ISSN 1064-2269, Journal of Communications Technology and Electronics, 2015, Vol. 60, No. 7, pp. 737–746. © Pleiades Publishing, Inc., 2015.
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ELECTRODYNAMICS
AND WAVE PROPAGATION

Numerical Electromagnetic Investigation of Patterns of the External Problem for the Huygens Element That Is the External Huygens Cube

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Received June 25, 2014

Abstract—Patterns of the external Huygens cube with the edge that is much less than a wavelength, a half of a wavelength, and a wavelength are given. The dependence of its pattern shape on the boundary condition on the second input is shown. The paradox of the external Huygens cube is noted. The influence of the boundary condition on the unexcited input on its pattern is considered.

DOI: 10.1134/S1064226915070086

INTRODUCTION

In paper [1], the frequency characteristics—the standing-wave ratio (SWR), loss, attenuation, and amplification—of the Huygens element radiating into the open space are numerically simulated. They are compared with the frequency characteristics of the internal Huygens cube. The dependence of the frequency characteristics of the external Huygens cube on the dimensions of the air cube is investigated. To estimate the limit characteristics that can be reached for real radiators, it is also necessary to analyze the directivity characteristics of the external Huygens cube, their frequency dependence for different variants of excitation of the inputs. Recall that, when a Huygens element is considered, a hypothetic radiator is kept in mind. The latter corresponds to an infinitely small element of the wave front of a linearly polarized plane electromagnetic wave. The Huygens element has been introduced into the antenna theory owing to the application of the equivalent surface (electric and magnetic) currents, which is an analog of the Huygens principle known from the optics [2–4].

The principle of the equivalent surface currents is as follows (Fig. 1). The sources of the electromagnetic wave that are in region I are surrounded by closed surface *C* and the electromagnetic field in region II is calculated taking into account the values of the equivalent surface electric and magnetic currents on closed surface *C*. Thus, surface *C* has the inner and outer boundaries. Its inner side is approached by the electromagnetic wave going from electromagnetic radiation sources located in region I, and the electromagnetic waves are radiated into the open space from the outer boundary. Just because of that, the Huygens element must have two inputs. One of these is situated on the side of the region I, and the other one is situated on the side of the region II. The boundary conditions for the Huygens element, which is a two-sided surface, should be both on the two sides and on the boundary of the Huygens element itself. So, we must have two sides where the short-circuit (SC) boundary conditions are assigned, two sides where the idler-circuit (IC) boundary conditions are assigned, and two sides where the electric and magnetic field ($\vec{E}$ and $\vec{H}$) intensities and vectors $\vec{S}$ of the power flux density (see Fig. 1) [2] are assigned. The infinitely small cube modeling the surface of the front of the linearly polarized plane electromagnetic wave that is formed during radiation into the open space was called an ideal external Huygens cube (see Fig. 1) [1]. Let us investigate the directivity characteristics of the outer Huygens cube and their frequency dependences for various variants of their input excitation with the help of universal electromagnetic program ANSYS HFSS v.15 [5]. In contrast to the ideal Huygens cube, the object modeled by us with electromagnetic program ANSYS HFSS v.15 has finite dimensions. This object was called the external Huygens cube [1].

1. INVESTIGATION OF THE EXTERNAL HUYGENS CUBE

Consider cube *A* of the dimensions 1 × 1 × 1 mm (Fig. 2) which is filled with a metal and located in the open space. Let us impose a boundary condition on faces *1* and *2* (Fig. 3) of the cube for tangential component E_τ of the electric field. This component equals

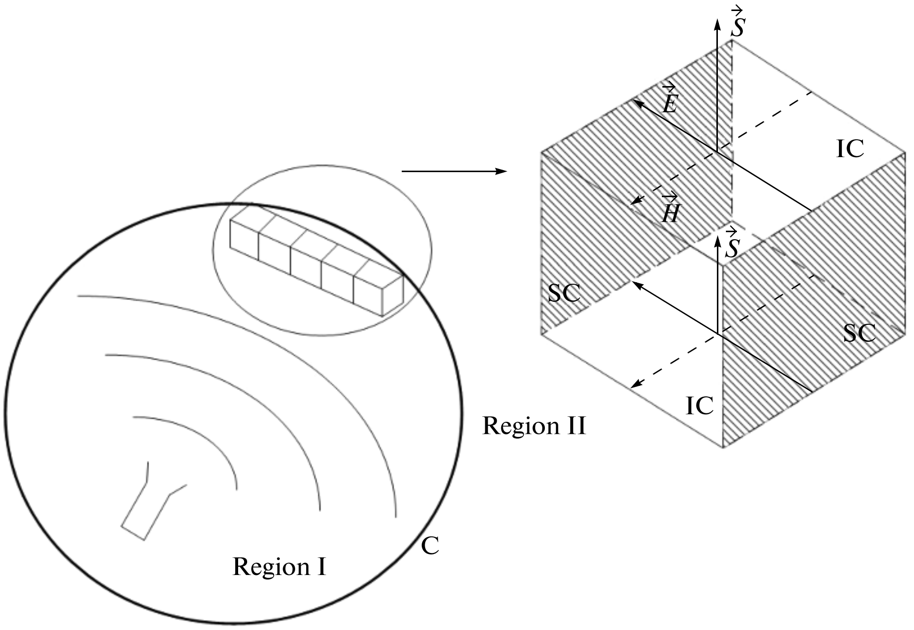

Fig. 1. Principle of equivalent surface currents and the principle of the external Huygens cube.

zero, which corresponds to a metal wall. We assume faces *1* and *2* to be SC walls [2–4].

Let us impose a boundary condition for tangential component H_τ of the magnetic field on faces *3* and *4* of cube *A* (Fig. 4). This component equals zero, which corresponds to a magnetic wall. We assume faces *3* and *4* to be IC walls [2–4].

On faces *5* and *6* of cube *A* (Figs. 5 and 6), we impose the boundary condition for excitation and matching of plane waves [2–4]. We denote face *5* as input *1* (Fig. 5) and face *6* as input *2* (Fig. 6). The polarizations of electric and magnetic fields $\vec{E}$ and $\vec{H}$ and the directions of Umov–Poynting vectors $\vec{S}$ [6–9] of these plane waves are shown in Figs. 7–9.

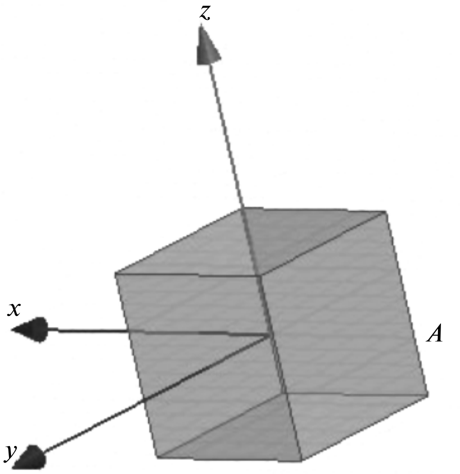

Fig. 2. Geometry of cube *A* filled with a metal.

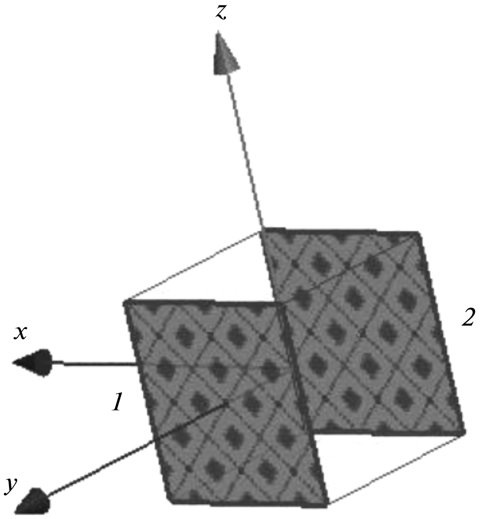

Fig. 3. Faces *1* and *2* of cube *A* where the SC boundary condition is specified.

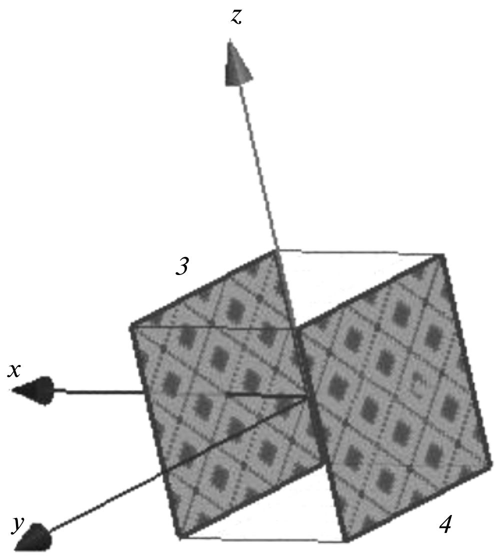

Fig. 4. Faces *3* and *4* of cube *A* where the IC boundary condition is specified.

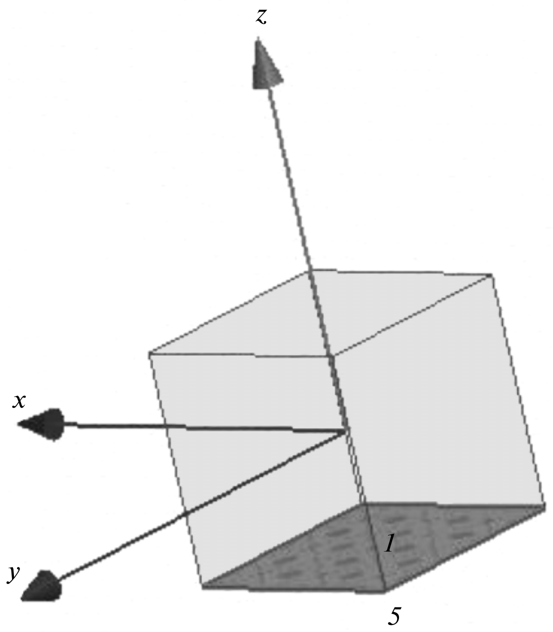

Fig. 5. Face *5* of cube *A* where the excitation and matching boundary condition is specified.

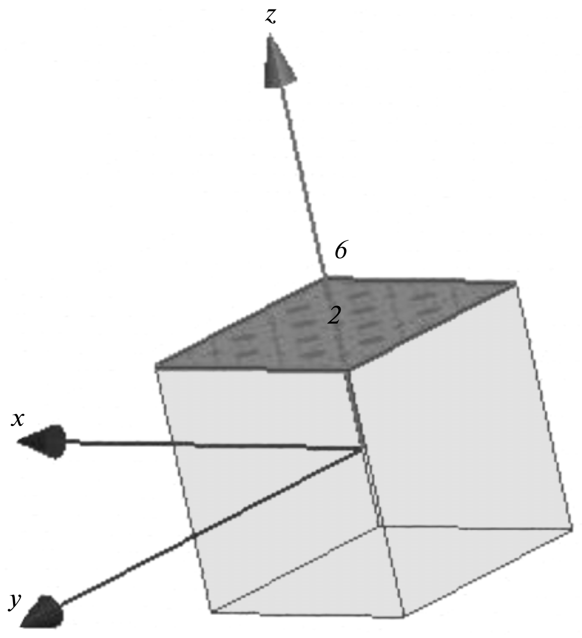

Fig. 6. Face *6* of cube *A* where the excitation and matching boundary condition is specified.

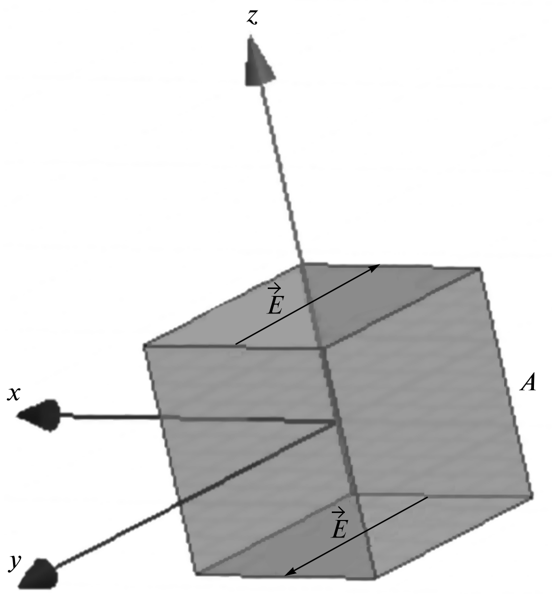

Fig. 7. Directions of the vectors of electric field intensities $\vec{E}$ for the incident waves on faces *5* and *6* of cube *A*.

To analyze the exterior problems, it is convenient to assign the densities of the surface electric and magnetic currents (Figs. 10–12) [10] rather than the values of electric and magnetic field intensities:

$$\vec{J}^{\,\mathrm{e}} = \vec{n} \times \vec{H}, \tag{1}$$

$$\vec{J}^{\,\mathrm{m}} = -\vec{n} \times \vec{E}, \tag{2}$$

where $\vec{J}^{\,\mathrm{e}}$ is the vector of the surface electric current density, $\vec{J}^{\,\mathrm{m}}$ is the vector of the surface magnetic current density, $\vec{H}$ is the vector of the magnetic field intensity, $\vec{E}$ is the vector of the electric field intensity, and $\vec{n}$ is the vector of the outward normal to the surface.

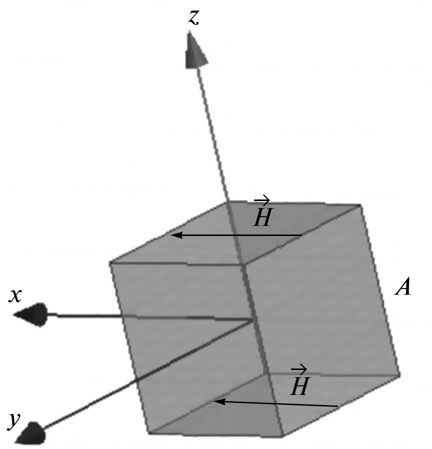

Fig. 8. Directions of the vectors of magnetic field intensities $\vec{H}$ for the incident waves on faces *5* and *6* of cube *A*.

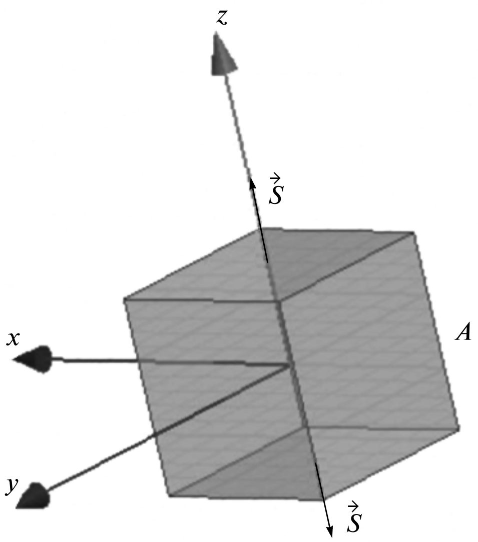

Fig. 9. Directions of the vectors of energy flow densities $\vec{S}$ of the (Umov–Poynting) electromagnetic fields for the waves incident on faces *5* and *6* of cube *A*.

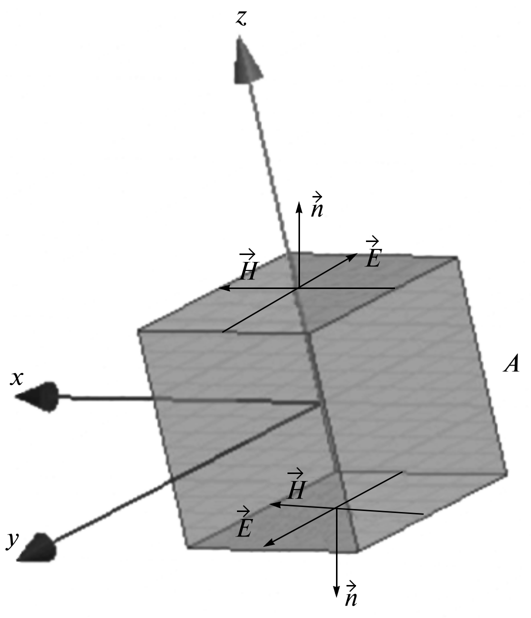

Fig. 10. Directions of external normal vectors $\vec{n}$ for faces *5* and *6* of cube *A*.

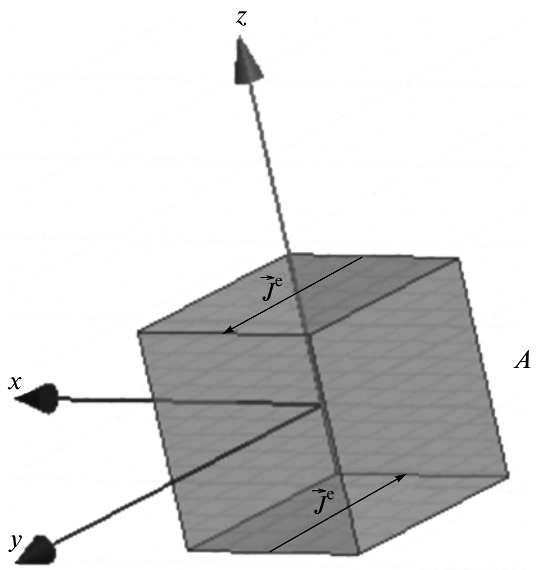

Fig. 11. Directions of the vectors of equivalent surface electric current densities $\vec{J}^{\mathrm{e}}$ for magnetic field intensities $\vec{H}$ for waves incident on faces *5* and *6* of cube *A*.

To consider the external electromagnetic problem, we place metal cube *A* with the boundary conditions specified above into air cube *B* (Fig. 13) on whose faces the radiation condition "Radiation" [5] is specified. External air cube *B* has the dimensions 5 × 5 × 5 mm and is filled with vacuum.

Figure 14 shows the faces of cube *B* where the absorption boundary conditions (Radiation) are specified.

We model the problem on scattering of electromagnetic waves in an external Huygens cube using 3D electromagnetic program complex ANSYS HFSS v.15 [5].

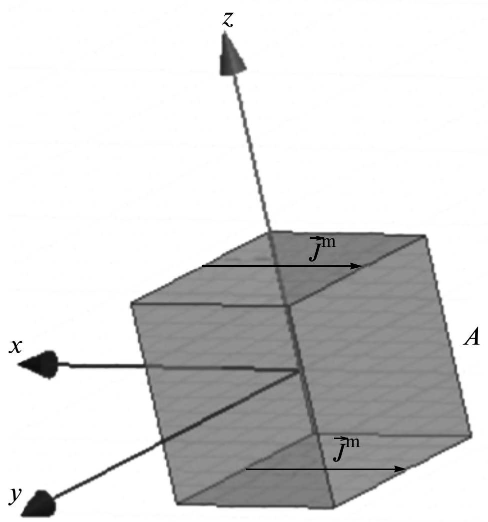

Fig. 12. Directions of the vectors of equivalent surface magnetic current densities $\vec{J}^{\,m}$ for electric field intensities $\vec{E}$ for waves incident on faces *5* and *6* of cube *A*.

2. RESULTS OF MODELING OF EXTERNAL HUYGENS CUBE PATTERNS

The calculation is performed in the frequency range 1–300 GHz with the step 5 GHz. The convergence for the absolute values of the elements of the scattering matrix is DeltaS = 0.02. The general number of tetrahedrons is 17266, the dimension of the obtained matrix is 109330, and 434 MB of RAM is used. The general time of calculation is 15 min 29 s on a server with 128 GB of RAM and the two Intel Xeon E5-2690 processors having the frequency 2.90 GHz.

We present the characteristics of the 3D patterns for inputs *1* and *2* of the external Huygens cube at the frequency of 1 GHz.

When input *1* of the external Huygens cube at the frequency of 1 GHz (Fig. 15) is excited, the pattern is a cardioid whose radiation maximum is opposite axis *z*. However, the maximum level is –62 dB. The main energy at the frequency of 1 GHz is absorbed at input *2* [2]. This effect can be called the paradox of the external Huygens cube. It consists in the fact that, for the quasi-static case, the directions of the pattern maximum and the main energy flow are opposite. Really, there is no paradox here, because the pattern is the characteristic of far-zone electromagnetic field radiation and the energy is transferred by near-zone fields at the 1 GHz for the given dimension 1 × 1 × 1 mm of the external Huygens cube.

Consider the pattern for the cophased equal-amplitude excitation of inputs *1* and *2*. The pattern for this case has the form shown in Fig. 16. For this case of excitation, the pattern is a toroid with the axis oriented along the *x* axis. This pattern can be expected, because

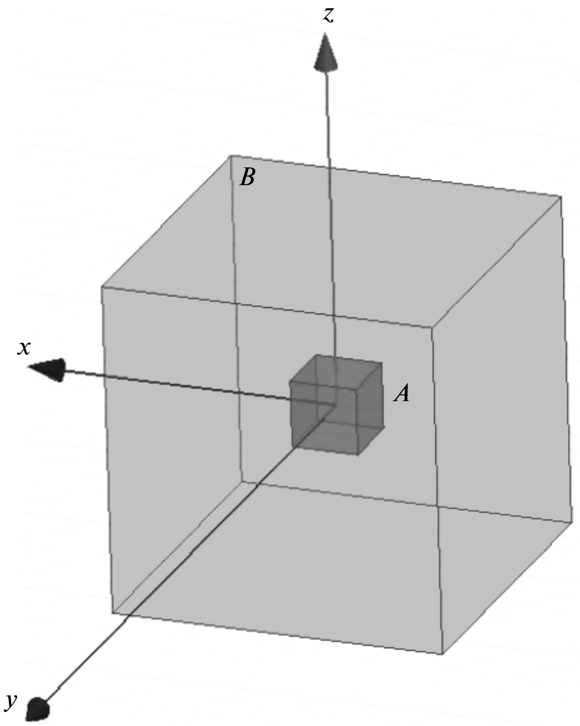

Fig. 13. Geometry of the problem for the external Huygens cube in the ANSYS HFSS v.15 program.

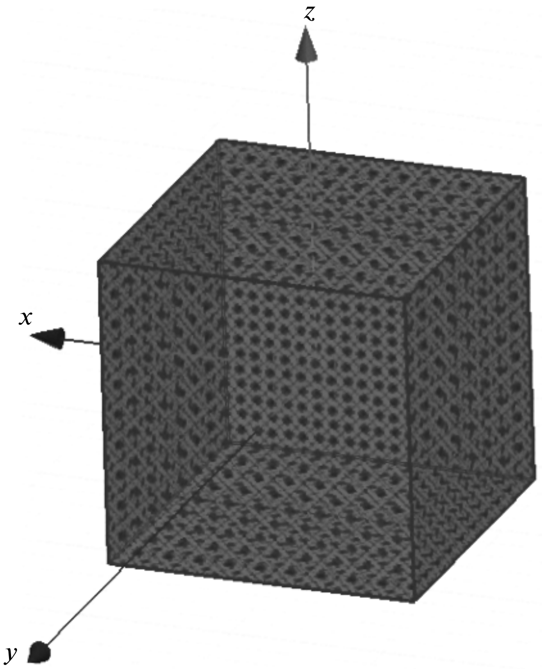

Fig. 14. (Radiation) Boundary absorption conditions on the faces of air cube *B*.

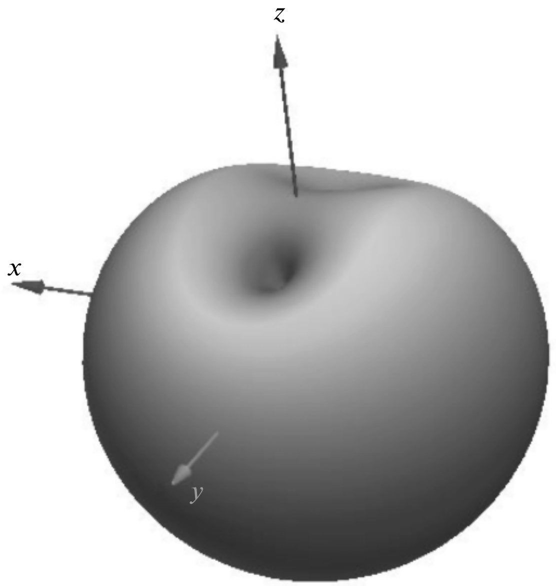

Fig. 15. Three-dimensional pattern observed when input *1* of the external Huygens cube is excited at the frequency of 1 GHz.

the equivalent surface electric and magnetic currents for such excitation look so as it is shown in Fig. 17. From here, it is seen that the vectors of equivalent surface electric currents are contrariwise directed. Since they have equal amplitudes and are situated at 1/300 of the wavelength at the frequency of 1 GHz, they practically absolutely compensate for each other. However, the vectors of surface magnetic current densities are identically directed. They also have identical amplitudes and are situated at the distance of 1/300 of the wavelength at the frequency of 1 GHz. Practically the same excitation is equivalent to the presence of a surface magnetic current whose density vector has an amplitude that is twice the amplitude at input *1* or *2*. This excitation is equivalent to the excitation of the magnetic dipole oriented against the *x* axis.

Now, let us consider the pattern in the case of the antiphase equal-amplitude excitation of inputs *1* and *2*. The pattern in this case looks like it is shown in Fig. 18. It is a toroid with the axis directed along the *y* axis. This is an expected view of the pattern, since the equivalent surface electric and magnetic currents for such an excitation look like it is shown in Fig. 19. From here it is seen that the vectors of the equivalent surface magnetic currents are oppositely directed. Since they have equal amplitudes and are located at the distance of 1/300 of the wavelength at the frequency of 1 GHz, they practically absolutely compensate for each other. However, the vectors of the surface electric currents are codirectional. They also have identical amplitudes and are situated at the distance of 1/300 of the wavelength at the frequency of 1 GHz. Practically the same excitation is equivalent to the surface electric current whose density vector has an amplitude that is twice the amplitudes at input *1* and *2*. This excitation is equivalent to the excitation of the electric dipole oriented along the *y* axis.

Usually the Huygens element is represented as the superposition of the electric and magnetic dipoles.

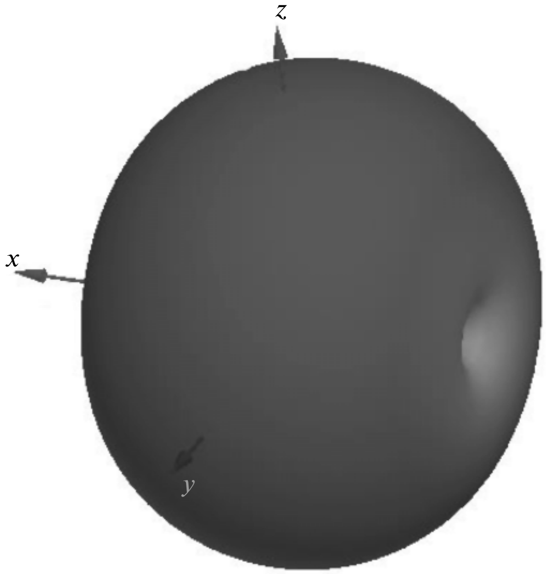

Fig. 16. Three-dimensional pattern observed when inputs *1* and *2* of the external Huygens cube are excited at the frequency of 1 GHz in the cophased equal-amplitude regime.

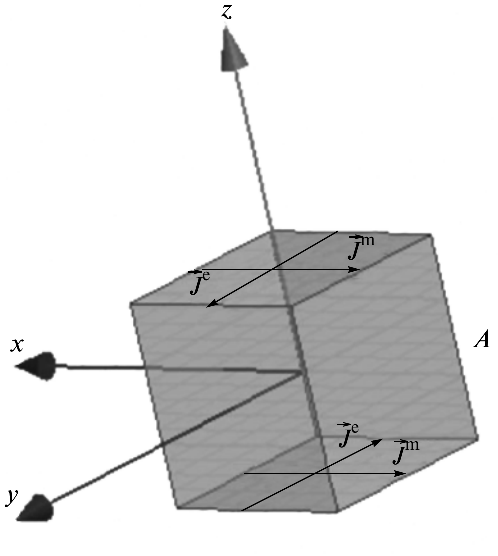

Fig. 17. Directions of the vectors of surface electric ($\vec{J}^{e}$) and magnetic ($\vec{J}^{m}$) current densities in the case of cophased excitation for waves incident on inputs *1* and *2* of cube *A*.

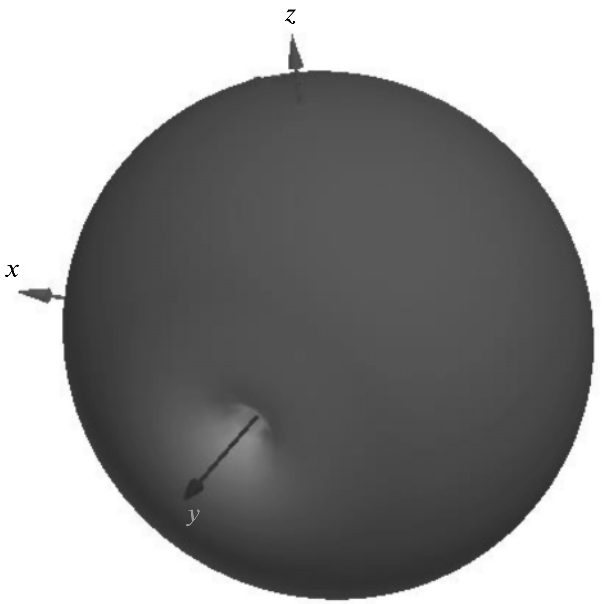

Fig. 18. Three-dimensional pattern for antiphased equal-amplitude excitation of inputs *1* and *2* of the external Huygens cube at the frequency of 1 GHz.

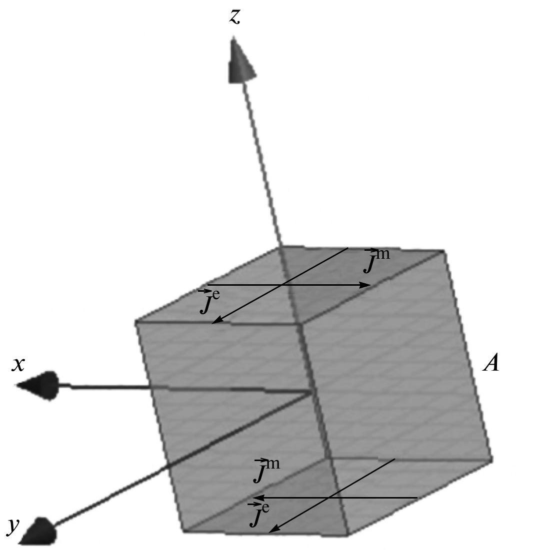

Fig. 19. Directions of the vectors of surface electric ($\vec{J}^{e}$) and magnetic ($\vec{J}^{m}$) current densities in the case of antiphased excitation for waves incident on inputs *1* and *2* of cube *A*.

The above patterns are the example of the representation of the electric and magnetic dipoles as the superposition of the excitation of the inputs of the external Huygens cube. It is clear from this reasoning why the reflected signal is sufficiently large and the radiated energy portion is insignificant in the case of small electric dimensions of the dipoles. In the case of the cophased (antiphased) excitation, the reflected energy for inputs *1* and *2* is the wave really transmitted from inputs *2* and *1*, respectively.

Note that it could be expected that the gain should be identical for the cophased and antiphased simultaneous excitation of two inputs. However, it is different and equals –64 dB (see Fig. 18) and –66 dB (see Fig. 16) for antiphased and cophased excitations, respectively.

We give the characteristics of the patterns for different variants of excitation of the inputs of the external Huygens cube at a frequency of 150 GHz, which corresponds to the dimension of the edge of the external Huygens cube equal to half the wavelength.

When input *1* of the external Huygens cube is excited at the frequency of 150 GHz, the pattern has the form shown in Fig. 20. The maximum of radiation is oriented against the z axis. The maximum level is 6.44 dB. Then, the energy absorbed at input *2* is –14.5 dB [1] of the energy supplied to input *1*. In contrast to the frequency of 1 GHz, the radiation energy dissipation is –0.48 dB [1].

Consider the view of the pattern in the case of the cophased equal-amplitude excitation of inputs *1* and *2*. The pattern has the form shown in Fig. 21. For this case of excitation, the pattern is a toroid with the symmetry axis oriented along the x axis. This is an expected form of the pattern, because the equivalent surface electric and magnetic currents for such an

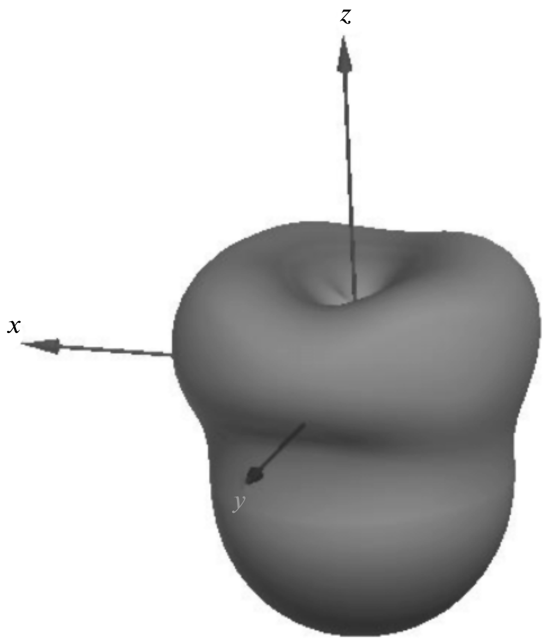

Fig. 20. Three-dimensional pattern observed when input *1* of the external Huygens cube is excited at the frequency of 150 GHz.

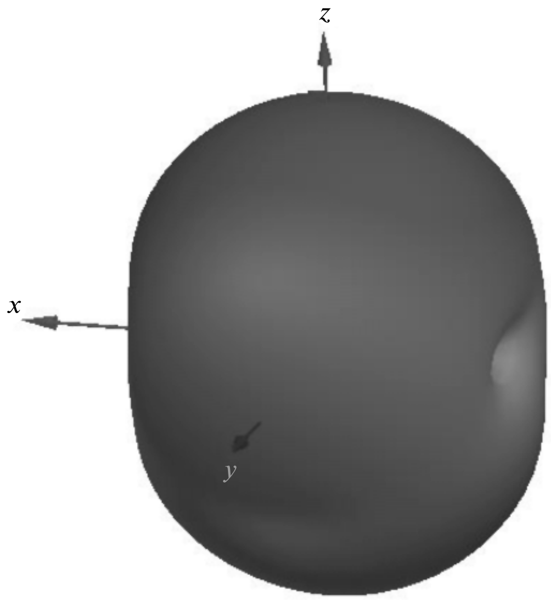

Fig. 21. Three-dimensional pattern observed when inputs *1* and *2* of the external Huygens cube are excited at the frequency of 150 GHz in the cophased equal-amplitude regime.

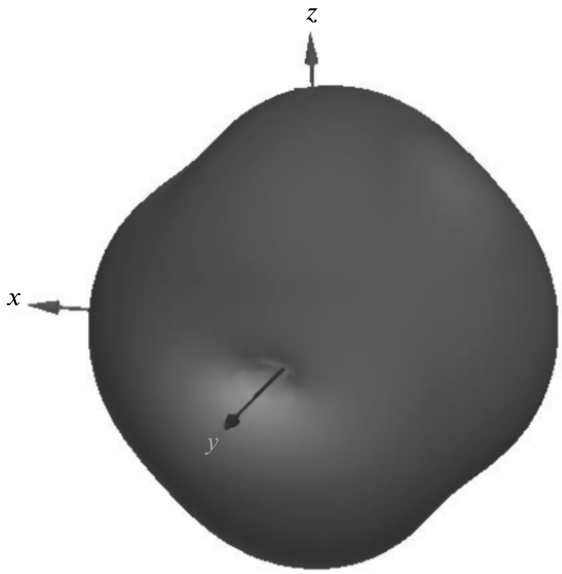

Fig. 22. Three-dimensional pattern observed when inputs *1* and *2* of the external Huygens cube are excited at the frequency of 150 GHz in the antiphased equal-amplitude regime.

excitation look as it is shown in Fig. 17. However, in contrast to the frequency of 1 GHz where the edge of the external Huygens cube is 1/300 of the wavelength, the edge of the external Huygens cube is now a half of the wavelength. Therefore, the pattern differs from a toroid. It is seen from Fig. 17 that the vectors of the equivalent electric currents are oppositely directed. For the dimension equal to the half of the wavelength, their dimensions and the shape of the surface where they are specified cannot be neglected. The pattern is the same as shown in Fig. 21. The pattern axis is oriented along the *x* axis in the same manner as at the frequency of 1 GHz.

Now, let us see the view of the pattern in the case of the antiphased equal-amplitude excitation of inputs *1* and *2*. The pattern looks like the plot given in Fig. 22. It now differs from a toroid as it is for the case of cophased excitation. It is seen from Fig. 19 that the vectors of the equivalent surface magnetic currents are now oppositely directed. It should be taken into account now that the dimensions and shape of the surface of the equivalent currents cannot be neglected for the dimension equal to a half wavelength. The pattern will be the same as shown in Fig. 22. The axis of the pattern is oriented along the *y* axis in the same way as it is at the frequency of 1 GHz.

Note that the level of the gain is different when two inputs are exited simultaneously in the cases of cophased and antiphase excitation. It is 2.59 dB in the case of the antiphased excitation (see Fig. 22) and 4.46 dB in the case of the cophased excitation (see Fig. 21).

We give also the characteristics of the pattern for different variants of excitation of the inputs of the external Huygens cube at the frequency of 300 GHz, which corresponds to the dimension of the edge of the external Huygens cube equal to the wavelength.

When input *1* of the external Huygens cube is excited at the frequency of 300 GHz, the pattern has the form shown in Fig. 23. The maximum of radiation is oriented against the *z* axis. The maximum level is 11.36 dB. The portion of the energy absorbed at input *2* at a frequency of 300 GHz is –41.79 dB [1] of the energy supplied to input *1*. In contrast to the frequency of 1 GHz, the radiation energy loss is –0.1 dB [1].

Consider the pattern in the case of cophased equal-amplitude excitation of inputs *1* and *2*. The pattern in this case looks so as it is shown in Fig. 24. The axis of the pattern is oriented along the *x* axis. The equivalent surface electric and magnetic currents in the case of such an excitation look as it is shown in Fig. 17. However, in contrast to the cases considered above, the edge of the external Huygens cube is a wavelength at the frequency of 300 GHz. The pattern takes the form shown in Fig. 24.

Now, let us see the pattern in the case of antiphased equal-amplitude excitation of inputs *1* and *2*. The pattern in this case looks like it is shown in Fig. 25. The equivalent surface electric and magnetic currents for such excitation look like it is shown in Fig. 19. The pattern is the same as shown in Fig. 25.

The levels of the gain in the case of simultaneous cophased and antiphased excitation of the two inputs should be expected to be the same. However, they are

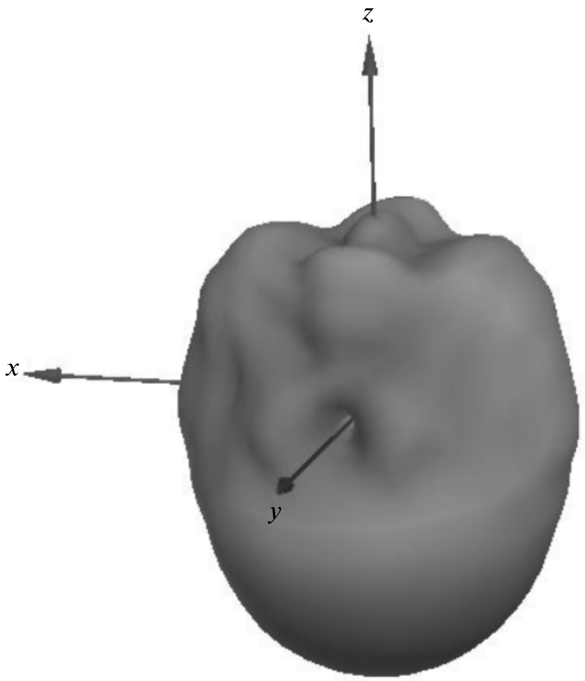

Fig. 23. Three-dimensional pattern observed when input *1* of the external Huygens cube is excited at the frequency of 300 GHz.

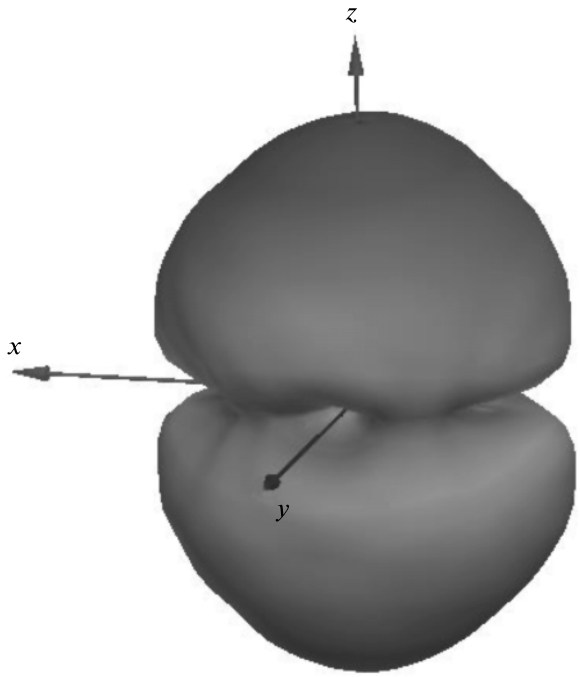

Fig. 24. Three-dimensional pattern observed when inputs *1* and *2* of the external Huygens cube are excited at the frequency of 300 GHz in the cophased equal-amplitude regime.

different. The gain level is 8.59 dB for the antiphased excitation (see Fig. 25) and 6.2 dB for the cophased excitation (see Fig. 24).

In contrast to the previous cases, the axis of the pattern for the cophased and antiphased excitations is now oriented along the z axis.

3. THE INFLUENCE OF THE BOUNDARY CONDITION AT INPUT 2 ON THE PATTERN WHEN INPUT 1 IS EXCITED

At the frequency of 1 GHz, the boundary condition at input *2* substantially affects the pattern of the external Huygens cube, because almost all the energy from input *1* comes to input *2* [1]. In the case of the boundary condition of the IC at input *2* [1], the pattern of the external Huygens cube looks as if inputs *1* and *2* are simultaneously excited in the cophased regime (see Fig. 16), i.e., the pattern coincides with the pattern of a magnetic dipole. In the case of the SC condition [1], the pattern of the external Huygens cube looks like the pattern of it in the case of the simultaneous antiphased excitation of inputs *1* and *2* (see Fig. 18), which coincides with the pattern of the electric dipole.

In contrast to the frequency of 1 GHz, the boundary conditions at input *2* at the frequencies of 150 and 300 GHz do not substantially affect the patterns, because the energy fraction arriving at input *2* is –14.5 and –41.79 dB, respectively.

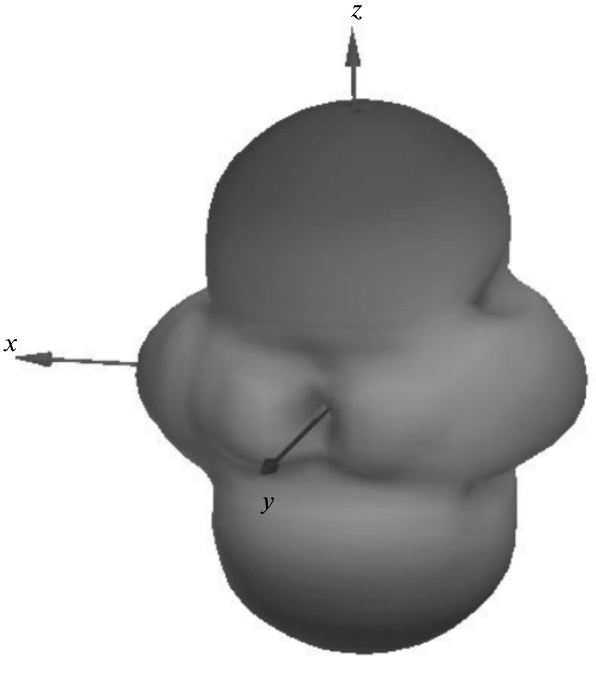

Fig. 25. Three-dimensional pattern observed when inputs *1* and *2* of the external Huygens cube are excited at the frequency of 300 GHz in the antiphased equal-amplitude regime.

CONCLUSIONS

The patterns of the external Huygens cube with different dimensions of the edge have been given. It has been shown how the shape of the pattern of the external Huygens cube depends on the boundary condition at input *2* when the dimension of the edge is substantially smaller than the wavelength. The paradox of the external Huygens cube has been noted. This paradox is in the fact that, when the dimension of the edge of the external Huygens cube is 1 mm, the direction of the maximum pattern and the direction of the main energy flow motion are opposite. The effect of the boundary condition at the input that is not exited on the pattern of the external Huygens cube has been considered.

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Translated by I. Efimova