

Syllabus

Title of a Course : Introduction to Functional Analysis
(6 ECTS)

Author, lecturer : A.Yu.Pirkovskii (pirkosha@gmail.com,
<https://www.hse.ru/staff/pirkovskii>)

Faculty of Mathematics

Meeting Minute # ___dated _____ 20__

1. Course Description

a) Pre-requisites : Calculus, linear algebra, metric spaces, the Lebesgue integral.

b) Abstract

Functional analysis studies infinite-dimensional vector spaces equipped with a norm (or, more generally, with a topology), operators between such spaces, and representations of algebraic structures on such spaces. The classical areas of Functional Analysis are the spectral theory of linear operators, the geometry of Banach spaces, distribution theory, operator algebra theory, etc. Among relatively new areas are noncommutative geometry à la Connes, operator space theory (a.k.a. «quantum functional analysis»), and locally compact quantum groups. Functional analysis has numerous applications in differential equations, harmonic analysis, representation theory, geometry, topology, calculus of variations, optimization, quantum physics, etc.

In this an introductory course, we plan to cover the very basics of Functional Analysis (the «irreducible minimum») only

2. Learning Objectives

3. Learning Outcomes

4. Course Plan

1. Normed and Banach spaces, bounded linear maps.
2. Hilbert spaces.
3. The Hahn-Banach Theorem, the Open Mapping Theorem, the Uniform Boundedness Principle.
4. Basic duality theory.
5. Elementary spectral theory.
6. Compact operators. The Hilbert-Schmidt Theorem.

5. Reading List

- a) **Required**
- b) **Optional**

6. Grading System : final grade = $0.7 \times (\text{cumulative grade}) + 0.3 \times (\text{exam grade})$,
where cumulative grade = $0.5 \times (\text{midterm grade}) + 0.5 \times (\text{exercise sheets grade})$.

The oral exam will be at the end of December and will include only the material of the 2nd module.

The midterm exam (also oral) will be at the end of October (or at the beginning of November) and will include only the material of the 1st module.

To get the maximum grade for the exercise sheets, you should solve 75% of all the exercises. If you solve more, you will earn bonus points.

You can also earn bonus points for working actively at the exercise classes and for solving «bonus exercises» (marked as «B» in the sheets).

7. Examination Type

8. Methods of Instruction

9. Special Equipment and Software Support : no requirements

10. Further reading