

NP-hardness of problems, connected with an auto-battler video-game genre

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Auto-battler video-game genre

The purpose of auto-battler video-games is to iteratively gather the team of heroes, who have some assigned attributes (e.g. speed of attack, damage, special abilities etc.) During the playthrough, player's team fights the others' players teams in a round-robin tournament.

Defining features

- There is no player involvement in the process of battle itself - his goal is to gather the optimal team
- Heroes has different subjective power levels
- The size of the team is restricted
- Gathering the set of specific heroes can strongly enhance them

Alliance system

Alliances

Each hero in the game is a member of at least one **alliance**. Having a several members of the same alliance in the team may grant specific bonuses to the members of said alliance or the whole team. Since hero can be a part of several alliances, which overlap, creating a complicated structure on the set of the heroes.

Alliances' impact

If a player gathered several heroes from one alliance, he gets an alliance-specific or army-wide buff. There might be several thresholds for a number of heroes needed for various bonuses inside one alliance.

An example

Heroes

We have four heroes:

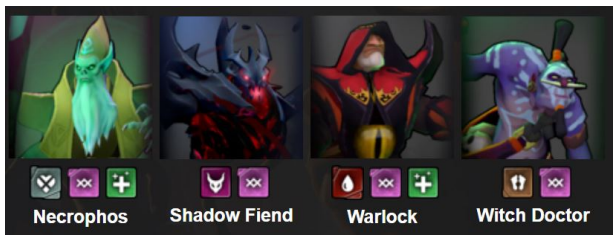
- Witch Doctor: {Troll; Warlock}
- Warlock: {Bloodbound, Warlock, Healer}
- Shadow Fiend: {Demon, Warlock}
- Necrophos: {Heartless, Warlock, Healer}

Alliances

We have the following alliances:

- Warlocks: 2 Warlocks heal an ally each time they use a spell. 4 Warlocks do the same, but with greater effect
- Healers: 2 Healers give a bonus the healing for the whole team
- Demon: Solitary Demon get a boost if effectiveness

An example



Optimal teams (in assumption of equivalence of effectiveness of all heroes and bonuses)

- 1: {Shadow Fiend}
- 2: {Necrophos; Warlock}
- 3: {Necrophos; Warlock; Shadow Fiend}
- 4: {Necrophos; Warlock; Shadow Fiend; Witch Doctor}

An auto-battler problem

Problem

What is the strongest team of given size?

We can assign numerical values to the strength of the heroes and to the bonuses. Therefore, we can formalize the auto-battler problem as an optimization problem.

Assumption

All bonuses are additive with each other.

Solution

An auto-battler problem can be written as Linear Programming problem

Formalization of a trivial case

Trivial no-alliance case

$$\max \sum_{i=1}^n x_i s_i$$

$$\sum_{i=1}^n x_i \leq m$$

$x_i \in \{0, 1\}$ – decision variable, controls if the hero number i is taken

m, s_i – constants

m - maximal size of a team

s_i - individual strength of hero number i

Trivial no-alliance case

It is maximized by picking m heroes with the highest s_i

Formalization of a general case

Objective function:

$$\max \sum_{i=1}^n x_i s_i + \sum_{i=1}^n \sum_{j=1}^t \sum_{k=1}^q e_{ijk} I_{ijk}$$

Input constraint

$$\forall j : \sum_{i=1}^n a_{ij} \leq q$$

Linear inequality constraints

$$\forall i, j, k : \sum_{i'=1}^n a_{i'j} x_{i'} - k \geq M(I_{ijk} - 1)$$

$$\sum_{i=1}^n x_i \leq m$$

$$\forall i, j, k : I_{ijk} \leq x_i$$

Decision variables:

$$I_{ijk}, \in \{0, 1\}, x_i \in \{0, 1\}$$

Constants:

$$n, m, t, q, s_i, e_{ijk}, a_{ij}, M$$

Hardness of the problems

Sirotkin, Ponomarenko, 2020

An auto-battler problem is NP-hard

Sirotkin, Ponomarenko, 2020

Decision version of an auto-battler problem is NP-complete if the alliance size is restricted with some constant q .

A solution of Dota Underlords video-game

Dota Underlords features

- 63 heroes
- Maximum size of a team is 10
- Maximum size of an alliance is 7

Hero			Alliance bonuses	Hero power	Total	
broodmother	insect	warlock	3.0	3	6.0	
disruptor	brawny	warlock	4.0	4	8.0	
dragon knight	dragon	human	knight	5	10.0	
lich	heartless	mage	4.0	5	9.0	
medusa	hunter	scaled	4.0	5	9.0	
necrophos	healer	heartless	warlock	4	8.0	
sand king	insect	savage	4.0	5	9.0	
svet	human	knight	scaled	4.0	4	8.0
troll warlord	troll	warrior	4.5	5	9.5	
witch doctor	troll	warlock	2.2	2	4.2	

Optimal team for Dota Underlords. Table of results

A solution of Dota Underlords video-game

Hero													Alliance bonuses	Hero power	Total			
broodmother	heartless	2	human	2	insect	2	scaled	2	troll	2	+0.3	warlock	2	warlock	4	3.0	3	6.0
		+0.3		+0.3		+0.3		+0.6					+0.6		+0.6			
disraptor	heartless	2	human	2	insect	2	scaled	2	troll	2	+0.4	warlock	2	warlock	4	4.0	4	8.0
		+0.4		+0.4		+0.4		+0.8					+0.8		+0.8			
dragon knight	heartless	2	human	2	insect	2	knight	2	scaled	2		troll	2	+0.5	warlock	2	warlock	4
		+0.5		+0.5		+0.5		+1.0		+1.0				+0.5		+0.5		+0.5
lich	heartless	2	human	2	insect	2	scaled	2	troll	2	+0.5	warlock	2	warlock	4	4.0	5	9.0
		+0.5		+0.5		+0.5		+1.0					+0.5		+0.5			
medusa	heartless	2	human	2	insect	2	scaled	2	troll	2	+0.5	warlock	2	warlock	4	4.0	5	9.0
		+0.5		+0.5		+0.5		+1.0					+0.5		+0.5			
necrophos	heartless	2	human	2	insect	2	scaled	2	troll	2	+0.4	warlock	2	warlock	4	4.0	4	8.0
		+0.4		+0.4		+0.4		+0.8					+0.8		+0.8			
sand king	heartless	2	human	2	insect	2	scaled	2	troll	2	+0.5	warlock	2	warlock	4	4.0	5	9.0
		+0.5		+0.5		+0.5		+1.0					+0.5		+0.5			
sven	heartless	2	human	2	insect	2	knight	2	scaled	2		troll	2	+0.4	warlock	2	warlock	4
		+0.4		+0.4		+0.4		+0.8		+0.8				+0.4		+0.4		+0.4
troll warlord	heartless	2	human	2	insect	2	scaled	2	troll	2	+1.0	warlock	2	warlock	4	4.5	5	9.5
		+0.5		+0.5		+0.5		+1.0					+0.5		+0.5			
witch doctor	heartless	2	human	2	insect	2	scaled	2	troll	2	+0.4	warlock	2	warlock	4	2.2	2	4.2
		+0.2		+0.2		+0.2		+0.4					+0.4		+0.4			

Optimal team for Dota Underlords. Extended table of results

Thank you for your attention!