HSE Masters Exam

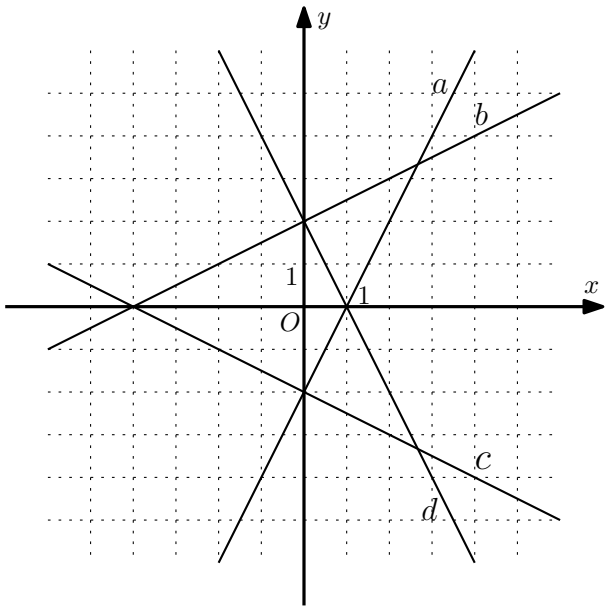
May 28, 2020

1. Convert the binary number 1011_2 to decimal.
(Answer: 11)

2. Find the remainder when $14 \cdot (34 - 8)$ is divided by 9. Write your answer as a nonnegative integer from 0 to 8.
(Answer: 4)

3. There is a new hotel in a city. Each guest brings \$40 income per day, and daily expenses for taxes and rent are \$300. Find the smallest number of guests per day for which the hotel will earn at least \$1000 per day.
(Answer: 33)

4. What is the label (a, b, c or d) of the graph of the linear function $y = \dfrac{x}{2} + 2$?
(Answer: b)



5. Given the parabola $y = x^2 - 4x - 3$,
a) find the y-coordinate of its intersection with the y-axis;
(Answer: -3)
b) find the x-coordinate of its vertex.
(Answer: 2)

6. If $\log_2 \dfrac{x}{\sqrt{x+5}} + \log_2 \sqrt{x+5} = 3$, what is the value of x?
(Answer: 8)

7. Solve the system of linear equations:

$$\begin{cases} 2y - 2x = -3x + 2 \\ x + 3y = 2y + 4 \end{cases}$$

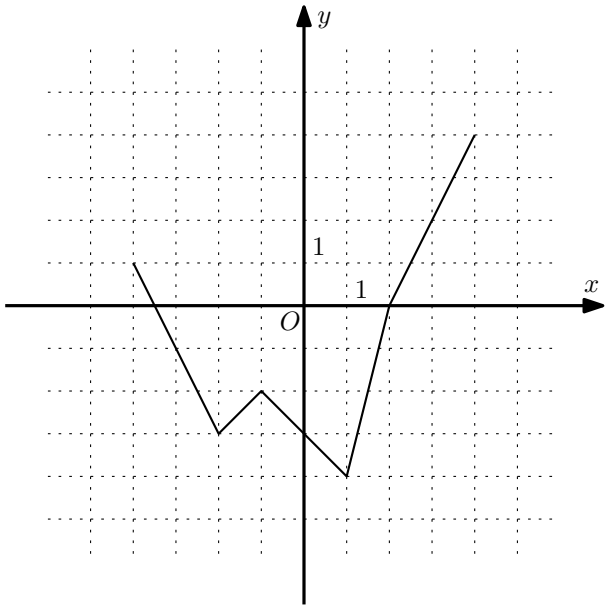
$x =$
$y =$
(Answer: $x = 6$, $y = -2$)

8. Solve the inequality $\dfrac{5}{-x+2} > 2$. Choose the right answer:

a) $(-\frac{1}{2}; 2)$, **b)** $(-2; \frac{1}{2}) \cup (2; +\infty)$, **c)** $(-\infty; -\frac{1}{2}) \cup (2; +\infty)$, **d)** $(-2; \frac{1}{2})$

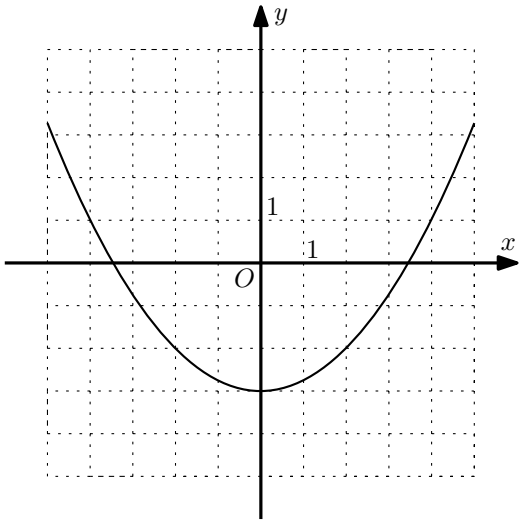
(Answer: a)

9. The complete graph of the function $y = f(x)$ is shown in the xy-plane below.
Find x such that the value $f(x)$ is the smallest possible.
What is the preimage of $y = 2$?



(Answer: 1; 3)

10. The graph of the function $f(x)$ is shown in the xy-plane below. What is the y-coordinate of the point where the graph of the function $g(x) = f(x-2) + 5$ intersects the y-axis?



(Answer: 3)

11. Given $f(x) = 2x^2 - 3x + 1$, find $f'(2)$.
(Answer: 5)

12. Calculate the binary number that equals $110111_2 + 11100_2 - 10100_2$. Write your answer as a binary number.
(Answer: 111111)

13. **X** and **Y** are two stations which are 225 kilometers apart. A train started at a certain time from **X** and travelled towards **Y**. At the same time another train started from **Y** and travelled

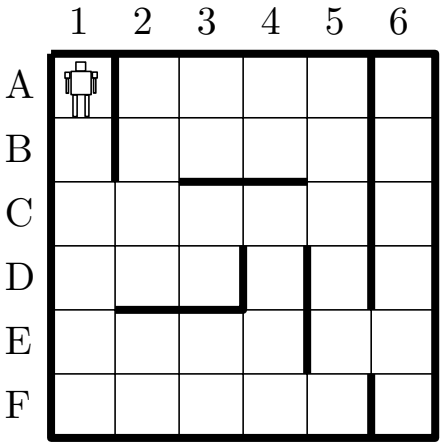
towards **X**. It moved 5 meters per second faster than the first train. The trains met after 75 minutes. Find the speed of the first train in kilometers per hour. Note that 1 kilometer = 1000 meters.

(Answer: 81)

14. Two years ago John made a deposit in a bank that pays 10% of interest once a year. Today he has $605 at the account. How many dollars did John deposit two years ago?

(Answer: 500)

15. A robot is placed in the maze below in A1 position. It is programmed to move in four directions ↑ (up), ↓ (down), → (right) and ← (left). If the robot can't make a move in a programmed direction, then it stays in the same place to make the next move. Where will the robot stop after executing the program ↓←↓→↓→↑→→→←↑↓↓ ? Give your answer as a pair of a letter and a number, e.g. B4.



(Answer: E4)

16. There are two operations:

A: multiply a given number by 3;

B: add 5 to a given number.

How to get 74 from 1 in the minimal number of steps using operations **A** and **B**? Write your answer as a sequence of symbols **A** and **B**. (Example: **ABB** turns 1 to 13).

(Answer: BABAB)

17. Find the minimal non-negative integer s such that at least one of the following conditions holds for all non-negative integers x, y:

$$x \leqslant s; \quad y < s; \quad 5x + 4y \neq 63.$$

(Answer: 7)

18. John goes up the stairs. He can hop either one step or two steps at a time. How many ways are there to get to the 8th step? E.g. there are 2 ways to get to the second step: to make two single steps or to hop to the second at once.

(Answer: 34)

19. For a positive real x, find $x^4 + \frac{1}{x^4}$ whenever $x + \frac{1}{x} = 5$.

(Answer: 527)

20. A polynomial $P(x)$ with integer coefficients satisfies the following:

$$P(5) = 25, \ P(7) = 49, \ P(9) = 81.$$

Find the minimal possible value of $|P(10)|$.

(Answer: 5)