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On L^2-dissipativity of linearized explicit finite-difference schemes with a regularization on a non-uniform spatial mesh for the 1D gas dynamics equations

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A R T I C L E I N F O

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A B S T R A C T

We deal with an explicit finite-difference scheme with a regularization for the 1D gas dynamics equations linearized at the constant solution. The sufficient condition on the Courant number for the L^2-dissipativity of the scheme is derived in the case of the Cauchy problem and a non-uniform spatial mesh. The energy-type technique is developed to this end, and the proof is both short and under clear conditions on matrices of the convective and regularizing (dissipative) terms. A scheme with a kinetically motivated regularization is considered as an application in more detail.

1. Introduction

The stability issue is crucial in theory of numerical methods for solving the gas dynamics equations, and often it is studied in a linearized statement [1–4]. In this paper, we deal with an explicit in time and three-point in space linear finite-difference scheme with general matrices of convective and regularizing (dissipative) terms. Such schemes arise from schemes with various regularizations for the 1D gas dynamics equations linearized with scaling at the constant background solution (with any velocity u_*); the well-known linearized Lax–Wendroff scheme [1] belongs to them. The sufficient condition on the Courant number for stability, more precisely, L^2-dissipativity is derived in the case of the Cauchy problem and a non-uniform spatial mesh. The energy-type technique is developed to this end, and the proof is both short and under clear conditions on the mentioned matrices. The same technique can be applied for initial–boundary value problems under suitable homogeneous boundary conditions. Notice that the derived condition is valuable for practical gas dynamics computations since it helps to reduce essentially the amount of numerical experiments to find the most suitable parameters of the schemes.

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The L^2-dissipativity of a linearized scheme from [5] with a complicated kinetically motivated regularization [6–8] is considered as an application of the general result, for zero and general u_*. The case of other possible regularizations, for example, based on [9,10] can be covered as well (results will be published elsewhere).

The technique essentially improves and generalizes its version suggested in [8] where only the case of the kinetic regularization, the uniform mesh and $u_* = 0$ was considered. Moreover, even in this particular case our condition on the Courant number is more clear and precise.

The alternative approach based on the spectral method [2] has recently been developed for the same purpose in the simplest case of the uniform spatial mesh in [11,12]. In particular, close sufficient conditions have been proved which are only slightly (at most twice) broader than in this paper. But the spectral method is inapplicable for non-uniform meshes and thus much more restrictive for further multidimensional studies.

2. An abstract explicit finite-difference scheme and its dissipativity

Let $\omega_h = \{x_k\}_{k\in\mathbb{Z}}$ be a non-uniform mesh on $\mathbb{R}$ with the steps $h_k = x_k - x_{k-1} > 0$ and $\omega_h^* = \{x_{k-1/2} = (x_{k-1} + x_k)/2\}_{k\in\mathbb{Z}}$ be the auxiliary mesh with the steps $h_{k+1/2} = x_{k+1/2} - x_{k-1/2}$. We assume that $h_{\min} := \inf_{k\in\mathbb{Z}} h_k > 0$. For functions y and v defined respectively on ω_h and ω_h^*, we introduce the averages, difference quotients and shifts

$$(sy)_{k-1/2} = \frac{y_{k-1} + y_k}{2}, \quad (\delta y)_{k-1/2} = \frac{y_k - y_{k-1}}{h_k}, \quad y_{-,k-1/2} = y_{k-1}, \quad y_{+,k-1/2} = y_k,$$
$$(s^* v)_k = \frac{h_k v_{k-1/2} + h_{k+1} v_{k+1/2}}{2h_{k+1/2}}, \quad (\delta^* v)_k = \frac{v_{k+1/2} - v_{k-1/2}}{h_{k+1/2}}.$$

We define a Hilbert space H of vector-functions $\mathbf{y}$: $\omega_h \to \mathbb{R}^n$ equipped with the inner product $(\mathbf{y}, \mathbf{z})_H = \sum_{k=-\infty}^{+\infty} (\mathbf{y}_k, \mathbf{z}_k)_{\mathbb{R}^n} h_{k+1/2}$ and the associated norm $\|\cdot\|_H$ (where $\|\mathbf{y}\|_H < \infty$ for $\mathbf{y} \in H$).

Let $\bar{\omega}^{\Delta t} = \{t_m = m\Delta t\}_{m\geqslant 0}$ be the uniform mesh in t on $[0, \infty)$ with the step $\Delta t > 0$. We set $\delta_t y^m = \frac{y^{m+1} - y^m}{\Delta t}$ and $y^{+,m} = y^{m+1}$ for y defined on $\bar{\omega}^{\Delta t}$.

We first consider an abstract explicit two-level in time and three-point in space linear finite-difference scheme

$$\delta_t \mathbf{y} + c_0 B \delta^* s\mathbf{y} - c_0^2 A \delta^* (\tau_* \delta \mathbf{y}) = 0 \ \text{ on } \ \omega_h \times \bar{\omega}^{\Delta t}, \tag{1}$$

where $\mathbf{y}^m \in H$ for $m \geqslant 0$, $\mathbf{y}^0$ is given, A and B are square matrices (of regularizing and convective terms) of order n, $c_0 > 0$ is a scaling parameter (a characteristic velocity) and $\tau_* > 0$ is a regularizing function depending on ω_h. The particular case $A = B^2$ and $\tau_* = \Delta t/2$ covers the linearized Lax–Wendroff scheme [1].

We are interested in the validity of the stability bound

$$\sup_{m\geqslant 0} \|\mathbf{y}^m\|_H \leqslant \|\mathbf{y}^0\|_H \quad \forall\, \mathbf{y}^0 \in H. \tag{2}$$

This bound is equivalent [12] to the H-dissipativity property

$$\|\mathbf{y}^{m+1}\|_H \leqslant \|\mathbf{y}^m\|_H \leqslant \cdots \leqslant \|\mathbf{y}^0\|_H \quad \forall\, \mathbf{y}^0 \in H, \ \forall\, m \geqslant 0.$$

Let τ_* and Δt be given by the standard-type formulas

$$\tau_* = \frac{\alpha h_+}{c_0} \ \text{ on } \ \omega_h^*, \quad \Delta t = \frac{\tilde{\beta} h_{\min}}{c_0}, \tag{3}$$

where $\alpha > 0$ is a parameter and $\tilde{\beta} > 0$ is a Courant-like number. The question is under what conditions on B, A and $\tilde{\beta}$ the stability bound (2) is valid.

Theorem 1. *Let the matrices B and A have the following properties*

$$B = B^*, \quad A = A^* \geqslant 0, \quad B^2 \leqslant a_0 A, \quad A \leqslant \bar{\lambda} I \tag{4}$$

for some $a_0 > 0$ and $\bar{\lambda} > 0$. The stability bound (2) is valid under the condition

$$\tilde{\beta} \leqslant 2\alpha \big/ \big(2\sqrt{\bar{\lambda}}\alpha + \sqrt{a_0}\big)^2 \equiv 1 \Big/ \Big(2\bar{\lambda}\alpha + 2\sqrt{a_0\bar{\lambda}} + \frac{a_0}{2\alpha}\Big). \tag{5}$$

Remark 1. The maximal eigenvalue $\lambda_{\max}(A)$ of A is clearly the best value of $\bar{\lambda}$ but often not available analytically.

Proof. 1. We begin with rewriting scheme (1) in the recurrent form

$$\mathbf{y}^+ = (I - c_0 \Delta t F)\mathbf{y}, \quad F := B\delta^* s - A\delta^*(c_0\tau_*\delta).$$

Then the bound

$$\|\mathbf{y}^+\|_H^2 = \|\mathbf{y}\|_H^2 + (c_0\Delta t)^2 \|F\mathbf{y}\|_H^2 - 2c_0\Delta t(F\mathbf{y}, \mathbf{y})_H \leqslant \|\mathbf{y}\|_H^2$$

is clearly equivalent to the following inequality

$$c_0 \Delta t \|F\mathbf{y}\|_H^2 \leqslant 2(F\mathbf{y}, \mathbf{y})_H. \tag{6}$$

Applying the Cauchy ε-inequality, we find

$$\begin{aligned}\|F\mathbf{y}\|_H^2 &= \|B\delta^* s\mathbf{y}\|_H^2 + \|A\delta^*(c_0\tau_*\delta\mathbf{y})\|_H^2 - 2\big(B\delta^* s\mathbf{y}, A\delta^*(c_0\tau_*\delta\mathbf{y})\big)_H \\ &\leqslant (1+\varepsilon^{-1})\|B\delta^* s\mathbf{y}\|_H^2 + (1+\varepsilon)\|A\delta^*(c_0\tau_*\delta\mathbf{y})\|_H^2 \quad \forall \varepsilon > 0.\end{aligned} \tag{7}$$

2. Let H_* and H_{*0} be Hilbert spaces of vector-functions $\mathbf{v}$: $\omega_h^* \to \mathbb{R}^n$ equipped with the inner products

$$(\mathbf{v}, \mathbf{w})_{H_*} = \sum_{k=-\infty}^{+\infty} (\mathbf{v}_{k-1/2}, \mathbf{w}_{k-1/2})_{\mathbb{R}^n} h_k, \quad (\mathbf{v}, \mathbf{w})_{H_{*0}} = \sum_{k=-\infty}^{+\infty} (\mathbf{v}_{k-1/2}, \mathbf{w}_{k-1/2})_{\mathbb{R}^n}$$

and the associated norms $\|\cdot\|_{H_*}$ and $\|\cdot\|_{H_{*0}}$ (where $\|\mathbf{v}\|_{H_*} < \infty$ for $\mathbf{v} \in H_*$ and $\|\mathbf{v}\|_{H_{*0}} < \infty$ for $\mathbf{v} \in H_{*0}$). According to [13] the following identities hold

$$(s\mathbf{y}, \mathbf{v})_{H_*} = (\mathbf{y}, s^*\mathbf{v})_H, \quad (\delta\mathbf{y}, \mathbf{v})_{H_*} = -(\mathbf{y}, \delta^*\mathbf{v})_H \quad \forall \mathbf{y} \in H, \mathbf{v} \in H_*; \tag{8}$$

note that here $s\mathbf{y}, \delta\mathbf{y} \in H_*$ and $s^*\mathbf{v}, \delta^*\mathbf{v} \in H$ as well as $\mathbf{y}_k \to 0$ and $\mathbf{v}_{k-1/2} \to 0$ as $|k| \to \infty$ (remind the assumption $h_{\min} > 0$).

Clearly $\delta^* s = s^*\delta$ and thus

$$\begin{aligned}\|\delta^* s\mathbf{z}\|_H^2 = \sum_{k=-\infty}^{+\infty} \frac{1}{h_{k+1/2}} \Big|\frac{1}{2}\big[(h_+\delta\mathbf{z})_{k+1/2} + (h_+\delta\mathbf{z})_{k-1/2}\big]\Big|^2 &\leqslant \frac{1}{2h_{\min}} \sum_{k=-\infty}^{+\infty} (h_+\delta\mathbf{z})_{k+1/2}^2 + (h_+\delta\mathbf{z})_{k-1/2}^2 \\ &= h_{\min}^{-1} \|h_+\delta\mathbf{z}\|_{H_{*0}}^2 \quad \forall \mathbf{z} \in H.\end{aligned} \tag{9}$$

It easy to see that also

$$\|\delta^*\mathbf{v}\|_H^2 \leqslant 4h_{\min}^{-1} \|\mathbf{v}\|_{H_{*0}}^2 \quad \forall \mathbf{v} \in H_{*0}. \tag{10}$$

3. Owing to identities (8), the formula $s^*\delta = \delta^* s$ and the property $B^* = B$ we also find

$$(B\delta^* s\mathbf{y}, \mathbf{y})_H = -(B\mathbf{y}, s^*\delta\mathbf{y})_H = -(B\delta^* s\mathbf{y}, \mathbf{y})_H,$$

i.e. $(B\delta^* s\mathbf{y}, \mathbf{y})_H = 0$. Therefore using the first formula (3) and the second identity (8), we get

$$(F\mathbf{y}, \mathbf{y})_H = (-A\delta^*(c_0\tau_*\delta\mathbf{y}), \mathbf{y})_H = \alpha\big(A(h_+\delta\mathbf{y}), h_+\delta\mathbf{y}\big)_{H_{*0}}. \tag{11}$$

4. Next applying inequalities (9)–(10) and then properties (4) we derive

$$\|B\delta^* s\mathbf{y}\|_H^2 \leqslant h_{\min}^{-1}\|Bh_+\delta\mathbf{y}\|_{H_{*0}}^2 \leqslant h_{\min}^{-1}a_0\big(A(h_+\delta\mathbf{y}), h_+\delta\mathbf{y}\big)_{H_{*0}},$$
$$\|A\delta^*(c_0\tau_*\delta\mathbf{y})\|_H^2 \leqslant 4\alpha^2 h_{\min}^{-1}\|A(h_+\delta\mathbf{y})\|_{H_{*0}}^2 \leqslant 4\alpha^2\bar{\lambda}h_{\min}^{-1}\big(A(h_+\delta\mathbf{y}), h_+\delta\mathbf{y}\big)_{H_{*0}}.$$

Using these inequalities in (7) and then choosing the optimal value $\varepsilon = \sqrt{a_0}/(2\alpha\sqrt{\bar{\lambda}})$, we get

$$\|F\mathbf{y}\|_H^2 \leqslant h_{\min}^{-1}\big(4\alpha^2\bar{\lambda} + 4\alpha\sqrt{a_0\bar{\lambda}} + a_0\big)\big(A(h_+\delta\mathbf{y}), h_+\delta\mathbf{y}\big)_{H_{*0}}.$$

This inequality and identity (11) imply the desired inequality (6) under condition (5). $\square$

An essential practical question is the optimal value of α. According to condition (5), this is $\alpha_{opt} = \frac{1}{2}\sqrt{a_0/\bar{\lambda}}$ ensuring the maximal possible value $1/(4\sqrt{a_0\bar{\lambda}})$ for $\tilde{\beta}$.

Note that the general case $B^2 \leqslant a_0 A$ for some $a_0 > 0$ could be easily derived from the particular one for $a_0 = 1$ by the change of parameters α and $\bar{\lambda}$ by α/a_0 and $a_0\bar{\lambda}$.

3. A finite-difference scheme with a kinetically motivated regularization for the 1D gas dynamics equations and its linearized L^2-dissipativity

Next we consider explicit two-level in time and three-point symmetric in space finite-difference scheme with a kinetically motivated regularization [6–8] and the entropy dissipative spatial discretization for the 1D gas dynamics equations from [5]:

$$\delta_t\rho + \delta^* j = 0,$$
$$\delta_t(\rho u) + \delta^*(jsu + sp) = \delta^* \Pi,$$
$$\delta_t E + \delta^*\big[(E^{(1)} + sp)(su - w) - 0.25h_+^2(\delta u)\delta p\big] = \delta^*(-q + \Pi su).$$

Here $\rho > 0$, u, $\varepsilon > 0$ are the density, velocity and the specific internal energy of the gas (the main sought functions defined on ω_h), $p = (\gamma - 1)\rho\varepsilon$ and $E = \frac{1}{2}\rho u^2 + \rho\varepsilon$ are the pressure (in the perfect polytropic gas case) and total energy. Moreover, the following formulas are in use

$$j = \rho_{\ln}(su - w), \quad w = \widehat{w} + \frac{\tau}{s\rho}(su)\delta(\rho u), \quad \widehat{w} = \frac{\tau}{s\rho}\big[(s\rho)(su)\delta u + \delta p\big],$$
$$\Pi = \mu\delta u + (s\rho)(su)\widehat{w} + \tau\big[(su)\delta p + \gamma(sp)_1\delta u\big],$$
$$-q = \widetilde{\varkappa}\delta\varepsilon + \tau(s\rho)\left(\delta\varepsilon - \frac{(sp)_1}{(s\rho)^2}\delta\rho\right)(su)^2,$$

where j is the regularized mass flux, w, $\widehat{w}$, Π and q are the two regularizing velocities, viscous stress and $\tau = \tau_h(su, s\varepsilon) > 0$ is a regularizing function (all defined on ω_h^*). In addition, the following formulas are applied

$$E^{(1)} = \tfrac{1}{2}\rho_{\ln}u_- u_+ + \rho_{\ln}\varepsilon^{\ln}, \quad (sp)_1 = (\gamma - 1)(s\rho)s\varepsilon, \quad \rho_{\ln} = 1/\ln(\rho_-;\rho_+), \quad \varepsilon^{\ln} = \varepsilon_-\varepsilon_+\ln(\varepsilon_-;\varepsilon_+);$$

the non-standard means $\rho_{\ln}$ and $\varepsilon^{\ln}$ exploit the difference quotient for the logarithmic function

$$\ln(a;b) = (\ln b - \ln a)/(b - a) \quad \text{for} \quad a \neq b, \quad \ln(a;a) = 1/a, \quad a > 0, \quad b > 0.$$

Let the artificial viscosity and scaled heat conductivity coefficients are given by

$$\mu = \alpha_S \tau sp = \alpha_S \tau s(\rho c^2)/\gamma, \quad \widetilde{\varkappa} = \gamma\alpha_P \tau sp = \alpha_P \tau s(\rho c^2)$$

(the usual QGD-formulas [7,8]), where $c = \sqrt{\gamma(\gamma-1)\varepsilon}$ is the sound velocity, $\alpha_S \geqslant 0$ and $\alpha_P \geqslant 0$ are parameters (the Schmidt and inverse Prandtl numbers). Note that a generalization and verification of this scheme were accomplished in [14,15].

The scheme is linearized at the constant background solution $\rho_* > 0$, u_* and $\varepsilon_* > 0$. To this end, similarly to the differential case [16] the solution of the scheme is written in the form

$$(\rho, u, \varepsilon) = (\rho_* + \rho_*\tilde{\rho}, u_* + \hat{c}_*\tilde{u}, \varepsilon_* + \hat{\varepsilon}_*\tilde{\varepsilon}) \text{ with } \hat{c}_* = \frac{c_*}{\sqrt{\gamma}}, \quad \hat{\varepsilon}_* = \sqrt{\gamma-1}\varepsilon_*, \quad c_* = \sqrt{\gamma(\gamma-1)\varepsilon_*},$$

where c_* is the background sound velocity. Let $\tau_* = \tau_h(u_*, \varepsilon_*)$ and $c_0 = c_*$. Omitting the second order terms with respect to the scaled perturbation $\mathbf{y} = (\tilde{\rho}, \tilde{u}, \tilde{\varepsilon})^T$ we obtain for it the linearized scheme like (1) with the matrices [12]

$$B = \begin{pmatrix} M & \frac{1}{\sqrt{\gamma}} & 0 \\ \frac{1}{\sqrt{\gamma}} & M & \sqrt{\frac{\gamma-1}{\gamma}} \\ 0 & \sqrt{\frac{\gamma-1}{\gamma}} & M \end{pmatrix}, \; A = \begin{pmatrix} M^2 + \frac{1}{\gamma} & 2\frac{M}{\sqrt{\gamma}} & \frac{\sqrt{\gamma-1}}{\gamma} \\ 2\frac{M}{\sqrt{\gamma}} & M^2 + \frac{\alpha_S}{\gamma} + 1 & 2\sqrt{\frac{\gamma-1}{\gamma}}M \\ \frac{\sqrt{\gamma-1}}{\gamma} & 2\sqrt{\frac{\gamma-1}{\gamma}}M & M^2 + \alpha_P + \frac{\gamma-1}{\gamma} \end{pmatrix}. \tag{12}$$

Here $M = u_*/c_*$, and $|M|$ is the Mach number. We rewrite formulas (3) in the form like in [12]

$$\tau_* = \frac{\widehat{\alpha}h_+}{c_* + |u_*|} \text{ on } \omega_h^*, \quad \varDelta t = \frac{\beta h_{\min}}{c_* + |u_*|}, \tag{13}$$

where β is the Courant number, i.e., with $\alpha = \widehat{\alpha}/(|M|+1)$ and $\tilde{\beta} = \beta/(|M|+1)$. Clearly $\alpha = \widehat{\alpha}$ and $\tilde{\beta} = \beta$ in the case $u_* = 0$.

Theorem 2. *For the scheme* (1), (12) and (13), *the sufficient L^2-dissipativity condition* (5) *is satisfied:*
(1) in the particular case $u_* = 0$, for

$$\beta \leqslant 1\Big/\Big(2\bar{\lambda}_0\alpha + 2\sqrt{\bar{\lambda}_0} + \frac{1}{2\alpha}\Big), \tag{14}$$

where

$$\bar{\lambda}_0 = \max\Big\{\frac{\alpha_S}{\gamma} + 1, \; \frac{\alpha_P + 1}{2} + \sqrt{\Big(\frac{\alpha_P - 1}{2}\Big)^2 + \frac{\gamma-1}{\gamma}\alpha_P}\Big\};$$

(2) in the general case, for

$$\beta \leqslant 1\Big/\Big(2\widehat{\alpha}\frac{\bar{\lambda}}{(|M|+1)^2} + 2\frac{\sqrt{\bar{\lambda}}}{|M|+1} + \frac{1}{2\widehat{\alpha}}\Big), \tag{15}$$

where

$$\bar{\lambda} = M^2 + \max\Big\{\frac{\alpha_S}{\gamma} + 1 + 2\frac{1+\sqrt{\gamma-1}}{\sqrt{\gamma}}|M|, \; \alpha_P + 1 + 2\sqrt{\frac{\gamma-1}{\gamma}}|M|\Big\}.$$

Proof. Clearly $B = B^*$ and $A = A^*$. Also we have $A \geqslant 0$, $B^2 \leqslant A$ (i.e., $a_0 = 1$) and, moreover, in the case $u_* = 0$ and the general one, respectively $\bar{\lambda}_0 = \lambda_{\max}(A)$ and $\lambda_{\max}(A) \leqslant \bar{\lambda}$, see [12]. Owing to Theorem 1 this implies the stated results. □

Notice that Theorem 2 remains valid for various other schemes based on the same regularization, in particular, see [5–8,17] since the constant background solution is considered.

The practically important point is that due to the applied regularization the right-hand side of (15) is uniformly bounded (from below and above) in $|M|$ as $|M| \to \infty$, for fixed $\widehat{\alpha}$, that is essential in computing super- and hypersonic flows. This property is not guaranteed at all for more simple regularizations.

Sufficient conditions close to (5) (for $a_0 = 1$), (14) and (15) have recently been derived by the spectral method in [12] in the particular case of the uniform spatial mesh. They contain no the above intermediate terms with $\sqrt{\bar{\lambda}}$ or $\sqrt{\bar{\lambda}_0}$, and thus they are only slightly (at most twice) broader than (5), (14) and (15).

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