National Research University Higher School of Economics

The International College of Economics and Finance

Syllabus for *Calculus*++, 1st year, 2019-2020

Утверждена Академическим советом ОП
Протокол № от __.__.20__

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Credit Number	
Contact Hours	69
Self-Study	96
Department Year of Study	The International College of Economics and Finance, 1st year
Delivery Format	This course does not include online components

1. Prerequisites, Learning objectives, Aims

It is expected that students attend classes and lectures of the main ICEF 1st year Calculus subject, and that they successfully completed "Algebra and Introduction to Calculus" subject at the level of secondary school programme.

By the end of this course the students should:

- be able to solve advanced problems from selected chapters of the main Calculus course.
- Know how to deal with double and triple integrals and use them for calculating volumes, surface areas and for solving various problems from probability theory.

2. List of topics and distribution of hours

- Module I: rational and irrational numbers, the method of mathematical induction, limits of sequences, properties of continuous functions, differentiable functions, derivatives.
- Module II: higher derivatives, the Taylor formula, convergence of infinite series, anti-derivatives and indefinite integrals.
- Module III: functions of two real variables, extreme values, constraint maximization, absolute maxima and minima, classical inequalities, double and triple integrals, calculation of volumes and surface areas.
- Module IV: topics from AP Calculus BC: differential and functional equations, Taylor series.

Topic Titles	Hours Contact Self-Study Online	Gained Skills	Assessment
Limit of a sequences	6 12 	Calculating limits of sequences and solving advanced related problems	Weakly quizzes
Properties of continuous functions	3 6 	Knowledge and application of main theorems (IVT, EVT)	Weakly quizzes
Differentiable functions	3 6 	Differentiation, application of mean value theorem	Weakly quizzes and written midterm examination
Higher derivatives, Taylor formula	6 8 	Applications of Taylor formula, estimation of approximation error	Weakly quizzes
Infinite series	6 8 	Investigation of convergence for infinite series	Weakly quizzes
Anti-derivatives	9 12 	Calculation of anti-derivatives, integration of rational functions	Weakly quizzes and written midterm examination
Functions of two and several variables	6 8 	Differentiation, extreme values, Lagrange method, Polar coordinates	Weakly quizzes
Double Integrals	9 12 	Writing down as iterated integrals, changing variables in double integrals	Weakly quizzes
Applications of double integrals	9 12 	Calculating volumes, masses, and areas of surfaces	Weakly quizzes and written midterm examination
Triple Integrals	6 8 	Writing down iterated integrals, changing variables	Weakly quizzes
Differential Equations	3 4 	Solving Bernoulli equations, second-order equations with constant coefficients	Weakly quizzes
AP Topics: Calculus BC	6 8 	Solving functional equations	Weakly quizzes and written final examination
Часов по видам учебных занятий:	**69** **96** 		

Итого часов:	**165**

3. Course content:

1. **Prologue.**
   Rational and irrational numbers. Properties of the Dirichlet function. The method of mathematical induction. *(Spivak, pp. 21-35, Stewart, pp. 97-102)*
2. **Sequences. Limit of a sequence.**
   The definition of a divergent sequence. The sandwich theorem. Sequences defined recursively. The Stolz lemma. Fundamental sequences and the Cauchy criterion. Advanced questions on convergence of sequences. Sequences of functions. Uniform convergence. *(Spivak, pp. 445-463, Binmore, pp. 26-52)*
3. **Properties of continuous functions.**
   Limit of a function. The extreme value theorem. The definition of a discontinuous function. *(Spivak, pp. 90-130, Binmore, pp. 74-90)*
4. **Differentiable functions.**
   The chain rule. Derivatives of inverse functions.
   (Spivak, pp. 147-184, Binmore, pp. 91-99)
5. **Higher derivatives. Taylor formula.**
   Calculating derivatives of *n*-th order. The Taylor formulae for elementary functions. Irrationality of the number *e*. Application of the Taylor formula for calculating limits.
   (Spivak, pp. 405-434, Stewart, pp. 97-102)
6. **Infinite series.**
   Telescoping series. The necessary condition for convergence. The Cauchy criterion. Convergence of the generalized harmonic series. Dirichlet's test.
   (Spivak, pp. 464-490, Stewart, pp. 727-765)
7. **Anti-derivatives.**
   Integration by parts. The method of undetermined coefficients. Decimation formulae. Integration of rational and trigonometric functions. Euler's substitutions.
   (Spivak, pp. 359-396, Stewart, pp. 487-524)
8. **Functions of two and several variables.**
   Polar, Cylinder and Spherical coordinates. The chain rule. Stationary points. The second derivative test. Lagrange method.
   (Stewart, pp. 902-939, 948-956, 970-988)
9. **Definite integrals.**
   Riemann sums. Non-integrable functions. Properties of definite integrals.
   (Spivak, pp. 250-299, Stewart, pp. 284-320)
10. **Double integrals.**
    Reduction of double integrals to iterated integrals. Changing the integration order in iterated integrals. Changing variables in double integrals. *(Stewart, pp. 998-1020)*
11. **Applications of double integrals.**
    Volumes and Surface Areas. Double integrals and probability. *(Stewart, pp. 1021-1040)*
12. **Triple integrals.**

Reduction of triple integrals to iterated integrals. Using cylinder and spherical coordinate for calculating triple integrals. *(Stewart, pp. 1041-1063)*

13. **Differentials Equations.**
    Separable differential equations. Bernoulli differential equations. Solution of differential equations in terms of power series. *(Stewart, pp. 1021-1040)*
14. **Calculus BC APT questions.**
    Infinite series. Functional equations. Taylor series.

4. Literature

1. Spivak M., Calculus, 3rd edition, Publish or Perish Inc., 1994.
2. Stewart J., Calculus, 7th edition, Brooks/Cole, Cengage Learning, 2012.
3. Binmore K.G., Mathematical Analysis, Cambridge University Press, 1977.
4. Maron I.A., Problems in Calculus of One Variable, Mir, 1973.

5. Assessment

There are weakly quizzes and 4 midterm exams during the academic year, whereby student performance is continuously assessed. The course ends with a final examination in the last week of April.

The overall Calculus++ mark is the weighted sum of the average quiz marks (10%), midterm exam marks (10% each), and the April exam mark (50%):

Final_Mark = 0.1*Midterm_1 + 0.1*Midterm_2 + 0.1*Midterm_3 + 0.1*Midterm_4 + 0.1*Quiz_Average + 0.5*April_Exam_Mark.

The Pass mark for Calculus++ course is 25 out of 100.

Sample examination papers are posted in ICEF Information system in due time.

Initially, all marks are computed in the scale “Out of 100”. Final marks are also transferred into “Out of 10” and “Out of 5” scales, according to the ICEF Grading Regulation (paragraph 3).

As a rule, no repeat examinations are scheduled for midterm and final exams. In exceptional circumstances repeat examinations can be scheduled for the April exam according to the HSE Interim and Ongoing Assessment Regulations (including Annex 8 for ICEF). After repeat examinations the final marks are computed according to ICEF Grading Regulations (paragraph 5).