Syllabus for Discrete Mathematics 2, Fall 2019-20, HSE and University of London Double Degree Program in Data Science and Business Analytics' 2

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1. **Course Description**
   a. **Title of Course**: Discrete Mathematics 2
   b. **Pre-requisites**: Discrete Mathematics 1, basic linear algebra, matrices and vectors manipulation, systems of linear equations, local and global minima and maxima within elementary calculus.
   c. Course Type: compulsory
   d. **Abstract:** The objective of this course is to make students familiar with basic models of linear and integer linear programming (discrete optimization models) and enumeration (branch-and-bound type) algorithms for solving a general mixed integer linear programming problem as well as its special cases (minimum spanning tree and its variations, linear assignment, and traveling salesman problem). A special attention of this course will be paid to informal understanding of duality theory for linear programming problems including total dual integrality theory, and totally unimodular matrices. The algorithmic line of this course will be represented by introduction to the theory and algorithms (exact and approximation) for pseudo-Boolean polynomials and their applications to data analysis problems.

2. **Learning Objectives**

   Students who complete this course successfully will learn or acquire:

   - Basic concepts, models and methods of discrete optimization, which are necessary for further learning computational methods, algorithms for big data, machine learning, algorithms and computational complexity, operations research, game theory, and combinatorial optimization.
   - Skills in using **discrete math 2** models and methods to formalize and solve applied problems in data analysis.

3. **Learning Outcomes**
   - Able to find mathematical structures in the object you work with
   - Learn basic math notions that are needed to work in broad areas of applied data analysis
   - Basic math skills for learning advanced courses in applied data analysis and computer science.

4. **Course outcomes:**

Outcomes of the course are related to the program outcomes of applied data analysis within double diploma program of LSE and NRU HSE. Upon the completion of the course students will be able to model, solve and make conclusions based on the following notions, problems and solution methods including exact and heuristic algorithms:

1. The linear programming (LP) problem and its applications.
2. The simplex method in graphical and tabular forms.
3. Shadow prices and their interpretations.
4. Sensitivity analysis in LP.
5. The primal and dual problems in LP and their properties.
6. The transportation problem, northwest and Vogel's heuristics.
7. The assignment problem (AP) and its applications.
8. The Hungarian algorithm for solving the AP.
9. The shortest path (SP) problem and its applications.
10. The SP algorithm for finding an optimal solution.
11. The minimum spanning tree (MST) problem, its variations, and applications.
12. The bottleneck MST.
12. The Kruskal's and Prim's algorithms for finding a MST.

13. The max-flow-min-cut problem (MFP) and its applications.
14. The Ford-Fulkerson’s algorithm for solving the MFP.
15. The traveling salesman problem (TSP) and its applications.
16. Computation of upper and lower tolerances for the following problems: LP, SP, MST, AP, MFP, TSP.
17. Applications of upper tolerances to the uniqueness (non-uniqueness) of optimal solution(s) for the LP, SP, MST, AP, MFP, TSP.
18. The branch and bound algorithm for the Asymmetric TSP (ATSP) based on the AP.
19. The branch and bound algorithm for the Symmetric TSP based on the 1-tree problem.
20. The following heuristics for solving the TSP: greedy (nearest neighbor), tolerance based greedy, nearest insertion, 2, 3-opt.
21. The integer LP (ILP) and its applications.
22. The branch and bound algorithm for the ILP based on the LP.
23. Pseudo-Boolean polynomials: theory and algorithms.

5. **Homework will be based on assignments performed in mini-groups (2 or 3 students per group)**:

Homework problems will be assigned starting from second week just once a week. **No late submission will be accepted**. Each student is expected to solve the problems individually. Each homework will be represented by the **mini-group printed report** containing an abstract, introduction, individual solutions by every member of the mini-group, summary, and conclusion, and references. The following sections of an assignment will be discussed by all members of the mini-group and reflecting the mini-group consensus. The common mistakes made in the homework will be discussed during the seminars.

Homework and exams consist of tasks that are equivalent or similar to those which have been studied at lectures and seminars. Any fact of cheating or breach of academic integrity will result in receiving a "0" (zero) for this work.

6. **Tests**

You will have two written tests and a final exam (comprehensive). Tests will be announced at least one week in advance. A student who misses an exam or test may make it up if prior permission is obtained from the Faculty of Computer Science administration.

7. **Student’s Presentation (at most 10 min = 8min + 2 min for questions)**:

The presentations will be based on team (individual) reports. Students will be credited for both their presentations and questions asked during the presentations.

8. **Guidelines for Knowledge Assessment**

There are weakly homework assignments. Students are encouraged to work together to help each other in understanding the course material and tackling the homework problems. However, it is essential that every student must write up his/ her own solution. Homework submitted after the general deadline will not be accepted.

9. **Grading**: DM 2 **Fall 2019-20**

Homework assignments and reports	30%
Two in-class tests	20%
Personal presentation of team reports	10%
Final exam (comprehensive)	40%

10. All grades will be posted online. Students should check their grades periodically and notify the instructor immediately if they notice any discrepancy.

Grade Scale

10	>=90%
9	>=80%
8	>=75%
7	>=70%
6	>=65%
5	>=60%
4	>=50%
3	>=40%NS
2	>=25%
1	>=15%
NS	<50%

11. Mid-Semester Progress Reports will be issued for all undergraduate students halfway through the semester. You will be issued an S/Satisfactory – which means you are passing the class with a 4 or better; an NS/Not Satisfactory – which means your grade is a 3 or lower; or an FA/Failure due to absences. If you are in the NS or FA categories, you should contact your instructor and advisor to discuss ways to improve your performance before it is too late.

12. Literature

1. Kenneth H. Rosen. Discrete Mathematics and Its Applications. McGraw Hill, 2009. (**R**)
2. Wayne L. Winston and Jeffrey B. Goldberg. Operations Research. Applications and Algorithms. Fourth Edition. Thomson Learning, Belmont, CA, 2004. (**W**)
3. B. Goldengorin. Notes 1, 2, 3, 4 (referred to as **N1, etc.**).
4. Endre Boros and Peter L. Hammer. Pseudo-Boolean Optimization. Rutgers University, 2001; http://rutcor.rutgers.edu/~boros/Papers/2002-DAM-BH.pdf (referred to as **BH)**
5. Blackboard, Course Documents will be published online (Week 1 referred to as W1).

Rounding of the cumulative grade, intermediate and final grades must be performed according to the following rules. Rounding down for marks between 1 and 5, rounding by the rules of arithmetic for marks between 5 and 6, and rounding up for all the other marks.
There is no possibility to get an extra point to compensate the low cumulative grade.

6. Methods of Instruction

The discipline is delivered through lectures seminars, including computer classes.