycle is D^2.

Repeating this procedure of energy transfer n times, we obtain the amplitude of the electric, $\vec{E}$, and magnetic, $\vec{H}$, fields of the reflected wave of $(\sqrt{D})^n$, and the amplitude of the energy flux density (the Poynting vector) $\vec{S}$ for the nth cycle equal to D^n.

Thus, we may conclude that the self-consistent solution in the system under consideration exists.

The self-consistent solution is understood in the sense that, in each cycle, the energy of the process under consideration decreases by the factor of D. This reduction in energy is connected with the loss for the radiation to the free space. In the quasi-static case, this loss is small (tends to zero with decreasing sizes of the system as compared to the wavelength).

2. ESTIMATING THE TIME FOR WHICH HALF OF THE ENERGY IS RADIATED FOR THE SELF-CONSITENT SOLUTION

For estimating the time $T_{1/2}$ for which half of the energy is radiated, we will use the condition

$$D^n = 1/2, \tag{4}$$

$$T_{1/2} = n(\tau_{\text{in}} + \tau_{\text{ext}}). \tag{5}$$

Leaning upon relationships (4) and (5), we can write the following expression:

$$T_{1/2} = \frac{-\ln 2}{\ln D}(\tau_{\text{in}} + \tau_{\text{ext}}). \tag{6}$$

Since, as $x \to 0$, we have [4]

$$\ln(1 - x) \approx -x, \tag{7}$$

taking into account (1), we can rewrite expression (6) in the form

$$T_{1/2} \approx \frac{\ln 2}{x}(\tau_{\text{in}} + \tau_{\text{ext}}). \tag{8}$$

Thus, for estimating the time $T_{1/2}$, for which half of the energy is radiated, we need to know the following quantities:

(i) the delay of passage of the electromagnetic wave through the internal Sestroretskii cube, τ_{in};

(ii) the delay of passage of the electromagnetic wave through the external Sestroretskii cube, τ_{ext};

(iii) the fraction of energy radiated to the free space by the external Sestroretskii cube for one cycle, x.

In the quasi-static case, the delay of an electromagnetic wave passing through the internal Sestroretskii cube, τ_{in}, can be estimated, using the results of [2]:

$$\tau_{\text{in}} \approx \frac{a}{2c}, \tag{9}$$

where a is the edge length of the internal Sestroretskii cube and c is the speed of light in vacuum.

The delay of an electromagnetic wave passing through the external Sestroretskii cube, τ_{ext}, in the quasi-static case, can be estimated, using the results of [1]:

$$\tau_{\text{ext}} \approx \frac{a}{0.852c}. \tag{10}$$

In the quasi-static case, x, the fraction of energy radiated to the free space for one cycle by the external Sestroretskii cube with an edge length $a = 1$ mm can be estimated using the following relationship [1]:

$$K_{\text{g}}(\text{dB}) = k + 20\log f, \tag{11}$$

where

$$k = 10\log\left(\frac{4\pi}{9}10^{-4}\right) = -38.55 \text{ dB} = K_{\text{g}} \text{ [dB]}, \tag{12}$$

where λ is the wavelength in the free space and f is the frequency in gigahertzs.

At the same time, the gain factor K_{g} can be written in the form [5]

$$K_{\text{g}} = \eta \mathrm{D}, \tag{13}$$

where D is the directivity of the external Sestroretskii cube and η is the efficiency of the external Sestroretskii cube.

In the quasi-static case, the directivity of the external Sestroretskii cube equals the directivity of a Huygens element, i.e., 3 [5, 6]:

$$\mathrm{D} = 3. \tag{14}$$

The efficiency of the external Sestroretskii cube is determined as

$$\eta = \frac{P_{\text{rad}}}{P_{\text{in}}}, \tag{15}$$

where P_{rad} is the energy radiated to the free space by the external Sestroretskii cube and P_{in} is the power arriving at the input of the external Sestroretskii cube.

Since the amplitude of the energy flux density (the Poynting vector) $\vec{S}$ in the first cycle was taken equal to 1, for the first cycle, we have $P_{\text{in}} = 1$ W and $P_{\text{rad}} = x$, and the efficiency for the first and subsequent cycles is

$$\eta = x. \tag{16}$$

Taking into account relationships (14)–(16), we can write expression (12) as

$$K_{\text{g}} = 3x. \tag{17}$$

Substituting expression (17) into (11) and taking into account that K_{g} [dB] $= 10\log K_{\text{g}}$, we obtain the following relationship for the fraction of energy radiated to the free space for one cycle by the external Sestroretskii cube with an edge length a of 1 mm:

$$x = \frac{4\pi \times 10^{-4}}{27} 10^{2\log f}. \quad (18)$$

Relationship (18) can be rewritten as follows:

$$x = \frac{4\pi \times 10^{-4}}{27} f^2. \quad (19)$$

Expression (19) can also be applied for an arbitrary edge length of the external Sestroretskii cube:

$$x = x_0 \left(\frac{f}{f_0}\right)^2, \quad (20)$$

where x_0 is the fraction of energy radiated by the external Sestroretskii cube at the frequency f_0. In particular, for the external Sestroretskii cube with an edge length a of 1 mm, we have

$$x_0 = \frac{4\pi \times 10^{-4}}{27}, \quad f_0 = 1\ \mathrm{GHz}, \quad a = \lambda_0/300, \quad (21)$$

where λ_0 is the wavelength in the free space for the frequency f_0.

Since it is true that $\frac{f}{f_0} = \frac{\lambda_0}{\lambda}$ [5], taking into account special case (21), we can rewrite expression (20) in the following form:

$$x = \frac{4\pi \times 10^{-4}}{27} \left(\frac{300a}{\lambda}\right)^2. \quad (22)$$

Thus, substituting into relationship (8) expressions (9), (10), and (22), we obtain the following expression for estimating the time $T_{1/2}$ for which half of the energy is radiated:

$$T_{1/2} \approx 1.674 \ln 2 \frac{3}{4\pi} \frac{\lambda^2}{ca}. \quad (23)$$

After uniting all number coefficients, relationship (23) for estimating the time $T_{1/2}$ for which half of the energy is radiated can be written in the form

$$T_{1/2} \approx 0.277 \frac{\lambda^2}{ca}. \quad (24)$$

It should be emphasized once more that relationship (24) is an estimate of the time $T_{1/2}$ for which half of the energy is radiated in the quasi-static case, when the ratio of the edge lengths a of the external and internal Sestroretskii cubes to the wavelength λ in the free space tends to zero.

Relationship (24) can be interpreted as follows: a wave train [7] of length λ in vacuum can be represented as a sum of local self-consistent solutions circulating in cubes with edge lengths a, and, on a finer decomposition into self-consistent solutions, they are more stable, because the time $T_{1/2}$ for which half of the energy is radiated increases.

For example, let us present a numerical estimate of the time $T_{1/2}$ for which half of the energy is radiated when the edge length of the Sestroretskii cube is 1 mm and the wavelength in the free space is 1 m:

$$T_{1/2} \approx 0.277 \frac{1^2}{3 \times 10^8 \times 10^{-3}} \approx 92.33 \times 10^{-8}\ [\mathrm{s}]. \quad (25)$$

As is evident from estimate (25), the time $T_{1/2}$ for which half of the energy is radiated in this case is rather small.

CONCLUSIONS

It has been shown that the fact that the absolute values of the scattering matrices of the external and internal Sestroretskii cubes in the quasi-static case are equal makes it possible to form a self-consistent solution in vacuum, which is simulated by uniting the external and internal Sestroretskii cubes. The expression obtained for the time $T_{1/2}$ for which half of the energy is radiated has confirmed that this time is very small, which is explained by a very high speed of light in vacuum. To increase this time, it is necessary to increase the wavelength with a simultaneous reduction in the spatial scale of energy localization.

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