

Heuristic Algorithm for the Cell Formation Problem

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Abstract In this chapter, we introduce a new heuristic for Cell Formation Problem in its most general formulation with grouping efficiency as an objective function. Suggested approach applies an improvement procedure to obtain solutions with high grouping efficiency. This procedure is repeated until efficiency can be increased for randomly generated configurations of cells. We consider our preliminary results for 10 popular benchmark instances taken from the literature. Also source instances with the solutions we got can be found in the [Appendix](#).

Keywords Cell formation · Grouping efficiency · Improvement heuristic

1 Introduction

Flanders (1925) [7] was the first who formulated the main ideas of the group technology. The notion of the Group Technology was introduced in Russia by Mitrofanov (1959) [9], though his work was translated to English only in 1966 (Mitrofanov, 1966 [10]). One of the main problems stated by the Group Technology is the optimal formation of manufacturing cells, that is, grouping of machines and parts

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into cells such that for every machine in a cell the number of the parts from this cell processed by this machine is maximized and the number of the parts from other cells processed by this machine is minimized. In other words, the intra-cell loading of machines is maximized and simultaneously the inter-cell movement of parts is minimized. This problem is called the Cell Formation Problem (CFP). Burbidge (1961) [3] suggested his Product Flow Analysis (PFA) approach for the CFP, and later popularized the Group Technology and the CFP in his book (Burbidge, 1975 [4]).

The CFP is NP-hard since it can be reduced to the clustering problem (Ghosh et al., 1996 [1]). That is why there is a great number of heuristic approaches for solving CFP and almost no exact ones. Our heuristic algorithm is based on sequential improvements of the solution. We modify the cell configuration by enlarging one cell and reducing another. The basic procedure of the algorithm has the following steps:

1. Generate a random cell configuration.
2. Improve the initial solution moving one row or column from one cell to another until the grouping efficiency is increasing.
3. Repeat steps 1–2 a predefined number of times (we use 2000 times for computational experiments in this chapter).

The chapter is organized as follows. In the next section, we provide the Cell Formation Problem formulation. In Sect. 3, we present our improvement heuristic that allows us to get good solutions by iterative modifications of cells which lead to increasing of the objective function. In Sect. 4, we report our computational results and Sect. 5 concludes the chapter with a short summary.

2 The Cell Formation Problem

The CFP consists in an optimal grouping of the given machines and parts into cells. The input for this problem is given by m machines, p parts and a rectangular machine-part incidence matrix $A = [a_{ij}]$, where $a_{ij} = 1$ if part j is processed on machine i . The objective is to find an optimal number and configuration of rectangular cells (diagonal blocks in the machine-part matrix) and optimal grouping of rows (machines) and columns (parts) into these cells such that the number of zeros inside the chosen cells (voids) and the number of ones outside these cells (exceptions) are minimized. A concrete combination of rectangular cells in a solution (diagonal blocks in the machine-part matrix) we will call a cells configuration.

For example, we are given the machine-part matrix (Waghodekar and Sahu, 1984 [13]) shown in Table 1. A feasible solution for this CFP is shown in Table 2.

There are a number of different objective functions used for the CFP. For our research we used so called grouping efficiency measure suggested by Chandrasekharan and Rajagopalan (1989) [6] and compare our computational results with the

Table 1 Machine-part 5×7 matrix from Waghodekar and Sahu (1984) [13]

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
m_1	1	0	0	0	1	1	1
m_2	0	1	1	1	1	0	0
m_3	0	0	1	1	1	1	0
m_4	1	1	1	1	0	0	0
m_5	0	1	0	1	1	1	0

Table 2 CFP solution

	p_7	p_6	p_1	p_5	p_3	p_2	p_4
m_1	1	1	1	1	0	0	0
m_4	0	0	1	0	1	1	1
m_3	0	1	0	1	1	0	1
m_2	0	0	0	1	1	1	1
m_5	0	1	0	1	0	1	1

results of Bhatnagar and Saddikuti (2010) [16] and Goldengorin et al. (2012) [8]. Grouping efficiency described by Chandrasekharan and Rajagopalan (1989) [6] by the following formula:

$$\eta = q\eta_1 + (1 - q)\eta_2, \tag{1}$$

where

$$\eta_1 = \frac{n_1 - n_1^{\text{out}}}{n_1 - n_1^{\text{out}} + n_0^{\text{in}}} = \frac{n_1^{\text{in}}}{n^{\text{in}}}$$

$$\eta_2 = \frac{mp - n_1 - n_0^{\text{in}}}{mp - n_1 - n_0^{\text{in}} + n_1^{\text{out}}} = \frac{n_0^{\text{out}}}{n^{\text{out}}}$$

- η_1 —a ratio showing the intra-cell loading of machines (or the ratio of the number of ones in cells to the total number of elements in cells).
- η_2 —a ratio inverse to the inter-cell movement of parts (or the ratio of the number of zeroes out of cells to the total number of elements out of cells).
- q —a coefficient ($0 \leq q \leq 1$) reflecting the weights of the machine loading and the inter-cell movement in the objective function. It is usually taken equal to $\frac{1}{2}$, which means that it is equally important to maximize the machine loading and minimize the inter-cell movement.
- n_1 —a number of ones in the machine-part matrix,
- n_0 —a number of zeroes in the machine-part matrix,
- n^{in} —a number of elements inside the cells,
- n^{out} —a number of elements outside the cells,
- n_1^{in} —a number of ones inside the cells,

- n_1^{out} —a number of ones outside the cells,
- n_0^{in} —a number of zeroes inside the cells,
- n_0^{out} —a number of zeroes outside the cells.

Efficiency values the solution in Table 2 is calculated below.

$$\eta = \frac{1}{2} \cdot \frac{16}{19} + \frac{1}{2} \cdot \frac{12}{16} \approx 79.60 \%$$

The mathematical programming model of the CFP with the grouping efficiency objective function can be described using boolean variables x_{ik} and y_{jk} . Variable x_{ik} takes value 1 if machine i belongs to cell k and takes value 0 otherwise. Similarly variable y_{jk} takes value 1 if part j belongs to cell k and takes value 0 otherwise. Machines index i takes values from 1 to m and parts index j —from 1 to p . Cells index k takes values from 1 to $c = \min(m, p)$ because every cell should contain at least one machine and one part and so the number of cells cannot be greater than m and p . Note that if a CFP solution has n cells then for k from $n + 1$ to c all variables x_{ik}, y_{jk} will be zero in this model. So we can consider that the CFP solution always has c cells, but some of them can be empty. The mathematical programming formulation is as follows.

$$\max \left(\frac{n_1^{\text{in}}}{2n^{\text{in}}} + \frac{n_0^{\text{out}}}{2n^{\text{out}}} \right) \quad (2)$$

where

$$n^{\text{in}} = \sum_{k=1}^c \sum_{i=1}^m \sum_{j=1}^p x_{ik} y_{jk}, \quad n^{\text{out}} = mp - n^{\text{in}}$$

$$n_1^{\text{in}} = \sum_{k=1}^c \sum_{i=1}^m \sum_{j=1}^p a_{ij} x_{ik} y_{jk}, \quad n_0^{\text{out}} = n_0 - (n^{\text{in}} - n_1^{\text{in}})$$

subject to

$$\sum_{k=1}^c x_{ik} = 1 \quad \forall i \in 1, \dots, m \quad (3)$$

$$\sum_{k=1}^c y_{jk} = 1 \quad \forall j \in 1, \dots, p \quad (4)$$

$$\sum_{i=1}^m \sum_{j=1}^p x_{ik} y_{jk} \geq \sum_{i=1}^m x_{ik} \quad \forall k \in 1, \dots, c \quad (5)$$

$$\sum_{i=1}^m \sum_{j=1}^p x_{ik} y_{jk} \geq \sum_{j=1}^p y_{jk} \quad \forall k \in 1, \dots, c \quad (6)$$

$$x_{ik} \in \{0, 1\} \quad \forall i \in 1, \dots, m \quad (7)$$

$$y_{jk} \in \{0, 1\} \quad \forall j \in 1, \dots, p \quad (8)$$

The objective function (5) is the grouping efficiency in this model. Constraints (6) and (7) impose that every machine and every part belongs to some cell. Constraints (8) and (9) guarantee that every non-empty cell contains at least one machine and one part. Note that if singleton cells are not allowed then the right sides of inequalities (8) and (9) should have a coefficient of 2. All these constraints can be linearized in a standard way, but the objective function will still be fractional. That is why the exact solution of this problem presents considerable difficulties.

3 Algorithm Description

Our algorithm is as follows.

1. First, we look for a potentially optimal range of cells.
 - a. Look over all the possible number of cells from 2 to maximal possible number of cells which is equal to $\min(m, p)$.
 - b. For every number of cells in this interval, we generate a fixed number of configurations (we use 500 in this chapter).
 - c. For every of these 500 configurations, we try to rearrange rows and columns between cells in order to increase efficiency value. These rearrangements are made until efficiency could be increased.
 - d. The best efficiency value from 500 results above are chosen. This would be our best bound for this particular number of cells.
 - e. Then we get the best efficiency value from bounds for cells number from 2 to $\min(m, p)$ and its 10 percent neighborhood is optimal cell range in that our following procedures will be made.
2. For every number of cells in our optimal interval, we do a more precise search.
 - a. For every number of cells in the optimal interval, we generate a larger number of configurations (we use 2000 in this chapter).
 - b. For every of these 2000 configurations, we try to rearrange rows and columns between cells in order to increase efficiency value. These rearrangements are made until efficiency could be increased.
 - c. The best efficiency value from these 2000 results above are chosen. This would be our best bound for this particular number of cells.
 - d. Then we get the best efficiency value for our optimal cell range and this result is our final for this problem instance.

The main idea of the improvement procedure (that rearranges rows and columns) is illustrated on Seifoddini and Wolfe (1986) [12] instance 8×12 (Table 3). To compute the grouping efficiency for this solution, we need to know the number of ones inside cells n_1^{in} , the total number of elements inside cells n^{in} , the number of

Table 3 Seifoddini and Wolfe (1986) [12] instance 8×12

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	0	0	0	0	0	0	0	0
2	1	0	1	1	1	1	1	0	0	1	0	0
3	0	0	1	1	1	1	1	1	1	0	0	0
4	0	0	0	0	0	1	1	1	1	1	0	0
5	0	0	0	0	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	1	1	1	0	1	0
7	0	0	0	0	0	0	0	0	0	0	1	1
8	0	0	0	0	0	0	0	0	0	0	1	1

Table 4 Moving part 4 from cell 2 to cell 1

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	0	0	0	0	0	0	0	0
2	1	0	1	1	1	1	1	0	0	1	0	0
3	0	0	1	1	1	1	1	1	1	0	0	0
4	0	0	0	0	0	1	1	1	1	1	0	0
5	0	0	0	0	0	0	1	1	1	1	0	0
6	0	0	0	0	0	0	1	1	1	0	1	0
7	0	0	0	0	0	0	0	0	0	0	1	1
8	0	0	0	0	0	0	0	0	0	0	1	1

zeros outside cells n_0^{out} and the number of elements outside cells n^{out} . The grouping efficiency is then calculated by the following formula:

$$\eta = q \cdot \frac{n_1^{in}}{n^{in}} + (1 - q) \cdot \frac{n_0^{out}}{n^{out}} = \frac{1}{2} \cdot \frac{20}{33} + \frac{1}{2} \cdot \frac{48}{63} \approx 68.4 \%$$

Looking at this solution (Table 3), we can conclude that it is possible for example, to move part 4 from the second cell to the first one. And this way the number of zeros inside cells decreases by 3 and the number of ones outside cells also decreases by 4. So it is profitable to attach column 4 to the first cell as it is shown on Table 4.

For the modified cells configuration we have:

$$\eta = \frac{1}{2} \cdot \frac{23}{33} + \frac{1}{2} \cdot \frac{51}{63} \approx 75.32 \%$$

As a result, the efficiency is increased almost for 7 percent. Computational results show that using such modifications could considerably improve the solution. The idea is to compute an increase in efficiency for each column and row when it is moved to another cell and then perform the modification corresponding to the maximal increase. For example, Table 5 shows the maximal possible increase in efficiency for every row when it is moved to another cell.

Table 5 Maximal efficiency increase for each row

	1	2	3	4	5	6	7	8	9	10	11	12	
1	1	1	1	1	0	0	0	0	0	0	0	0	-6.94 %
2	1	0	1	1	1	1	1	0	0	1	0	0	+1.32 %
3	0	0	1	1	1	1	1	1	1	0	0	0	+7.99 %
4	0	0	0	0	0	1	1	1	1	1	0	0	-0.07 %
5	0	0	0	0	0	0	1	1	1	1	0	0	+0.77 %
6	0	0	0	0	0	0	1	1	1	0	1	0	+0.77 %
7	0	0	0	0	0	0	0	0	0	0	1	1	-4.62 %
8	0	0	0	0	0	0	0	0	0	0	1	1	-4.62 %

4 Computational Results

In all the experiments for determining a potentially optimal range of cells, we use 500 random cell configurations for each cells number and for obtaining the final solution we use 2000 random configurations. An Intel Core i7 machine with 2.20 GHz CPU and 8.00 Gb of memory is used in our experiments. We run our heuristic on 10 CFP benchmark instances taken from the literature. The computational results are presented in Table 6. For every instance, we make 50 algorithm runs and report minimum, average and maximum value of the grouping efficiency obtained by the suggested heuristic over these 50 runs. We compare our results with the best known values taken from Goldengorin et al. (2012) [8] and Bhatnagar and Saddikuti (2010) [16]. Our final solutions are presented in Tables 7–26.

5 Concluding Remarks

In this chapter, we present a new heuristic algorithm for solving the CFP. The high quality of the solutions is achieved due to the enumeration of different numbers of cells and different cell configurations and applying our improvement procedure. Since the suggested heuristic works fast, we apply it for thousands of different configurations. Thus, a big variety of good solutions is covered by the algorithm and the best of them has high grouping efficiency.

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Table 6 Computational results

#	Source	$m \times p$	Efficiency value, %			Time, sec	
			Bhatnagar and Saddikuiti		Our		
			Min	Avg	Max		
1	Sandbothe (1998) [14]	10×20	95.40	95.93	95.66	95.66	0.36
2	Ahi et al. (2009) [15]	20×20	92.62	93.85	95.99	95.99	0.62
3	Mosier and Taube (1985) [11]	20×20	85.63	88.71	90.16	90.22	0.88
4	Boe and Cheng (1991) [2]	20×35	88.31	88.05	93.47	93.55	1.62
5	Carrie (1973) [5]	20×35	90.76	95.64	95.78	95.79	1.54
6	Ahi et al. (2009) [15]	20×51	87.86	94.11	95.36	95.45	3.1
7	Chandrasekharan and Rajagopalan (1989) [6]	24×40	98.82	100.00	100	100	1.8
8	Chandrasekharan and Rajagopalan (1989) [6]	24×40	95.33	97.48	97.75	97.76	2.42
9	Chandrasekharan and Rajagopalan (1989) [6]	24×40	93.78	96.36	96.84	96.89	2.56
10	Chandrasekharan and Rajagopalan (1989) [6]	24×40	87.92	94.32	96.11	96.21	3.3

Appendix: Source Instances and Solutions

Table 7 Sandbothe (1998) [14]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0
4	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
9	0	0	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	1	1

Table 8 Ahi et al. (2009) [15]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0	1	0	0	1
2	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
3	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
4	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	1	0
6	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
7	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0
8	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0
9	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	1	0	0	0
10	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1
11	0	1	0	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	1	0
12	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1
13	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
14	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0
15	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0
16	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0
17	0	0	0	0	0	1	1	0	0	0	0	0	1	0	1	0	0	0	0	0
18	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1
19	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0
20	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0

Table 9 Mosier and Taube (1985) [11]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	0	0	0	0	1	0	0	1	0	1	0	1	0	0	1	0	0	0	0
2	0	1	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1	0	0	1
3	0	0	1	1	0	0	0	1	0	0	1	0	1	1	0	1	0	1	0	0
4	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0
5	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
6	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0
7	1	0	0	0	0	1	1	0	1	0	0	0	0	1	0	1	0	1	1	1
8	0	0	0	1	0	1	0	0	1	1	0	0	0	1	1	0	0	1	0	1
9	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	1	0	0	1	0
10	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1	0	0	0	1	0
11	0	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0	0	1	1	0
12	0	0	0	1	0	0	0	0	0	0	1	1	0	1	0	0	1	1	0	0
13	0	1	0	0	0	0	0	0	0	1	1	0	1	0	0	1	1	1	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	1	0
15	0	0	1	0	1	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0
16	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0
17	0	0	0	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0
18	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0
19	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0
20	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0

Table 10 Boe and Cheng (1991) [2] 35 parts and 20 machines

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
2	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0
3	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0
4	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	0
5	0	0	1	0	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
6	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0
7	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0
8	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	1	0	0	1	1	0	0	1	1	0	0	1	0
10	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
11	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0
12	0	1	0	1	0	0	1	0	0	0	0	0	1	1	0	1	0	1	0	0
13	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0
14	1	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	1
15	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
16	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
17	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
18	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
19	0	1	0	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	1
20	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0
21	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0
22	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0
23	1	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	1
24	1	1	0	1	0	0	1	0	0	0	1	0	1	1	0	0	0	1	0	0
25	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
26	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	1	0	0	0	1
27	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0
29	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	0	0	0	0	1	1	0	0	0	1	0	0	0	1	1	0	0	1	0
31	0	1	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0
32	1	0	0	0	0	0	1	0	1	0	1	0	0	0	0	1	0	0	1	0
33	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
34	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	1	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	0	0	0

Table 11 Carrie (1973) [5] 35 parts and 20 machines

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
2	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0
3	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
4	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	0
5	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0
7	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	0
10	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
11	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	0
12	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0
13	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0
14	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	1
15	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
16	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
17	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
18	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
19	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	1
20	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
21	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	0
22	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0
23	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	1
24	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0
25	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
26	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	1
27	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	0
29	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
30	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	0
31	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
32	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0
33	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
34	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0

Table 12 Ahi et al. (2009) [15] 51 parts and 20 machines

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0
3	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
4	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
5	1	0	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0
6	1	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0
7	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
8	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0
10	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0
11	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1
12	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0
15	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0
16	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1
17	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0
18	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0
19	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1
20	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
21	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
22	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0
23	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0
24	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0
25	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0
26	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1
27	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1
28	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
29	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
30	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0
31	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0
32	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1
33	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0
34	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
35	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
36	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0
37	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0
38	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1
39	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	1	0	0
40	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0
41	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1
42	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0
43	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0
44	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
45	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0
46	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0
47	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1
48	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0
49	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
50	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
51	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0

Table 13 Chandrasekharan and Rajagopalan (1989) [6] 40 parts and 24 machines

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
3	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1
4	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
5	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
8	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0
10	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
11	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
12	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
13	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
14	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
15	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
16	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0
17	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0
18	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
19	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
21	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
22	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
23	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
24	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
25	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1
26	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
27	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
28	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
30	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
31	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
32	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1
33	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0
34	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
35	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
36	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
37	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
38	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
39	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

Table 14 Chandrasekharan and Rajagopalan (1989) [6] 40 parts and 24 machines

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
3	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1
4	0	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
5	0	0	0	0	1	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
8	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0
10	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	1	0	0	0
11	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
12	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
14	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
15	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
16	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0
17	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
18	0	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
19	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0
21	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
22	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
23	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
24	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
25	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
26	0	0	0	0	0	1	0	1	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0
27	0	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
28	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
30	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
31	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
32	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	1
33	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0
34	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
35	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
36	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
38	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
39	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0

Table 15 Chandrasekharan and Rajagopalan (1989) [6] 40 parts and 24 machines

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	1	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
3	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1
4	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
5	0	0	0	0	1	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
8	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
10	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	1	0	0	0
11	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
12	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
14	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
15	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
16	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0
17	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
18	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
19	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0
20	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0
21	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
22	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
23	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
24	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
25	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
26	0	0	0	0	0	1	0	1	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0
27	0	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
28	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
30	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
32	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1
33	1	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0
34	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
35	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
36	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
38	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
39	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0

Table 16 Chandrasekharan and Rajagopalan (1989) [6] 40 parts and 24 machines

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	1	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
3	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	1	1
4	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
5	0	0	0	0	1	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
8	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
10	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
12	0	0	1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
13	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0
14	1	1	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0
15	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0
17	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0
18	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
19	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0
20	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0
21	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
22	0	1	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
24	0	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
26	0	0	0	0	0	1	0	1	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	0	0	0	0	0
28	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
30	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
32	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1
33	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
34	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
35	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0	0	0
36	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
38	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

Table 17 Sandbothe (1998) [14] solution

	1	4	3	6	2	5	12	13	11	9	14	8	7	15	17	16	18	10	19	20
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	0	0
5	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	1	0	1	1	1	0	0	0	1	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1

Table 18 Ahi et al. (2009) [15] solution

	8	7	6	5	16	13	14	10	3	18	17	12	1	9	20	15	11	2	4	19
13	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
4	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
17	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
16	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
15	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	1	0
19	0	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0
8	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0	1	1	1	1	1	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
14	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
11	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	1
2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1

Table 19 Mosier and Taube (1985) [11] solution

	14	11	12	6	9	19	20	1	18	15	13	4	8	3	7	10	2	17	5	16
14	1	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0
19	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
12	1	1	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0
8	1	0	0	1	1	0	1	0	1	1	0	1	0	0	0	1	0	0	0	0
4	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
1	0	1	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1
9	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
7	1	0	0	1	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	1
5	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0
10	0	1	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0
11	1	0	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	1	0
3	1	1	0	0	0	0	0	0	1	0	1	1	1	1	0	0	0	0	0	1
20	0	0	0	0	0	1	0	1	0	0	0	1	1	1	0	0	0	0	0	0
6	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
18	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	1
2	0	0	0	0	0	0	1	0	0	1	1	0	1	1	1	1	1	1	1	0
13	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	1	1	1	0	1
17	1	0	0	1	0	0	0	0	0	1	0	1	0	0	1	0	0	0	1	1
15	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1
16	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

Table 20 Boe and Cheng (1991) [2] 35 parts and 20 machines solution

	12	19	15	11	7	8	17	3	16	5	2	13	4	18	14	10	6	9	20	1
21	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
33	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
9	1	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
30	0	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1
28	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1
17	0	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1
20	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	1
31	0	0	0	0	1	0	0	1	0	0	1	0	0	1	1	0	0	0	0	0
29	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0
2	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
12	0	0	0	0	1	0	0	0	1	0	1	1	1	1	1	0	0	0	0	0
24	0	0	0	1	1	0	0	0	0	0	1	1	1	1	1	0	0	0	0	1
13	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
7	0	0	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
18	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0
22	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
16	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
14	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	1	1
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
19	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0
25	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1
23	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	1	1	1
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
32	0	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1
35	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	1

Table 21 Carrie (1973) [5] 35 parts and 20 machines solution

	15	16	19	11	12	1	7	20	17	3	8	6	9	10	14	2	18	4	13	5
11	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	1
22	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
14	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	1
19	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
31	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0
16	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1
34	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1

Table 22 Ahi et al. (2009) [15] 51 parts and 20 machines solution

	14	10	17	4	9	20	15	13	7	2	16	1	12	19	5	6	8	11	3	18
39	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
41	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
48	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
44	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0
47	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	1	0	0	1	0	1	0	0	0	1	0	0	0	0	0
49	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	1
13	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0
26	0	0	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0
28	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	0	0	0	0
25	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	1
21	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0
22	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0
20	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0
11	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0
10	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1
5	0	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0
9	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0
43	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
46	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0
14	0	0	0	0	0	0	0	0	1	0	1	0	0	1	1	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0	0	0
19	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	1	0	0	0	0
12	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	1
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
38	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0
32	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0
45	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
42	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1

Table 23 Chandrasekharan and Rajagopalan (1989) [6] 40 parts and 24 machines solution

	21	13	22	1	18	6	8	12	15	4	16	3	20	5	2	11	19	17	10	9	7	14	23	24
17	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1

Table 24 Chandrasekharan and Rajagopalan (1989) [6] 40 parts and 24 machines solution

	4	24	23	14	7	2	3	20	10	5	11	19	6	12	8	18	16	15	22	1	13	21	17	9
29	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
38	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
39	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
21	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
2	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
3	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
12	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
24	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1	0	1	0	0	0	0	0	0
14	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0
36	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	1	0	1	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
33	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
6	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
40	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

Table 25 Chandrasekharan and Rajagopalan (1989) [6] 40 parts and 24 machines solution

	17	9	10	6	12	18	8	15	4	14	23	16	7	24	22	13	21	2	5	19	11	1	3	20	
40	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	
20	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
31	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
29	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
7	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
6	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	
30	0	0	0	1	1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	
27	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
37	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	
26	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
18	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
19	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1
38	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	
39	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	
21	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	
28	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	
25	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	
32	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	
16	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	1	0	0	
33	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	1	0	0	
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	
14	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
24	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

Table 26 Chandrasekharan and Rajagopalan (1989) [6] 40 parts and 24 machines solution

	9	10	17	16	15	21	1	3	20	14	7	23	24	8	6	18	12	5	11	19	2	22	13	4	
39	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
40	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0
6	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
20	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
31	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	1	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
37	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
15	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
25	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	1	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	0
27	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0
35	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0	0
10	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
36	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0
14	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0
17	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
16	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
11	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
8	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
19	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
21	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1

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