Physics of Josephson junctions, brief overview

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Outline

1. Josephson effect, introduction
2. Photonic transport of heat across a Josephson junction
3. Experiments in our lab in Helsinki

Superconductivity

1. Electrons form Cooper pairs
2. Cooper pairs form Bose - Einstein condensate — hence they are described by a single "order parameter" $\Delta(\boldsymbol{r})e^{i\varphi(\boldsymbol{r})}$, which is basically the wave function of a Cooper pair
3. Dissipationless "superconducting" current can flow through the superconductor

$$\boldsymbol{j}_S(\boldsymbol{r}) = \frac{\pi\sigma_N\Delta(\boldsymbol{r})}{2e}\nabla\varphi(\boldsymbol{r}).$$

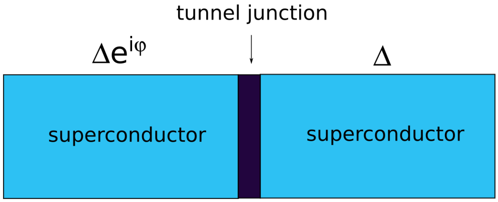
4. Current through a wire of length L and cross section $\mathcal{S}$

$$I_S(\varphi) = \frac{\pi\sigma_N\mathcal{S}\Delta}{2e}\frac{\varphi(L)-\varphi(0)}{L} = \frac{\pi}{2}\frac{\Delta}{eR_N}\varphi.$$

Josephson current

B. D. Josephson, "Possible new effects in superconductive tunnelling". Phys. Lett. **1**, 251 (1962).

Nobel Prize in Physics in 1973

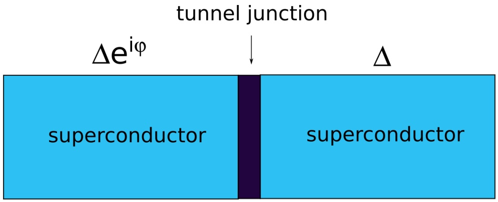


Dissipationless superconducting current $I_S(\varphi) \equiv I_J(\varphi) = I_C \sin\varphi$

Josephson relation between phase and voltage $V = \frac{\hbar\dot{\varphi}}{2e}$

Josephson tunnel junction

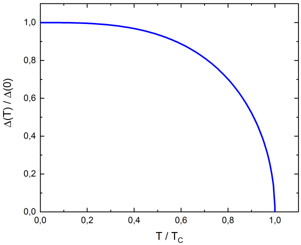
V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963).

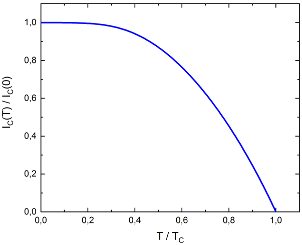


Tunnel junction (transmission probability of all conducting channels $\tau_n \ll 1$)

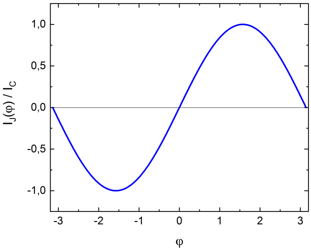
$$I_J(\varphi) = \frac{\pi\Delta(T)}{2eR_N}\tanh\frac{\Delta(T)}{2k_BT}\sin\varphi$$

Josephson tunnel junction



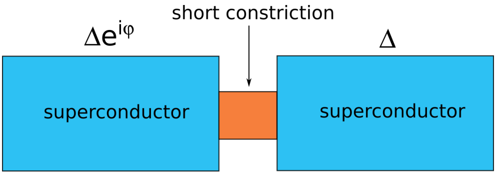


$$I_J(\varphi) = \frac{\pi\Delta(T)}{2eR_N}\tanh\frac{\Delta(T)}{2k_BT}\sin\varphi$$



Short Josephson junction

W. Haberkorn, H. Knauer and J. Richter, Phys. Status Solidi A **47**, K161 (1978).



Short constriction between superconduting leads

$$E_{\mathrm{Th}} = \frac{\hbar}{t_{\mathrm{flight}}} \gg \Delta$$

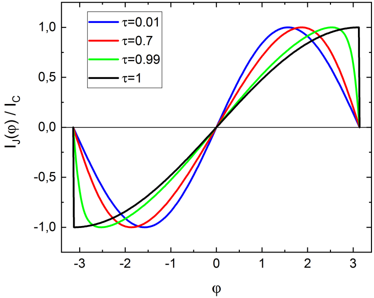
Josephson current of a short junction

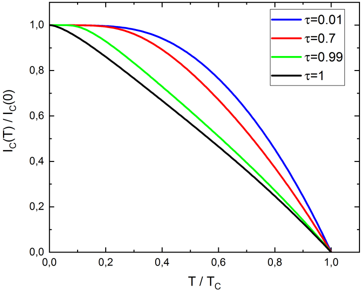
$$I_J(\varphi) = \sum_n \frac{e\Delta(T)}{2\hbar} \frac{\tau_n \sin\varphi}{\sqrt{1 - \tau_n \sin^2\frac{\varphi}{2}}} \tanh \frac{\Delta(T)\sqrt{1 - \tau_n \sin^2\frac{\varphi}{2}}}{2k_B T}.$$

Normal state resistance is given by Landauer formula

$$\frac{1}{R_N} = \frac{2e^2}{h} \sum_n \tau_n.$$

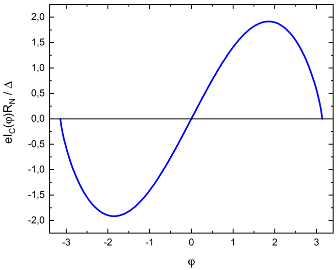
Short Josephson junction

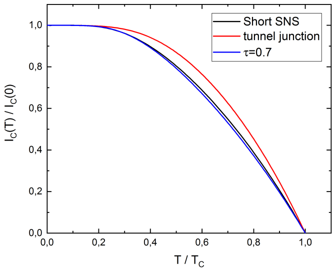




$$I_J(\varphi) = N_{\rm ch}\frac{e\Delta(T)}{2\hbar}\frac{\tau\sin\varphi}{\sqrt{1-\tau\sin^2\frac{\varphi}{2}}}\tanh\frac{\Delta(T)\sqrt{1-\tau\sin^2\frac{\varphi}{2}}}{2k_BT}$$

Short SNS junction



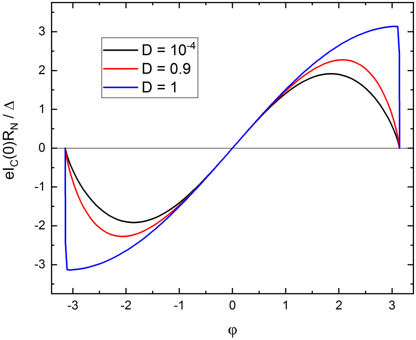


Dorokhov distribution of channel transmissions

$$\sum_n f(\tau_n) \to \frac{\pi\hbar}{2e^2 R_N} \int_0^1 d\tau_n \frac{f(\tau_n)}{\tau_n\sqrt{1-\tau_n}}$$

$$I_J(\varphi, T=0) = \frac{\pi\Delta}{2eR_N} \cos\frac{\varphi}{2} \ln\left(\frac{1+\sin\frac{\varphi}{2}}{1-\sin\frac{\varphi}{2}}\right).$$

Short SINIS junction without impurities, $T = 0$



$$I_J(\varphi) = \frac{\Delta}{eR_N} K\left(\sqrt{\frac{1-t^2}{1-t^2\sin^2\frac{\varphi}{2}}}\left|\sin\frac{\varphi}{2}\right|\right)\sin\varphi, \quad t = \frac{D}{2-D}$$

Product $I_C(0)R_N$

Tunnel junction $I_C(0)R_N = \frac{\pi\Delta}{2e}$

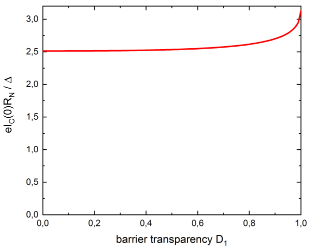
Short transparent junction $I_C(0)R_N = \frac{\pi\Delta}{e}$

Short junction with transparency τ $I_C(0)R_N = \frac{1}{1+\sqrt{1-\tau}}\frac{\pi\Delta}{e}$

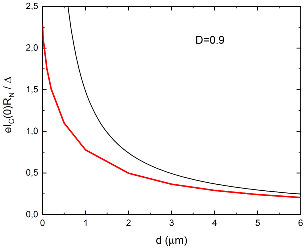
Long junction without impurities $I_C(0)R_N = \frac{\pi}{2}\frac{D}{2-D}\frac{\hbar v_F}{eL} = \frac{\pi}{2}\frac{D}{2-D}\frac{E_{\rm Th}}{e}$

Short SNS junction $I_C(0)R_N = 2.083\frac{\Delta}{e}$

Short junction without impurities



Long junction with $D = 0.9$



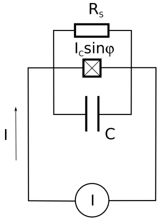
RCSJ model

Resistively and capacitively shunted Josephson junction (RCSJ model)

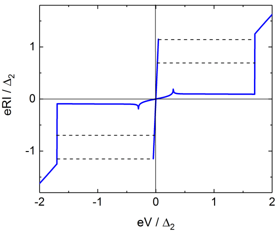
$$C\frac{\hbar\ddot{\varphi}}{2e} + \frac{1}{R_S}\frac{\hbar\dot{\varphi}}{2e} + I_C \sin\varphi = I$$

Josephson frequency $\omega_0 = \sqrt{2eI_C/\hbar C}$, damping rate $\gamma_d = 1/R_S C$

schematics

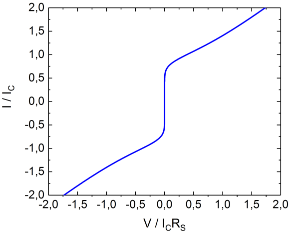


underdamped junction
$\omega_0 > 1/R_S C$



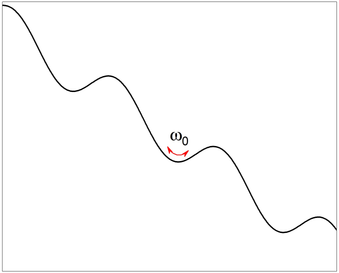
hysteretic I-V curve

overdamped junction
$\omega_0 < 1/R_S C$



$V = R_S\sqrt{I^2 - I_C^2}$ at $T = 0$

Classical motion in tilted Josephson potential



$$C\frac{\hbar\ddot{\varphi}}{2e} + \frac{1}{R_S}\frac{\hbar\dot{\varphi}}{2e} + I_C \sin\varphi = I$$

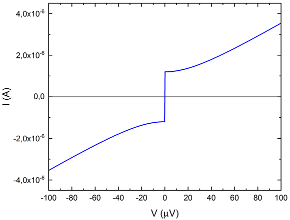
$$U(\varphi) = -E_J\cos\varphi - \frac{\hbar I}{2e}\varphi$$

$$\text{Kinetic energy} \;\; E_K = \frac{C}{2}\left(\frac{\hbar\dot{\varphi}}{2e}\right)^2 = \frac{Q^2}{2C}, \quad Q = CV = C\frac{\hbar\dot{\varphi}}{2e}$$

$$\text{Potential energy} \;\; U(\varphi) = -E_J\cos\varphi - \frac{\hbar I}{2e}\varphi$$

Josephson energy $E_J = \frac{\hbar I_C}{2e}$, Charging energy $E_C = \frac{e^2}{2C}$, Josephson freq. $\omega_0 = \sqrt{8E_J E_C}/\hbar$

microwaves off ($I_{\rm mw} = 0$)



microwaves on

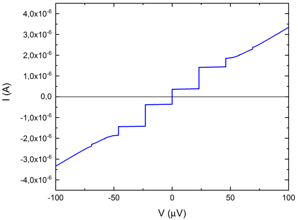


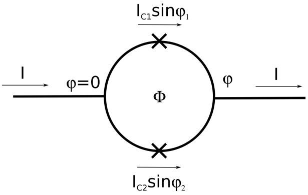
Figure: Paramters: $I_C = 1.2\ \mu$A, $R_S = 30\ \Omega$, $\omega_{\rm mw}/2\pi = 11.5$ GHz, $I_{\rm mw} = 1.5 I_C$.

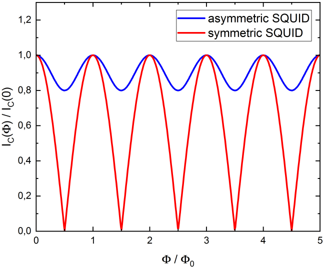
Example: overdamped Josephson junction under microwave irradiation

$$\frac{1}{R_S}\frac{\hbar\dot\varphi}{2e} + I_C \sin\varphi = I + I_{\rm mw}\cos\omega_{\rm mw}t$$

Shapiro steps occur at $V_n = \frac{\hbar\omega_{\rm mw}}{2e} n$ — voltage standard

Critical current of a SQUID





$I = I_{C1}\sin\varphi_1 + I_{C2}\sin\varphi_2$; in the bulk of the superconducting ring $I_S = \alpha\left(\nabla\varphi - \frac{2e}{\hbar c}\boldsymbol{A}\right)$. Taking the integral around the ring and assuming that $\nabla\varphi$ is small, we get

$$2\pi n = \varphi_1 - \varphi_2 - \frac{2e}{\hbar c}\Phi + \alpha 2\pi R\mathcal{A}\nabla\varphi \Rightarrow \varphi_1 - \varphi_2 \approx \frac{2\pi\Phi}{\Phi_0}$$

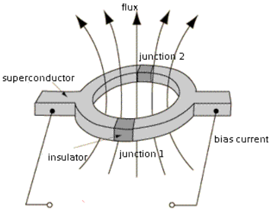
Hence

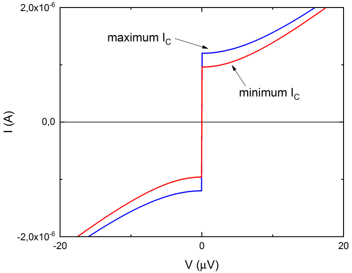
$$I = I_{C1}\sin\left(\varphi + \frac{\pi\Phi}{\Phi_0}\right) + I_{C2}\sin\left(\varphi - \frac{\pi\Phi}{\Phi_0}\right), \quad \varphi = \frac{\varphi_1 + \varphi_2}{2}$$

$$I_C = \left|I_{C1}e^{i\pi\Phi/\Phi_0} + I_{C2}e^{-i\pi\Phi/\Phi_0}\right| = \sqrt{I_{C1}^2 + I_{C2}^2 + 2I_{C1}I_{C2}\cos\frac{2\pi\Phi}{\Phi_0}}$$

SQUID as a magnetometer

SQUID stands for superconducting quantum interference device



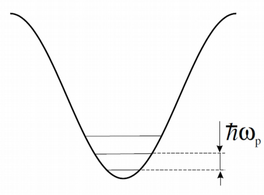


$$I_C = \sqrt{I_{C1}^2 + I_{C2}^2 + 2I_{C1}I_{C2}\cos\frac{2\pi\Phi}{\Phi_0}}, \quad \Phi_0 = \frac{\pi\hbar c}{e} = 2.07 \times 10^{-15}\text{ Wb}.$$

SQUID is the best magnetometer

Qubit

Qubit is a two level quantum system



Classical Hamiltonian of an underdamped junction

$$H = \frac{Q^2}{2C} + E_J(1 - \cos\varphi) \tag{1}$$

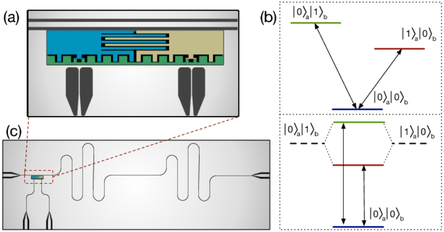
Quantum Hamiltonian of an underdamped junction

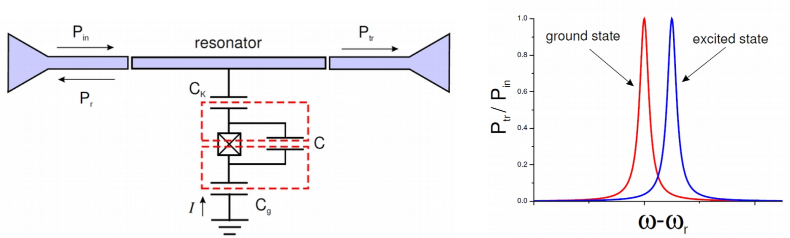
$$\hat{H} = -4E_C \frac{\partial^2}{\partial\varphi^2} + E_J(1 - \cos\varphi), \quad \hat{Q} = -ie\frac{\partial}{\partial\varphi} \tag{2}$$

Energy levels close to the bottom of the potential

$$E_n = \sqrt{8E_J E_C}\left(n + \frac{1}{2}\right) - \frac{E_C}{12}(6n^2 + 6n + 3) \tag{3}$$

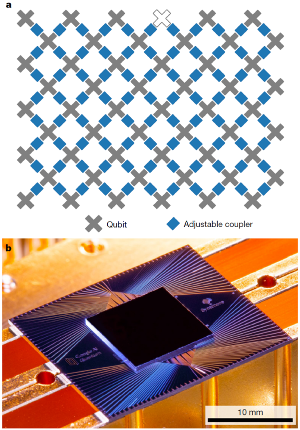
Transmon qubit





Coherence time up to 100 μs

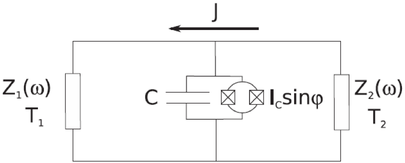
Quantum computer from Google



Quantum supremacy using a programmable superconducting processor
Nature 2019

Photonic heat transport across a Josephson junction

George Thomas, Jukka P. Pekola, and Dmitry S. Golubev, Phys. Rev. B 2019



Linear approximation

$J = \int_0^\infty \frac{d\omega}{2\pi}\, \hbar\omega\, \tau(\omega)\, [N_2(\omega) - N_1(\omega)]$

$\tau(\omega) = \frac{4\mathrm{Re}\left[\frac{1}{Z_1(\omega)}\right]\mathrm{Re}\left[\frac{1}{Z_2(\omega)}\right]}{\left|-i\omega C + \frac{1}{Z_1(\omega)} + \frac{1}{Z_2(\omega)} + \frac{1}{Z_J(\omega)}\right|^2}$

$Z_J(\omega) = -i\hbar\omega/2eI_C$

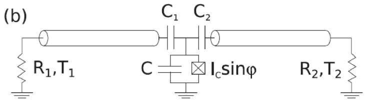
For a symmetric SQUID one finds $I_C = I_C(0)|\cos(\pi\Phi/\Phi_0)|$

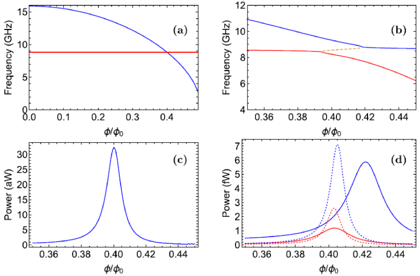
$$\frac{1}{Z_J(\omega)} = \frac{2eI_C(0)}{-i\hbar\omega}\,|\cos(\pi\Phi/\Phi_0)|\,.$$

Two resonators coupled via a Josephson junction

Two identical resonators $\omega_1 = \omega_2$
Coupling constants between resonator and the junction, Josephson frequency

$$g_{1,2} = \sqrt{\frac{Z_r \omega_{1,2}^3 C_{1,2}^2}{\pi(C_1 + C_2 + C)}}, \quad \omega_J = \sqrt{\frac{2eI_C}{\hbar(C_1 + C_2 + C)} \left|\cos\frac{\pi\Phi}{\Phi_0}\right|}$$

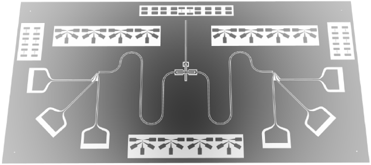


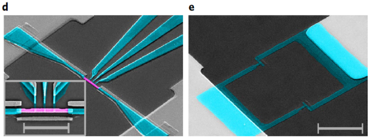


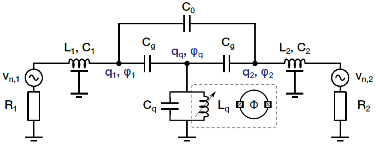
Peak in the heat power appears if $\omega_1 = \omega_2 = \omega_J$

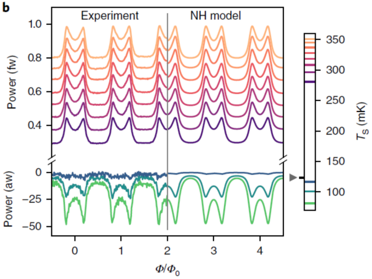
Experiments in our lab

Heat transport through a Josephson junction
Ronzani *et al*, Nat. Phys. 2018









Thermometers