Conditioning of Imprecise Probabilities Based on Generalized Credal Sets

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The outline of the paper

1. The intersection of generalized credal sets can be considered as a generalization conjunction rules in the theory of imprecise probabilities and in the theory of belief functions. Thus, it is interesting to look at what happens if we will use the same approach for updating information used for belief functions and based on Dempster's rule.
2. We analyze this approach in the paper and and show that it can be realized in two possible ways, and in each way leads for Dempster's conditioning for belief functions.
3. The proposed approach looks simpler than the traditional in the theory of imprecise probabilities and allows us better to implement learning process based on statistical data.
4. The obtained results look promising for using in robust statistics models.

Monotone Measures: Some Useful Constructions

Let X be a finite universal set and 2^X be the power set of X. The set function $\mu : 2^X \to [0,1]$ is called

- normalized if $\mu(\emptyset) = 0$ and $\mu(X) = 1$;
- monotone if $A \subseteq B$, $A, B \in X$ implies $\mu(A) \leqslant \mu(B)$;
- a monotone measure if μ is normalized and monotone;
- a belief function if there is a set function $m : 2^X \to [0,1]$ with $\sum_{B \in 2^X} m(B) = 1$ called the basic belief assignment (bba) such that $\mu(A) = \sum_{B \in 2^X | B \subseteq A} m(B)$.

Relations and operations on monotone measures:

- $\mu_1 \leqslant \mu_2$ for set functions on 2^X if $\mu_1(A) \leqslant \mu_2(A)$ for all $A \in 2^X$.
- μ^d is called the dual of μ if $\mu^d(A) = 1 - \mu(A^c)$ for all $A \in 2^X$, where A^c denotes the complement of A.
- We write $\mu = a\mu_1 + (1-a)\mu_2$ if $a \in [0,1]$, μ, μ_1, μ_2 are set functions and $\mu(A) = a\mu_1(A) + (1-a)\mu_2(A)$ for all $A \in 2^X$.

Some definitions from the theory of belief functions

Let Bel be a belief function with the bba m, then $A \in 2^X$ is called a focal element for Bel if $m(A) > 0$. A belief function is called categorical if it has the only one focal element B. This function is denoted by $\eta_{\langle B \rangle}$ and clearly

$$\eta_{\langle B \rangle}(A) = \begin{cases} 1, & B \subseteq A, \\ 0, & \text{otherwise}, \end{cases} \quad A \in 2^X.$$

Every belief function Bel with the bba m can be represented as a convex sum of categorical belief functions as

$$Bel = \sum_{B \in 2^X} m(B) \eta_{\langle B \rangle}.$$

Modeling Uncertainty with Probabilities

A belief function P is a probability measure if its body of evidence consists of singletons, i.e.

$$P = \sum_{i=1}^{n} a_i \eta_{\langle \{x_i\} \rangle}, \tag{1}$$

where $\sum_{i=1}^{n} a_i = 1$, $a_i \geqslant 0$, $i = 1, ..., n$.
Notation: M_{pr} is set of all possible probability measures on 2^X.
Assume that an experiment is described by $P \in M_{pr}$ and a function $f : X \to \mathbb{R}$ shows us the award that we get after conducting the experiment. Then, in a frequentist's view, the value

$$E_P(f) = \sum_{i=1}^{n} f(x_i) P(\{x_i\}) = \sum_{i=1}^{n} a_i f(x_i)$$

gives us the expected award. This award can be considered as the mean value of awards obtained during the series of the same independent experiments.

Modeling Uncertainty with Imprecise Probabilities

If the measure P is not exactly known, then it is possible to describe an experiment by a set of probability measures $\mathbf{P}$, and we know the lower bound $\underline{E}_{\mathbf{P}}(f)$ and upper bound $\overline{E}_{\mathbf{P}}(f)$ of the expected award defined by

$$\underline{E}_{\mathbf{P}}(f) = \inf_{P \in \mathbf{P}} E_P(f), \; \overline{E}_{\mathbf{P}}(f) = \sup_{P \in \mathbf{P}} E_P(f).$$

Let K be the linear space of all possible real valued functions on X. Then functionals $\underline{E}_{\mathbf{P}}$ and $\overline{E}_{\mathbf{P}}$ on K have the following properties:

1. $\underline{E}_{\mathbf{P}}(af + c) = a\underline{E}_{\mathbf{P}}(f) + c$, $\overline{E}_{\mathbf{P}}(af + c) = a\overline{E}_{\mathbf{P}}(f) + c$ for every $a \geqslant 0$, $c \in \mathbb{R}$ and $f \in K$;
2. $\underline{E}_{\mathbf{P}}(f_1 + f_2) \geqslant \underline{E}_{\mathbf{P}}(f_1) + \underline{E}_{\mathbf{P}}(f_2)$, $\overline{E}_{\mathbf{P}}(f_1 + f_2) \leqslant \overline{E}_{\mathbf{P}}(f_1) + \overline{E}_{\mathbf{P}}(f_2)$, for every $f_1, f_2 \in K$;
3. $\underline{E}_{\mathbf{P}}(f_1) \leqslant \underline{E}_{\mathbf{P}}(f_2)$ for every $f_1, f_2 \in K$ with $f_1(x) \leqslant f_2(x)$ for all $x \in X$;
4. $\overline{E}_{\mathbf{P}}(f) = -\underline{E}_{\mathbf{P}}(-f)$ for every $f \in K$.

Modeling Uncertainty with Imprecise Probabilities

Credal Sets

Credal sets are non-empty sets of probability measures which are assumed to be closed and convex. There is a bijection between credal sets and functionals $\underline{E}_{\mathbf{P}}$.

Lower Previsions

Let $K' \subseteq K$, then every $\Phi : K' \to \mathbb{R}$ is called a *lower prevision* if its values $\Phi(f)$ can be viewed as lower bounds of $E_P(f)$. A $\Phi : K' \to \mathbb{R}$ is called *non-contradictory* or *consistent*, if it defines the credal set

$$\mathbf{P}(\Phi) = \{P \in M_{pr} | \forall f \in K' : E_P(f) \geqslant \Phi(f)\}.$$

Otherwise, Φ is called contradictory lower prevision. We will assume that for every lower prevision $\Phi : K' \to \mathbb{R}$

$$\Phi(f) \leqslant \max_{x \in X} f(x).$$

Upper and lower probabilities

- If we describe uncertainty by monotone measures, then the set K' consists of characteristic functions $1_A(x) = 1$ if $x \in A$ and $1_A(x) = 0$ otherwise.
- Thus, every Φ on $K' = \{1_A | A \in 2^X\}$ is conceived as a set function $\mu(A) = \Phi(1_A)$, $A \in 2^X$.
- μ is called a lower probability if its values are viewed as lower bounds of probabilities.
- A lower probability μ is called non-contradictory, if it defines the credal set
$$\mathbf{P}(\Phi) = \{P \in M_{pr} | P \geqslant \mu\}.$$
Otherwise, if $\mathbf{P}(\Phi)$ is empty, then μ is called contradictory.
- Analogously, we can define monotone measures conceived as upper probabilities.

Contradiction correction based on generalized credal sets

Assume $\Phi : K' \to \mathbb{R}$ is an upper prevision. Following (Bronevich & Rozenberg, ECQUARY-2017), Φ can be represented as the convex sum of two functionals:

$$\Phi = (1-a)\Phi^{(1)} + a\Phi^{(2)}, \tag{2}$$

where $a \in [0,1]$, $\Phi^{(1)}$ is a non-contradictory upper prevision and $\Phi^{(2)}$ is a contradictory upper prevision.

The exact lower bound of a in representation (2) is called the amount of contradiction in Φ, denoted by $Con(\Phi)$.

Since $V_{\min}(f) \leqslant \Phi^{(2)}(f)$, $f \in K'$, we can compute $Con(\Phi)$ by

$$Con(\Phi) = 1 - a \longrightarrow \max$$
$$\exists P \in M_{pr} : aE_P(f) + (1-a)V_{\min}(f) \leqslant \Phi(f) \text{ for all } f \in K'.$$

Contradiction correction based on generalized credal sets

An upper prevision Φ is called fully contradictory if $Con(\Phi) = 1$. We can identify the fully contradictory information with the full ignorance that can be modeled by the upper prevision

$$V_{\max}(f) = \max_{x \in X} f(x).$$

If the upper prevision Φ is not fully contradictory, then $Con(\Phi) = a \in [0, 1)$ and Φ can represented as

$$\Phi = (1 - a)\Phi^{(1)} + aV_{\min},$$

where $\Phi^{(1)}$ is a non-contradictory upper prevision.

Contradiction correction:

$$\Phi' = (1 - a)n.ext(\Phi^{(1)}) + aV_{\max},$$

where $n.ext(\Phi^{(1)})(f)$ is the natural extension of $\Phi^{(1)}$.

Generalized credal sets

Lower generalized credal sets (LG-credal sets) consist of upper probabilities defined by

$$P = a_0\eta_{\langle X\rangle} + \sum_{i=1}^{n} a_i\eta_{\langle\{x_i\}\rangle}, \tag{3}$$

where $\sum_{i=0}^{n} a_i = 1$, and $a_i \geqslant 0$, $i = 1, ..., n$.
We see that the set function $\eta_{\langle X\rangle}$ is the counterpart of $V_{\min}$, since $\eta_{\langle X\rangle}(A) = V_{\min}(1_A)$, $A \in 2^X$.
Any P from (3) can be represented as a convex sum of $\eta_{\langle X\rangle}$ and a probability measure

$$P^{(1)} = \frac{1}{1-a_0}\sum_{i=1}^{n} a_i\eta_{\langle\{x_i\}\rangle}.$$

We see that $Con(P) = a_0$. We denote M^d_{cpr} the set of all possible P defined by (3).

Generalized credal sets

Definition 1

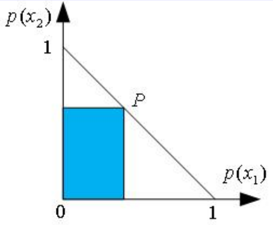
A non-empty subset $\mathbf{P} \subseteq M_{cpr}^d$ is called a lower generalized credal set (LG-credal set) iff the following conditions hold

1. $P_1 \in \mathbf{P}$ implies $P_2 \in \mathbf{P}$ for every $P_2 \leqslant P_1$ in M_{cpr}^d;
2. $\mathbf{P}$ is a convex subset of M_{cpr}^d, i.e. $P_1, P_2 \in \mathbf{P}$ implies $aP_1 + (1-a)P_2 \in \mathbf{P}$ for any $a \in [0,1]$;
3. $\mathbf{P}$ is a closed subset of $\mathbb{R}^n$ (every P defined by (3) is considered as a point $(a_1, ..., a_n)$ of $\mathbb{R}^n$).

Let $\mathbf{P}$ be a LG-credal set, then the set of all maximal elements in $\mathbf{P}$ w.r.t. $\leqslant$ is called the profile of $\mathbf{P}$. We denote it by $profile(\mathbf{P})$. We identify an usual credal set $\mathbf{P}' \subseteq M_{pr}$ with the LG-credal set $\mathbf{P}$ whose profile is $\mathbf{P}'$.
The amount of contradiction in every LG-credal set $\mathbf{P}$ can be computed by $Con(\mathbf{P}) = \inf\{Con(P) | P \in \mathbf{P}\}$.

Examples of generalized credal sets

Let $X = \{x_1, x_2\}$ and any $P \in M_{pr}$ on 2^X is defined by probabilities $P(\{x_i\}) = p(x_i)$, $i = 1, 2$.



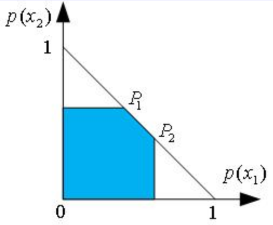


Fig. 1. Left: LG-credal set (blue rectangle), whose profile is the probability measure P.
Right: LG-credal set, whose profile is the usual credal set $\{aP_1 + (1-a)P_2 | a \in [0,1]\}$.

Generalized credal sets

We can extend any $P \in M^d_{cpr}$ to K by

$$\overline{E}_P(f) = a_0 V_{\min}(f) + (1 - a_0) E_{P^{(1)}}(f) = a_0 \min_{x \in X} f(x) + \sum_{i=1}^{n} a_i f(x_i).$$

Every upper prevision $\Phi : K' \to \mathbb{R}$ can be described by the LG-credal set $\mathbf{P}(\Phi)$ defined by

$$\mathbf{P}(\Phi) = \left\{ P \in M^d_{cpr} | \forall f \in K' : \overline{E}_P(f) \leqslant \Phi \right\}. \tag{4}$$

Proposition 1

Let $\Phi : K' \to \mathbb{R}$ be an upper prevision with $Con(\Phi) = a_0 < 1$, and let $\mathbf{P}(\Phi)$ be defined by (4), then the contradiction correction is produced by

$$\Phi''(f) = \sup\{\overline{E}_{P^d}(f) | P \in \mathbf{P}(\Phi), Con(P) = a_0\},$$

where $\overline{E}_{P^d}(f) = a_0 \max_{x \in X} f(x) + \sum_{i=1}^{n} a_i f(x_i)$.

The conjunctive rule for imprecise probabilities

Assume that sources of information are described by upper previsions $\Phi_1, ..., \Phi_m$ on $K' \subseteq K$. If these sources of information are assumed to be reliable, then we can use the conjunctive rule:

$$\Phi(f) = \min_{i=1,...,m} \Phi_i(f),\ f \in K'.$$

Analogously the conjunctive rule is defined for upper probabilities.

If we describe these sources of information by LG-credal sets $\mathbf{P}(\Phi_i)$, $i = 1, ..., m$, then

$$\mathbf{P}(\Phi) = \bigcap_{i=1}^{m} \mathbf{P}(\Phi_i).$$

Let us notice that in the traditional theory of imprecise probabilities the conjunctive rule is defined only in the case, when sources of information are not contradictory. In this case, the intersection of the corresponding usual credal sets is not empty.

Conditioning based on conjunctive rule

Let P be a probability measure on 2^X and we know that an event $B \subseteq X$ occurs, then we can update probabilities using the conditional probability measure

$$P_B(A) = P(A \cap B)/P(B),\ A \in 2^X.$$

The conditional probability measure P_B is not defined if the event B fully contradicts to the probability measure P, i.e. $P(B) = 0$.
Such a result can be viewed as an aggregation of two sources of information. The first source is the prior information described by a probability measure P, and the second source certifies that the event B occurred. This can be modeled by the upper probability $\eta^d_{\langle B \rangle}$. Thus, we can implement the conjunctive rule:

$$\mu = \min\{P, \eta^d_{\langle B \rangle}\}.$$

Conditioning based on conjunctive rule

Proposition 2

Let $P = \sum_{i=1}^{n} a_i \eta_{\langle \{x_i\} \rangle}$ be in M_{pr} and $\mu = \min\{P, \eta^d_{\langle B \rangle}\}$ for a $B \in 2^X \backslash \{\emptyset\}$. Then the LG-credal set $\mathbf{P}(\mu) = \{P \in M^d_{cpr} | P \leqslant \mu\}$ has the profile $\{P^*_B\}$, where $P^*_B = a_0 \eta_{\langle X \rangle} + \sum_{x_i \in B} a_i \eta_{\langle \{x_i\} \rangle}$, where $a_0 = 1 - \sum_{x_i \in B} a_i$.

We can generalize Proposition 2 as follows.

Proposition 3

Let $P = a_0 \eta_X + \sum_{i=1}^{n} a_i \eta_{\langle \{x_i\} \rangle}$ be in M^d_{cpr} and $\mu = \min\{P, \eta^d_{\langle B \rangle}\}$ for a $B \in 2^X \backslash \{\emptyset\}$. Then the LG-credal set $\mathbf{P}(\mu) = \{P \in M^d_{cpr} | P \leqslant \mu\}$ has the profile $\{P^*_B\}$, where $P^*_B = b_0 \eta_{\langle X \rangle} + \sum_{x_i \in B} a_i \eta_{\langle \{x_i\} \rangle}$ and $b_0 = 1 - \sum_{x_i \in B} a_i$.

Conditioning based on conjunctive rule

Let us look at the result of the conditioning based on the conjunctive rule, described by the profile $\{P_B^*\}$:

$$P_B^* = a_0\eta_{\langle X\rangle} + \sum_{x_i\in B} a_i\eta_{\langle\{x_i\}\rangle}.$$

It contains the fully contradictory part: $a_0\eta_{\langle X\rangle}$, and the non-contradictory part: $(1-a_0)P_B = \sum_{x_i\in B} a_i\eta_{\langle\{x_i\}\rangle}$, where P_B is the conditional probability measure given B.
Consider an arbitrary LG-credal set $\mathbf{P}\subseteq M_{cpr}^d$. Then the conditioning of $\mathbf{P}$ given $B\in 2^X$ is the subset of M_{cpr}^d defined by $\mathbf{P}_B^* = \{P_B^*|P\in\mathbf{P}\}$.

Lemma 1

Let $\mathbf{P}\subseteq M_{cpr}^d$ be a LG-credal set and $B\in 2^X$. Then $\mathbf{P}_B^* = \mathbf{P}\cap(M_{cpr}^d)_B^*$, where $(M_{cpr}^d)_B^* = \{P_B|P\in M_{cpr}^d\}$.

Conditioning based on conjunctive rule

Proposition 4

The subset $\mathbf{P}_B^*$ of M_{cpr}^d defined above is the LG-credal set.

Thus, by Proposition 4, we define for every LG-credal set $\mathbf{P}$ and every event $B \in 2^X$ the conditional LG-credal set $\mathbf{P}_B^*$. Then we need to make the correction and to have the consistent information after that. Notice that the way, earlier discussed in this presentation, is not suitable for us, because it does not give us the usual result for probability measures. This correction for measures in M_{cpr}^d should be

$$\varphi(a_0\eta_{\langle X\rangle} + \sum\nolimits_{i=1}^{n} a_i\eta_{\langle\{x_i\}\rangle}) = \frac{1}{1-a_0}\sum_{i=1}^{n} a_i\eta_{\langle\{x_i\}\rangle},$$

and for LG-credal sets we have the following two major possibilities:

1. $\varphi^{(1)}(\mathbf{P}) = \{\varphi(P) | P \in \mathbf{P}, Con(P) = Con(\mathbf{P})\}$;
2. $\varphi^{(2)}(\mathbf{P}) = \{\varphi(P) | P \in profile(\mathbf{P})\}$.

Conditioning based on conjunctive rule

Remark 1

Clearly, $\varphi^{(1)}(\mathbf{P})$ is the usual credal set for every LG-credal set $\mathbf{P}$. Assume that $\mathbf{P}$ is a LG-credal set with the profile $\mathbf{P}' \subseteq M_{pr}$. Then the updating $\varphi^{(1)}(\mathbf{P}_B^*)$ for $B \in 2^X$ exists if $\overline{E}_{\mathbf{P}'}(1_B) > 0$ and the conditioning $\varphi^{(1)}(\mathbf{P}_B^*)$ is called the maximal likelihood conditioning because

$$\varphi^{(1)}(\mathbf{P}_B^*) = \left\{ P_B | P \in \mathbf{P}', P(B) = \overline{E}_{\mathbf{P}'}(1_B) \right\}.$$

Remark 2

Because the profile of a LG-credal set is not necessarily a convex set, the set $\varphi^{(2)}(\mathbf{P})$ is not a usual credal set in general. It is sufficient to know the extreme points of $\mathbf{P}$ that belong to $profile(\mathbf{P})$.
Let $\mathbf{P}$ be a LG-credal set with the profile $\mathbf{P}' \subseteq M_{pr}$ whose extreme points are $\{P_1, ..., P_k\}$. Then all extreme points in $profile\{\mathbf{P}_B^*\}$ are in the set $\{(P_1)_B^*, ..., (P_k)_B^*\}$.

Conditioning based on conjunctive rule

Example 1

Assume that $X = \{x_1, ..., x_5\}$ and $\mathbf{P}$ is the LG-credal set whose profile is the usual credal set described by extreme points
$P_1 = (0.5, 0.3, 0.1, 0.1, 0)$, $P_2 = (0.2, 0.3, 0.1, 0.1, 0.3)$,
$P_3 = (0.4, 0.2, 0.2, 0.1, 0.1)$. Consider the event $B = \{x_1, x_2\}$. Then
$(P_1)^*_B = (0.5, 0.3, 0, 0, 0)$, $(P_2)^*_B = (0.2, 0.3, 0, 0, 0)$,
$(P_3)^*_B = (0.4, 0.2, 0, 0, 0)$. Because $(P_1)^*_B \geqslant (P_2)^*_B$ and $(P_1)^*_B \geqslant (P_3)^*_B$, the LG-credal set $\mathbf{P}^*_B$ has the profile $\{(P_1)^*_B\}$. Thus, in this case $\varphi^{(1)}(\mathbf{P}^*_B) = \varphi^{(2)}(\mathbf{P}^*_B) = \{(P_1)_B\}$.
Analogously, let $C = \{x_1, x_2, x_3\}$ then $(P_1)^*_C = (0.5, 0.3, 0.1, 0, 0)$, $(P_2)^*_C = (0.2, 0.3, 0.1, 0, 0)$, $(P_3)^*_C = (0.4, 0.2, 0.2, 0, 0)$. Because $(P_1)^*_C \geqslant (P_2)^*_C$, the LG-credal set $\mathbf{P}^*_C$ has the profile $\{a(P_1)^*_C + (1-a)(P_3)^*_C | a \in [0,1]\}$. In this case, $\varphi^{(1)}(\mathbf{P}^*_C) = \{(P_1)_C\}$ and $\varphi^{(2)}(\mathbf{P}^*_C) = \{a(P_1)_C + (1-a)(P_3)_C | a \in [0,1]\}$.

Conditioning based on conjunctive rule

Remark 3

Let $\mathbf{P}'$ be an usual credal set. Then the following updating rule is often used in the theory of imprecise probabilities:

$$(\mathbf{P}')_B = \{P_B | P \in \mathbf{P}'\} \text{ for } B \in 2^X.$$

This rule is defined iff $\underline{E}_{\mathbf{P}'}(1_B) > 0$.
It is well-known that if $\mathbf{P}'$has a finite set of extreme points $\{P_1, ..., P_k\}$, then extreme points of $(\mathbf{P}')_B$ are in the set $\{(P_1)_B, ..., (P_k)_B\}$. Thus,

$$(\mathbf{P}')_B = \left\{ \sum_{i=1}^{k} a_i (P_i)_B \middle| \sum_{i=1}^{k} a_i = 1,\ a_i \geqslant 0,\ i = 1, ..., k \right\}.$$

The difference is that the conditioning based on conjunctive rule throws out probability measures that are not plausible enough for a given event B.

Conditioning based on conjunctive rule

Conditioning of upper prevision $\Phi : K' \to \mathbb{R}$ given B

$$\Phi_B^*(f) = \begin{cases} 0, & f = 1_{B^c}, \\ \Phi(f), & f \in K'' \backslash \{1_{B^c}\}, \end{cases}$$

where $K'' = K' \cup \{1_{B^c}\}$.

Conditioning of upper probabilities

Proposition 5

Let μ be an upper probability on 2^X, $B \in 2^X$, and let μ_B^* be an upper probability on 2^X defined by

$$\mu_B^*(A) = \begin{cases} \mu(A \cap B), & A \neq X, \\ 1, & A = X. \end{cases}$$

Then $(\mathbf{P}(\mu))_B^* = \mathbf{P}(\mu_B^*)$.

Conditioning based on conjunctive rule

Proposition 6

Let μ be an upper probability on 2^X and $\mu(B) > 0$ for some $B \in 2^X$. Then $(\mathbf{P}(\mu))^*_B = \{(1-\mu(B))\eta_X + \mu(B)P | P \in \mathbf{P}(\mu_B)\}$, where $\mu_B(A) = \mu(A \cap B)/\mu(B)$, $A \in 2^X$.

Corollary 1

Let μ be an 2-alternating upper probability[a] on 2^X and $\mu(B) > 0$ for some $B \in 2^X$, and we use the notation from Proposition 5. Then

$$\varphi^{(1)}((\mathbf{P}(\mu))^*_B) = \varphi^{(2)}((\mathbf{P}(\mu))^*_B) = \{P \in M_{pr} | P \leqslant \mu_B\}.$$

[a] A monotone measure μ is called 2-alternating if $\mu(A) + \mu(B) \geqslant \mu(A \cap B) + \mu(A \cup B)$ for all $A, B \in 2^X$.

Conditioning based on conjunctive rule

Remark 4

Let us remind that plausibility functions are the dual of belief functions. We see that the conditioning based on generalized credal sets coincides with the conditioning based on Dempster's rule that for a plausibility function Pl on 2^X gives the result $Pl_B(A) = Pl(A \cap B)/Pl(B)$ for $A \in 2^X$ and $B \in 2^X$ such that $Pl(B) > 0$.

Thanks for your attention

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